24.118 – Paradox and Infinity
Problem Set 10: Gödel's Theorem (Part I)

You will be graded both on the basis of whether your answers are correct and on the basis of whether they are properly justified. There is no word limit.

Problems:

1. In *Excursion II: Gödel's Incompleteness Theorems*, Rucker characterizes a system P and explains what it is to give a formal proof of a sentence of the language of arithmetic in P. Give a formal proof of

$$(\forall x)[t = s \rightarrow (t + x = s + x)]$$

    in P. (10 points)

2. By setting forth a Gödel-style coding system, find a formula $\Phi(x)$ of the language of arithmetic such that $\Phi(n)$ says something true if and only if n is (the Gödel-code of) a sequence of symbols containing '='. (5 points)

3. By setting forth a Gödel-style coding system, find a formula $\Phi(x, y)$ of the language of arithmetic such that $\Phi(n, m)$ says something true if and only if: (a) n is (the Gödel-code of) a sequence of symbols ξ, (b) m is (the Gödel-code of) a sequence of symbols χ, and (c) ξ is longer than χ. (5 points)

4. Give an informal sketch of a computer program that determines whether a sequence of symbols of the language of arithmetic is a term. (Write your sketch in such a way that it can be understood by someone with no knowledge of any particular programming language.) (5 points)

5. Give an informal sketch of a computer program that determines whether a sequence of symbols of the language of arithmetic is a formula. (Write your sketch in such a way that it can be understood by someone with no knowledge of any particular programming language.) (5 points)