AN OUTLINE OF A METHOD FOR PROGRAMME EVALUATION[1]

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Section I. General Discussion of the Method

The purpose of the present note is to outline a method of programme evaluation. Various recent discussions on the so-called "investment criteria" had also the same purpose in view. But the discussion has been essentially single-project oriented, with the implicit assumption that a programme could be regarded as a linear sum of various individual projects. Thus an optimal programme is assumed to be determined, once the priorities of the individual projects are optimally ascertained, each independently of the rest.

This, at its best, however, is an insufficient method which, in the absence of decomposability of the programme, could be seriously misleading, excepting in those cases where projects are few and represent a not too significant addition to the existing capital stock. But it is not an equivalent or a substitute for a programme approach.

The method presented here has the following characteristics:

a) It deals with a whole constellation of inter-related projects, rather than a marginal project. With a marginal project it is admissible to use a partial equilibrium approach, involving the cost-benefit ratio or any such criterion, although it may be social cost and

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[1] I am greatly indebted to Professor P. N. Rosenstein-Rodan for the suggestion of the problems and his stimulating comments. I am also very much grateful to the participants at the M.I.T. India Seminar; in particular, to Professors R. S. Eckaus and R. M. Solow for valuable suggestions. The errors that persist are all mine.

social benefit which are involved rather than private cost and private benefit. But the interesting point to note is that any method to determine "social" as distinguished from private benefit must transcend the possibilities of partial equilibrium approach, thus rendering the usual discussion an inexact one, or simply replacing one set of unknowns by another. An inter-related group of projects necessarily demands a more general approach, which emphasizes inter-sectoral dependence, etc. In certain cases, the use of "shadow prices" to calculate cost-benefit ratios may obviate the necessity for a full-scale programme approach if the shadow prices can be approximated in relatively simple ways. For a further discussion of shadow prices, see the paper entitled "The Use of Shadow Prices in Programme Evaluation," M.I.T. India Project (mimeographed).

b) Secondly, the method is dynamic, inasmuch as the development of the economy over several periods of time is an essential part of it, while most of the programme evaluation techniques yield results for a single period of time.

c) Thirdly, the method uses an explicit characterization of the projects involving the ensemble of technical data, i.e. the gestation lags, the depreciation rates, the intersectoral capital-output ratios, the degrees of intersectoral dependence in current production, etc. This is an extension of the ordinary methods where all the relevant information is generally subsumed under one or two headings, i.e. the capital-income ratio or the capital-labor ratio.

d) The balance of payments problem may be taken account of by introducing a side-condition that the excess of total import requirements over total exports should not surpass a certain preassigned magnitude. If the side-condition is effective, it necessarily implies a non-zero shadow rate of exchange.

While these are the main characteristics of the method, let us state explicitly the possibilities with regard to the choice of the basic criterion. Several alternatives present themselves:

a) If the savings coefficient is already known, our criterion may be stated as one of maximizing the sum of incomes over the specified time horizon. In this case, no separate provision for terminal equipment is needed, because whatever maximizes total income also maximizes total investment, since one bears a well-defined relation to the other; the same holds a fortiori for total consumption over the whole period.

b) If the savings rate is an unknown of the problem, then the criterion may be stated as maximizing the sum of consumption over the whole period, subject to a provision for terminal equipment. In this case, our unknowns are not merely the distribution of total investment, but also the over-all rate of investment in the economy in each time period.

The choice of criterion (a) has the advantage that the planning problem is then decomposed into two consecutive problems: (a) the determination of the over-all savings rate; (b) the determination of the composition of investment. The choice of the savings rate already reflects the decisions regarding the future. It should be understood that the situation (a) holds even though the savings coefficient is not fixed but varies in a predictable manner over time. If it varies with the level of income, then we have a non-linear system which is still a well-determined one. In what follows we shall assume criterion (a) on considerations of simplicity.

The procedure for determining this maximum consists in using an arbitrary parameter that indicates how net total investment is distributed between two sectors, which, for example's sake, we call the "programme sector" and the rest of the economy. This bisector classification is a simplifying device and by no means an essential part of it. As a matter of fact, these two sectors here represent any two sectors that together make up the whole economy. In a more disaggregated approach it will be necessary to have 'n' sectors where $n > 2$. Although the computational difficulties are increased, the method outlined here is equally applicable to the more general situation. In the two sector case, there is only one independent allocation coefficient, 'λ', which indicates how net total investment is to be distributed between two sectors, while in the 'n'-sector case we have (n-1) such as 'λ's'. In the two sector case, the single 'λ' is to be so determined as to satisfy our basic criterion, while in the 'n'-sector case, the criterion requires the determination of an optimal configuration of (n-1) λ's.

The following algebraic model gives an answer to the above problem of maximization on a first level of approximation. This model will be extended in Section III to take into account the following questions:

a) The direct (nonmarket) technological externalities which make output or increment of output in any particular sector dependent not merely on the capital or increment of capital invested in these respective sectors but also on capital invested in other sectors.

b) The changes in the flow coefficients (a_{ij}), which are the Leontief coefficients for cross-deliveries, normally associated with an expanding size of the industry. The simplest way of introducing this factor is to make the input-output relationship "linear" rather than "proportional" as is normally done. Thus, if $X_{ij} = a_{ij} X_j + \bar{K}_{ij}$, where $\bar{K}_{ij}$ is a constant, then $\frac{X_{ij}}{X_j}$ rises with increasing X_j if $K_{ij} < 0$; it falls with increasing X_j if $K_{ij} > 0$. The latter situation corresponds to the phenomenon of increasing returns. Introducing this two-parameter production function renders the Leontief system nonhomogeneous, but it can still be handled in an easy way. For more complicated situations, we may introduce cost functions in each input which are either linear or proportional in facets. If proportionality in facets is assumed as realistic, then there must exist certain nodal points of output at which the coefficients change discontinuously. Thus the variability of coefficients is introduced in a way that does not presuppose abandoning completely the traditional apparatus of input-output analysis.

c) Depreciation rates may also be assumed to be variable over time. Thus we may assume relatively lower rates for the initial years and enlarged ones for later years. Secondly, we need not adhere to the method of straightline depreciation which in a growing economy understates the amount of net investible resources. Thus the usual Domar type of question may be taken care of by changing the depreciation procedure. The more intractable point regarding depreciation that arises in the context of quality change does not appear here because we normally abstract from technical progress in this context.

Section II. An Algebraic Model

(1) $I(t) = S(t) - D(t)$ Where $I(t)$ is investment at time 't'
$S(t)$ is gross savings at time 't'
and $D(t)$ is depreciation of capital stock at 't'.

In those cases where there is a planned balance of payments deficit, that may also be introduced on the right hand side as an additive factor. For simplicity we ignore it for the time being.

We use the following notation:

$V_k(t)$ = Gross output of k^{th} industry at time 't'.

b_k = Output (gross)-capital ratio of the k^{th} industry (direct capital coefficient).

$K_k(t)$ = Capital stock of the k^{th} industry.

$d_k(t)$ = The rate of depreciation of a unit of capital stock in the k^{th} industry.

s = The savings coefficient for the whole economy. We may, if we so prefer, assume this coefficient to be variable from sector to sector. Further, if we are not interested in explicit solutions, we may assume savings coefficients to be variable. This means that the savings ratio diminishes with increase in income. For purposes of numerical extrapolation, this does not raise any additional difficulties.

Equation (1) may then be written as:

$$I(t) = \left[sY(t) - \overline{d_1K_1(t) + d_2K_2(t)}\right]$$

$$= \left[s\Sigma\overline{V(t) - \Sigma\Sigma V_{kk'}(t)} - \overline{d_1K_1(t) + d_2K_2(t)}\right]$$

$$= \left[s\Sigma\overline{b_iK_i(t) - \Sigma\Sigma a_{ij}b_jK_j(t)} - \overline{d_1K_1(t) + d_2K_2(t)}\right]$$

Now $\lambda_1 I(t)$ is the fraction of net investment that goes to Sector I while $\lambda_2 I(t)$ is the fraction that goes to Sector II, with the natural restriction that $\lambda_1 + \lambda_2 = 1$.

Thus $\lambda_1 I(t) = \lambda_1 [s \; \Sigma \overline{b_i K_i(t) - \Sigma\Sigma a_{ij} b_j K_j(t)} - \overline{d_j K_j(t) + d_2 K_2(t)}]$

and $\lambda_2 I(t) = (1-\lambda_1) \; [\ldots \text{same as above} \ldots]$

In the presence of gestation lags, there are several ways of indicating evolution of productive capacity over time. We may take the following two cases:

a) $K_1(t + \ell_1) - K_1(t + \ell_1 - 1) = \lambda_1 I(t)$

$K_2(t + \ell_2) - K_2(t + \ell_2 - 1) = \lambda_2 I(t)$

when ℓ_1 and ℓ_2 are the lags of the two sectors.

b) A more explicit approach to the problem may be to consider the following case which distinguishes between investment in execution and investment that is finished; (which means, the <u>net</u> rate of increase in capital stock, i.e. addition to capital stock—attrition of capital).[1]

$$I(t) = \frac{1}{\ell_1} \int_t^{t+\ell_1} I'_1(t)^{dt}$$

where $I'_1(t)$ is investment that is finished.

$$= \frac{1}{\ell_1} \int_0^{\ell_1} K_1(t)^{dt} = \frac{1}{\ell_1} [K_1(t + \ell_1) - K_1(t)]$$

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[1]This, however, is not a very satisfactory method of dealing with problems of depreciation in the context of gestation lags. But for the purpose of the present paper, the simplicity of this presentation is an advantage which is well worth retaining. The problem of depreciation will be considered separately in another paper.

Now we have the following system of equations:

$$K_1(t_1 + \ell_1) - K_1(t) = \ell_1 \lambda_2 [s\{\sum b_i K_i(t) - \sum\sum a_{ij} b_j K_j(t)\} - d_1 K_1(t) + d_2 K_2(t)]$$

$$K_2(t_2 + \ell_2) - K_2(t) = \ell_2 \lambda_1 [s \sum b_i K_i(t) - \sum\sum a_{ij} b_j K_j(t) - d_1 K_1(t) + d_2 K_2(t)]$$

This is a system of linear difference equations of order 'ℓ' where $\ell = \max (\ell_1, \ell_2)$. The number of initial conditions needed to start the system is at most $(2 \times \ell)$.

In certain singular cases, the system may be "collapsed" to yield a single difference equation in aggregate capital stock, the order remaining the same as in the "noncollapsed" state.

Once the $K_i(t)$'s are known as solutions of the system of difference equations as outlined above, the timepath of 'Y' and hence the integral of 'Y' over the planning horizon is known too. Thus the criterion (a) will then imply 'max y' where $y = \int_0^h Y(t)^{dt}$. Thus the decision variables λ's will have to be chosen in such a way as to reach the above maximum.

The criterion (b) will imply: $\max C = \int_0^h C(t)^{dt}$ subject to $K_{n+1} = K_n + 1$. In this case the decision variables are not merely the λ's but include the savings coefficient as well. This naturally is a more complicated problem. The converse to this problem has been considered by Mr. Little who assumes the following criterion: $\max K_{n+1}$ subject to a $C(t) = \overline{C}(t)$, a prescribed function of time.[2]

Assuming continuous derivatives, etc. the maximum in (a) is attained where:

$$\frac{d\overline{\overline{y}}}{d\lambda_1} = 0, \qquad \frac{d^2\overline{\overline{y}}_2}{d\lambda_1} < 0.$$

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[2] I.M.D. Little, "Reflections on the Planning Experience in India," India Project, M.I.T. (mimeographed).

In practice, the above formalism has hardly much importance, for firstly it is quite unlikely that the functions involved will have the necessary continuity properties and secondly, the explicit solution of the difference equations may be quite a job in itself. Thus the technique of "numerical extrapolation" will have to be employed to trace the development over time. This technique is further considered in an appendix.

The method of numerical extrapolation has the additional advantage that the coefficients need not be assumed to be constant over time. While it is still possible to handle in a somewhat general way a system of linear difference equations with variable coefficients over time, the practical difficulties may be great and the purist may also insist at the same time on convergence proofs, etc. No such problem arises if we adopt what has been called the technique of "numerical extrapolation." Thus the technique suggested above may take into account such delayed effects as are normally associated with investments in social overhead capital, etc.

The demand considerations relating to final consumer goods are not gone into in detail in the model presented above. But they may also be introduced as additional constraints in a multisectoral model. In that case, the set of decision variables will be 'n-r-1' where 'r' is the number of additional equations introduced to take care of certain requirements on the composition of consumption. Thus, let us assume a situation where minimum amounts of consumption of certain commodities have been specified by the planner. Then, a number of decision variables will have to assume a set of values such that technology would enable these required amounts of consumption output to be produced. This limits the range of variability of the set of λ's, but there would be a certain amount of freedom so long as the number of restrictions 'μ' is less than (n-1). Instead of using the elimination procedure, we may use a more symmetric procedure such as the technique of Lagrange multipliers which maximizes a target function involving λ's subject to the various a priori restrictions.[3]

Section III

We now introduce the changes in our model announced towards the end of Section I.

It should be noted that the introduction of technological interactions requires the use of a new matrix of coefficients, which is different from the Leontif matrices so far used. The two Leontief matrices are the matrix of flow coefficients (a_{ij}) and the matrix of investment coefficients (b_{ij}). The Leontief matrix (a_{ij}) is quite explicit in our system of calculations, but the second Leontief matrix is hidden behind the 'b_k's". Of course $1/b_k$'s are nothing but the column sums of Leontief's second matrix. Thus:

$$\frac{1}{b_k} = \sum_{i=1}^{n} C_{ik} \;.$$

where C_{ik}'s are the intersectoral capital-output ratios.

Now let us assume that $V_i(t) \neq f(K_i)$, and instead $V_i(t) = f(K_1, K_2, \ldots, K_n)$. For simplicity we have $V_i(t) = g_{1i}K_1 + g_{2i}K_2 + \ldots + g_{ni}K_n$

$$\text{where } (g_{ij}) = \begin{bmatrix} g_{1i} & \cdots & g_{1n} \\ g_{2i} & \cdots & g_{2n} \\ g_{ni} & \cdots & g_{nn} \end{bmatrix}$$

[3] The discussion in this paper is exclusively devoted to a closed economy. In an open economy, where target setting involves questions of import substitution, more complicated problems may arise. For this, see the author's "The Logic of Investment Planning," Chs. V-VII (North Holland Publishing Company, Amsterdam, 1959).

Now only $g_{ii} = \frac{1}{b_i}$ while the other coefficients g_{ij}, $i \neq j$ are the nonmarket influence exercised by the i^{th} industry over j^{th} industry. These influences must necessarily be nonmarket influences. To the extent they are taken care of by the market mechanism through the prices and quantities of investment goods and intermediate goods output, they have no place here. The reason for that is the use of two other matrices, (a) and (b), which relate to observable market transactions. Leontief's use of constant coefficients for these matrices, however, precludes any emergence of pecuniary external economies, because relative prices remain constant. Thus Leontief can only take account of quantity effects, and not of price effects. Pecuniary external economies are, however, considered in our system because (a) we do not assume the technological coefficients to remain unchanged, they change in facets, and (b) because we have more than one primary factor. It is easy to see that either of these factors is sufficient to introduce pecuniary external economies into the picture. It should, however, be noted that for the system as a whole it is misleading to call such price induced effects "external."[4]

The rows of the 'g' matrix indicate the influence exerted by one sector over all the other sectors, while the columns indicate the influences received by one sector from all the other sectors. In ordinary discussion the matrix 'g' is a diagonal matrix so that all the other elements are necessarily zero. The literature on external economies, however, indicates the importance of

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[4] Although the pecuniary external economies are internal to the system as a whole and merely reflect the laws of general interdependence of the economy, since the private investor is not in a position to estimate these changes accurately, the investment equilibrium of the economy is affected. On this point, see T. Scitovsky, "Two Concepts of External Economies," J.P.E.

assuming that some off-diagonal elements are not necessarily zero. This does not mean that we have any fool-proof method of estimating these coefficients. In the first place it is necessary to consider whether these coefficients are "identifiable," in the sense the term is used in econometric literature. What kind of a priori restrictions on the 'structure' of the system are necessary in order to render them identifiable? This is all the more important if we have technical progress, because, then, the distinction between technological external economies and the over-all effect of technical progress is a somewhat blurred one in practice. But, conceptually the literature on economic development has often maintained, and rightly, that certain industries act more frequently as transmitters of growth via the effect that they have on the productivity of labour, thus providing an instance of technological externality. Although labour is not formally in our equation, its influence is taken account of through the shape of the equations or the values of the coefficients. The off-diagonal elements crucial to the present argument are those referring to the 'g' matrix. The presence or absence of off-diagonal elements in the other matrices (a and b) are indicative of triangularity in the processes of production and capital formation.[5] It is generally held that there are certain sectors of the economy which are important from the point of view of radiating influence on all the other sectors, and they are normally classified as belonging to the "infrastructure."

Having discussed the general nature of this new matrix, we now rework our set of difference equations for this modified case. We assume n = 2 for

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[5]The triangularity in the (a) matrix is significant also from a computational point of view. This is so because only the matrix (I-a) is needed for inversion.

the sake of exposition. Thus the equations are now as follows:

$$K_1(t+\ell_1) - K_1(t) = \ell_1\lambda_1 \Big[s\Big\{ \overline{g_{11}K_1(t) + g_{21}K_2''(t) + g_{12}K_1(t)} + \overline{g_{22}K_2(t)} - (\overline{a_{12}V_2(t) + a_{21}V_1(t)})\Big\} - \overline{d_1K_1(t) + d_2K_2(t)}\Big]$$

$$= \ell_1\lambda_1 \Big[s\Big\{ \overline{(g_{11} + g_{12})\,K_1(t) + (g_{21} + g_{22})\,K_2(t)} - (\overline{a_{12}g_{12}K_1(t) + a_{12}g_{22}K_2(t) + a_{21}g_{11}K_1(t)} + \overline{a_{21}g_{21}K_2(t)}\Big\} - \overline{d_1K_1(t) + d_2K_2(t)}\Big]$$

$$K_1(t+\ell_1) = \lambda_1\ell_1 \Big[\Big\{ s(\overline{g_{11} + g_{12}}) - s(\overline{a_{12}g_{12} + a_{21}g_{11}}) - d_1 \Big\} K_1(t) + \Big\{ s\,(\overline{g_{21} + g_{22}}) - s(\overline{a_{12}g_{22} + a_{21}g_{21}}) - d_2 \Big\} K_2(t) \Big] + K_1(t)$$

$$K_2(t+\ell_2) = \lambda_2\ell_2 \Big[\ldots \text{ same as above } \ldots \Big] + K_2(t).$$

Thus we have a system of two difference equations, the order being $\ell = \max(\ell_1, \ell_2)$, as in the previous case. Once again, we may try to solve the case explicitly or attempt the method of numerical extrapolation as mentioned earlier.

While the method proposed above formally takes into account the technological externalities so far as the evolution of output and productive capacity are concerned, there are very difficult problems of estimation involved: as remarked previously, the 'g' coefficients are not easily determined.[6]

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[6]In this connection, it is interesting to note that the parametrization device generally connected with the dual of a programming becomes very complicated in the presence of externalities. The problem has been discussed against the background of statistical considerations. Nothing has been done in the literature in this dynamic setting. For a discussion of the static question, see F. M. Bator, "The Anatomy of Market Failure," Q.J.E., 1958.

We now turn to the second of the major extensions which were announced on page 5. This relates to the way in which the variability in time of the coefficients of the two Leontief matrices may be incorporated in our model. One way of introducing such variability is to assume these coefficients to be autonomous functions of time. In other words, the sole reason why the coefficients change is technical progress. Thus differences between coefficients at two different points of time are indicative of "structural change," due to innovations, etc. This, however, is a cheap way of generalization unless we can foresee the nature and extent of such technical progress, which is bound to be quite difficult to predict. To the extent technical progress is correctly foreseen, we may incorporate them into our model without difficulty. While technical progress is not easily foreseen, the variability introduced into the picture via the increasing outputs of different industries over time can be more easily projected. These changes reflect the economies of scale which become important when the industry has reached a certain size as well as the Allyn Young type of external economies due to greater size of the market. A cross-section study of the production functions of comparable industries in different countries at different stages of growth may indicate how the relevant coefficients change when the size of output increases. A study of this nature has already been undertaken by Chenery.[7] Such a study is quite indispensable from the operational point of view, if variability of coefficients is to be introduced into planning questions. At this stage, the transition in our analysis should be carefully paced. Thus, our first extension consists only in introducing linearity. At the next stage, we

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[7]Chenery, H.B., Patterns of Industrial Growth, (Paper presented at the Washington Meeting of the Econometric Society, December 1959).

postulate facetwise proportionality or linearity, as the case may be.

We can show how the introduction of linearity already enables some extension of our traditional results. Assume two sectors, manufacturing and social overhead capital. Social overhead capital enjoys increasing returns to scale so that input-output ratio falls as output expands. Then, we have:

$$X_1 = a_{11}X_1 + a_{12}X_2 - \bar{\bar{a}}_{12} + F_1$$

$$X_2 = a_{21}X_1 + a_{22}X_2 - \bar{\bar{a}}_{22} + F_2$$

Then, $\{X\} = [A]\{X\} + \{F - \bar{\bar{a}}\}$ where $\{X\}$ is the columnvector of gross output levels. $[A]$ is a matrix of marginal input-output coefficients. $\{F - \bar{\bar{a}}\}$ stands for adjusted final demand.

Then, $\{X\} = [I-A]^{-1}\{F - \bar{\bar{a}}\}$.

This differs from the traditional estimates of $\{X\}$ for any given amounts of $\{F\}$ by a factor $[I - A]^{-1}\{\bar{\bar{a}}\}$ which may be sizeable depending on $\{\bar{\bar{a}}\}$

We may also introduce some inequalities such as for values $X < \bar{\bar{X}}$, $X_{ij} = a_{ij} X_j$, but for values $> \bar{\bar{X}}$, $X_{ij} = a_{ij}X_j$. This is what we mean by proportionality in facets. Even here, we had best postulate proportionality in facets (stages) rather than continuous variability. This implies that at any point of time there is a proportional relationship between each input and output, although the coefficient of proportionality need not be the same as on any earlier occasion. Such piecewise variation of the coefficients is not quite easy to handle explicitly. Since the prices are changing between the various nodal points, the procedure of numerical extrapolation in this case must distinguish explicitly between <u>value</u> variables and <u>volume</u> variables. This raises the familiar problems of index number construction which under such concepts as real income and investment is somewhat ambiguous.

If, however, we are interested only in numerical extrapolation, not in an explicit solution, all that we need to do is to work on a set of fixed coefficients for one well-defined facet. Beyond that, a different set of coefficients will be needed, and the procedure may be repeated. This sounds slightly artificial because in reality the facets are not that precisely marked, but the advantage in the handling of the problem is very great on this assumption.

Another way of handling this problem of piecewise linearity may be to assume that substitutability operates only on the margin, that is to say, we may assume that the increment of capital stock may be used in various ratios with complimentary factors, while once a choice has been made, we have a certain unique ratio in which the factors must be employed. Thus we have layers of capital stock and corresponding layers of technique and the relative importance of a given type of technique decreases in proportion as the importance of the corresponding layer of capital stock becomes less important. This comes about in two ways:

a) The capital stock of a special type depreciates.

b) It is not replaced by an old type but one appropriate to the changed conditions of the system.

This second approach is very interesting from the theoretical point of view, and it may be shown to be quite consistent with the first point of view, although computational problems suggested by its approach are not quite simple. Two things must be noted about the method of piecewise linearity:

a) At each point of time, we must ascertain whether the conditions relating to the consistency of the various coefficients are satisfied. In case these coefficients turn out to be inconsistent, i.e. the Hawkins-Simons conditions of the system are not satisfied, this is presumably because the system determining the coefficients has more equations than unknowns. This <u>over-determination</u> arises because changes in the coefficient in one industry may well entail certain changes in other sets of coefficients, which may not be immediately apparent. Thus by postulating constancy somewhere we are dealing with a structure implicitly over-determined. The reason why such over-determination will not arise in this approach is that we allow for induced changes in the coefficients in a piecewise manner via the price effects.

b) Secondly, if the coefficients are changing as output increases, relative prices will, of course, be changing. This raises, of course, all the familiar index number difficulties in determining real income over time. Since index number problems are theoretically "insoluble," we may have to ascertain <u>limits</u> within which such discrepancies will lie and then proceed as we would have done otherwise. A practical resolution of this difficulty may be indicated along the lines of successive iteration. This means that we plan for the subperiod for which prices are more or less constant, having sufficient regard for the terminal capital equipment. Then, repeat the procedure for the next subperiod, having regard for the terminal equipment at the end of this period. In this way, we can avoid some of the difficulties in practice. This is, of course, analogous to the procedure on which chain indexes are constructed.

The last point relates to the way depreciation should be calculated. With straightline depreciation, "depreciation" (amortization) exceeds "replacement" in a growing economy, but in the context of numerical extrapolation, there is no reason why we should use straightline depreciation. We may calculate "depreciation" in such a way that the difference between depreciation and replacement does not exist. Where we are concerned more with "real" conditions rather than with financial practices, such a procedure should not evoke much criticism.

Appendix

The Technique of Numerical Extrapolation

The technique of numerical extrapolation may be illustrated in the following way:

a) Specify the initial condition:

For simplicity, we assume both the lags to be the same, i.e. $\ell_1 = \ell_2 = 3$. Then the number of arbitrary initial conditions equals 6. These are $K_1(0)$, $K_1(1)$, $K_1(2)$, and $K_2(0)$, $K_2(1)$, $K_2(2)$.

b) The data of the system are: $[a_{ij}]$, $\{b\}$, $\{d\}$, s $[g]$

c) The unknowns of the problem are: $K_1(3)$, $K_1(4)$, $K_1(5)$, and $K_2(3)$, $K_2(4)$, $K_2(5)$. They may be determined from our set of equations. Thus $K(3)$, $K(4)$, $K(5)$, are determined. In the second round, the data are the unknowns of the first stage, the constants may or may not remain unchanged, and the whole procedure will be repeated. Thus all the successive points in time will be reached, and the time path of all the variables will be ascertained in a stepwise fashion.

In the above example, lags have been assumed to be the same in both the sectors. The more general situation, involving different gestation lags, may also be considered without introducing any difficulties.