8.044 Statistical Physics I
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044: Statistical Physics I — Spring Term 2008

Problem Set #1

Please put your name and recitation number clearly on the top page of your problem set.

Problem 1: Probability, Example from playing cards

Consider a standard deck of playing cards. There are 52 cards in total. These cards are divided among 4 suits (clubs, diamonds, hearts, spades). And within each suit, there are 13 types of cards (ace, 2, 3,...,10, jack, queen, king).

Suppose you are dealt a 5-card hand from a full, well-shuffled deck.

a) How many different hands are possible? (Here, the ordering of the cards within the hand is not important.)

b) What is the probability of being dealt a flush (all 5 cards have the same suit)?

c) What is the probability of being dealt one pair (two cards of the same type) and only one pair?

d) What is the probability of *not* getting one pair (and only one pair) until the n^{th} hand? Here, each successive hand is dealt from a full, well-shuffled deck.

Problem 2: A Continuous Random Variable, a Harmonic Oscillator

Take a pencil about 1/3 of the way along its length and insert it between your index and middle fingers, between the first and second knuckles from the end. By moving those fingers up and down in opposition you should be able to set the pencil into rapid oscillation between two extreme angles. Hold your hand at arms length and observe the visual effect. We will examine this effect.

Consider a particle undergoing simple harmonic motion, $x = x_0 \sin(\omega t + \phi)$, where the phase ϕ is completely unknown. The amount of time this particle spends between x and $x + dx$ is inversely proportional to the magnitude of its velocity (its speed) at

x. If one thinks in terms of an ensemble of similarly prepared oscillators, one comes to the conclusion that the probability density for finding an oscillator at x, $p(x)$, is proportional to the time a given oscillator spends near x.

a) Find the speed at x as a function of x, ω, and the fixed maximum displacement x_0.

b) Find $p(x)$. [Hint: Use normalization to find the constant of proportionality.]

c) Sketch $p(x)$. What are the most probable values of x? What is the least probable? What is the mean (no computation!)? Are these results consistent with the visual effect you saw with the oscillating pencil?

Problem 3: A Discrete Random Variable, Quantized Angular Momentum

In a certain quantum mechanical system the x component of the angular momentum, L_x, is quantized and can take on only the three values $-\hbar$, 0, or $\hbar$. For a given state of the system it is known that $< L_x >= \frac{1}{3}\hbar$ and $< L_x^2 >= \frac{2}{3}\hbar^2$. [$\hbar$ is a constant with units of angular momentum. No knowledge of quantum mechanics is necessary to do this problem.]

a) Find the probability density for the x component of the angular momentum, $p(L_x)$. Sketch the result.

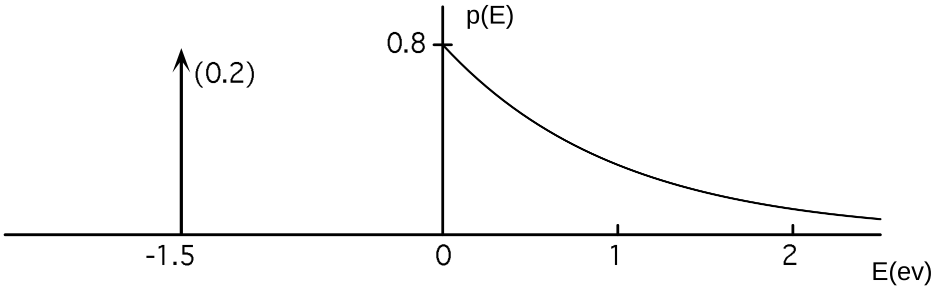
b) Draw a carefully labeled sketch of the cumulative function, $P(L_x)$.

Problem 4: A Mixed Random Variable, Electron Energy

The probability density $p(E)$ for finding an electron with energy E in a certain situation is

$$\begin{aligned} p(E) &= 0.2\,\delta(E+E_0) \qquad && E<0 \\ &= 0.8(\frac{1}{b})e^{-E/b} && E>0 \end{aligned}$$

where $E_0 = 1.5$ eV and $b = 1.0$ eV.



a) What is the probability that E is greater than 1.0 eV?

b) What is the mean energy of the electron?

c) Find and sketch the cumulative function $P(E)$.

Problem 5: Bose-Einstein Statistics

You learned in 8.03 that the electro-magnetic field in a cavity can be decomposed (a 3 dimensional Fourier series) into a countably infinite number of modes, each with its own wavevector $\vec{k}$ and polarization direction $\vec{\epsilon}$. You will learn in quantum mechanics that the energy in each mode is quantized in units of $\hbar\omega$ where $\omega = c|\vec{k}|$. Each unit of energy is called a photon and one says that there are n photons in a given mode. Later in the course we will be able to derive the result that, in thermal equilibrium, the probability that a given mode will have n photons is

$$p(n) = (1-a)a^n \qquad n = 0,\, 1,\, 2,\, \cdots$$

where $a < 1$ is a dimensionless constant which depends on ω and the temperature T. This is called a Bose-Einstein density by physicists; mathematicians, who recognize that it is applicable to other situations as well, refer to it as a geometric density.

a) Find $< n >$. [Hint: Take the derivative of the normalization sum, $\sum a^n$, with respect to a.]

b) Find the variance and express your result in terms of $< n >$. [Hint: Now take the derivative of the sum involved in computing the mean, $\sum n a^n$.] For a given mean, the Bose-Einstein density has a variance which is larger than that of the Poisson by a factor. What is that factor?