14.471 Public Economics I
Fall 2007

Solutions for Problem Set #3 (14.471)

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1 Flow of Funds

See the EXCEL spreadsheet available in the assignments section. The average marginal tax rate is 8.24%.

2 Tax Avoidance

2.1 Part (a)

The individual solves

$$\max_{C_1, C_2} \quad \log(C_1) + \tfrac{1}{1+\delta}\log(C_2)$$
$$s.t. \quad C_1 + \tfrac{C_2}{1+r(1-\tau)} = W_1$$

Her demands for consumption in each period are

$$C_1^* = \frac{1+\delta}{2+\delta} W_1; \quad C_2^* = \frac{1+r(1-\tau)}{2+\delta} W_1$$

Her lifetime indirect utility function is

$$V(W_1, r(1-\tau)) = \log(1+\delta) - \frac{2+\delta}{1+\delta}\log(2+\delta) + \frac{1}{1+\delta}\log(1+r(1-\tau)) + \frac{2+\delta}{1+\delta}\log(W_1)$$

Call W_{new} the amount of period 1 endowment that would make the individual as happy in the presence of the capital tax as she was in the no-tax world. We equate the indirect utilities in each situation:

$$\begin{aligned}
V(W_{new}, r(1-\tau)) &= V(W_1, r) \\
\frac{1}{1+\delta}\log(1+r(1-\tau)) + \frac{2+\delta}{1+\delta}\log(W_{new}) &= \frac{1}{1+\delta}\log(1+r) + \frac{2+\delta}{1+\delta}\log(W_1) \\
\frac{2+\delta}{1+\delta}[\log(W_{new}) - \log(W_1)] &= \frac{1}{1+\delta}[\log(1+r) - \log(1+r(1-\tau))] \\
\log(\frac{W_{new}}{W_1})^{\frac{2+\delta}{1+\delta}} &= \log(\frac{1+r}{1+r(1-\tau)})^{\frac{1}{1+\delta}} \\
(\frac{W_{new}}{W_1})^{\frac{2+\delta}{1+\delta}} &= (\frac{1+r}{1+r(1-\tau)})^{\frac{1}{1+\delta}} \\
\frac{W_{new}}{W_1} &= (\frac{1+r}{1+r(1-\tau)})^{\frac{1}{2+\delta}}
\end{aligned}$$

Note that the change in endowment that would make the individual indifferent is then $W_{new} - W_1$ which is the compensating variation.

2.2 Part (b)

$\frac{dC_1}{d\tau} = 0$, since preferences over consumption in periods 1 and 2 are Cobb-Douglas. We should not conclude, however, that there is no efficiency loss as the result of the introduction of the capital tax. Although the total change in consumption in period 1 is zero, it is the substitution effect that leads to the efficiency costs of taxation.

2.3 Part (c)

(i) The lifetime budget constraint is:

$$C_1 + \frac{C_2}{1+r} = (1-\tau_1)(Y_1 - A) - \beta(A) + \frac{(1-\tau_2)(Y_2 + A)}{1+r}$$

(ii) The first order condition for A is found by differentiating the lifetime budget constraint with respect to A and setting equal to zero (i.e. the individual just maximizes her after-tax wealth):

$$\beta'(A) = \frac{1-\tau_2}{1+r} - (1-\tau_1)$$

That is, the individual chooses A until the marginal cost of tax avoidance equals the marginal benefit of tax avoidance. The optimal level of tax avoidance does not depend on the utility function because the amount of avoidance only affects total wealth, not how much the individual can consume in period 1 relative to period 2. If the individual could not

borrow then A might depend on the utility function, since tax avoidance would affect how much she could consume in period 1.
(iii) We just use the first order condition above to get

$$A = \frac{\tau_1 - \tau_2}{2\gamma}$$

The elasticity of tax avoidance with respect to $1 - \tau_1$ is

$$\frac{\partial A}{\partial (1 - \tau_1)} \cdot \frac{(1 - \tau_1)}{A} = \frac{-(1 - \tau_1)}{\tau_1 - \tau_2}$$

From this we can reach the rather intuitive conclusion that reductions in first period taxes decrease tax avoidance and, by a similar argument, reductions in second period taxes increase it.

3 Budget Balance and Chamley-Judd

3.1 Part (a)

The government budget constraint in period t is

$$g_t = \tau_t w_t l_t + \kappa_t (r_t - \delta) k_t.$$

3.2 Part (b)

The consumers' sequential budget constraint is

$$c_t + k_{t+1} = w_t (1 - \tau_t) l_t + R_t k_t$$

with $R_t \equiv 1 + (1 - \kappa_t)(r_t - \delta)$. The resource constraint in period t is

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1 - \delta) k_t.$$

Let's subtract the consumers' budget constraint from the resource constraint. Then we are left with

$$g_t = F(k_t, l_t) + (1 - \delta) k_t - (1 - \tau_t) w_t l_t - R_t k_t.$$

Assuming CRS technology so that $F(k_t, l_t) = w_t l_t + r_t k_t$, this reduces to

$$g_t = \tau_t w_t l_t + \kappa_t (r_t - \delta) k_t,$$

which is the government budget constraint.

3.3 Part (c)

A competitive equilibrium is a sequence of quantities $\{c_t, l_t, k_t\}$, prices $\{w_t, r_t\}$ and taxes $\{\tau_t, \kappa_t\}$ such that

- in each period t, consumers solve

  $$\max_{c_t, l_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

  subject to

  $$c_t + k_{t+1} = w_t(1 - \tau_t) l_t + R_t k_t$$

  with $R_t \equiv 1 + (1 - \kappa_t)(r_t - \delta)$, taking prices and k_0 as given,

- in each period t, firms solve

  $$\max_{l_t, k_t} F(k_t, l_t) - w_t l_t - r_t k_t$$

  taking prices and k_0 as given, and

- the resource constraint is satisfied in each period t.

By the result from (b), government budget balance is implied.

3.4 Part (d)

From the consumers' problem in (c), we obtain the FOCs

$$(1 - \tau_t) w_t = -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)}$$

and

$$R_t = \frac{u_c(c_{t-1}, l_{t-1})}{\beta u_c(c_t, l_t)}.$$

Substituting this in the consumers' budget constraint, we obtain the sequential implementability constraint

$$u_c(c_t, l_t)c_t + u_l(c_t, l_t)l_t = \frac{1}{\beta}u_c(c_{t-1}, l_{t-1})k_t - u_c(c_t, l_t)k_{t+1}.$$

Note that, in period 0, the implementability constraint is

$$u_c(c_0, l_0)c_0 + u_l(c_0, l_0)l_0 = \frac{1}{\beta}u_c(c_0, l_0)(R_0 k_0 - k_1),$$

where R_0 is not pinned down. Since the consumers' problem is convex, the first order conditions and hence the implementability constraints are necessary and sufficient for consumer optimality.

Clearly, for any given sequence of quantities $\{c_t, l_t, k_t\}$ that satisfy the sequence of implementability and resource constraints, we can construct a competitive equilibrium by choosing prices

$$w_t = F_l(k_t, l_t)$$

and

$$r_t = F_k(k_t, l_t).$$

This ensures that the second condition for a competitive equilibrium in (c) is satisfied. Then we set taxes such that the consumers' first order conditions hold given quantities and prices, so that

$$\kappa_t = 1 - \left(\frac{u_c(c_{t-1}, l_{t-1})}{\beta u_c(c_t, l_t)} - 1\right) \Big/ (r_t - \delta)$$

and

$$\tau_t = 1 + \frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} \Big/ w_t.$$

Government budget balance is implied by the resource constraint being satisfied and the result from (b).

3.5 Part (e)

The planning problem is to maximize consumers' utility subject to the sequence of resource and implementability constraints. The corresponding Lagrangian (ignoring particularities for $t = 0$) is

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \Big[u(c_t, l_t) + \mu_t \left(u_c(c_t, l_t)c_t + u_l(c_t, l_t)l_t - \frac{1}{\beta} u_c(c_{t-1}, l_{t-1})k_t + u_c(c_t, l_t)k_{t+1} \right) \\ &\quad + \lambda_t \left(F(k_t, l_t) + (1-\delta)k_t - c_t - g_t - k_{t+1} \right) \Big]. \qquad (1)\end{aligned}$$

Defining $W(c, l, \mu) \equiv u(c, l) + \mu(u_c(c, l)c + u_l(c, l)l)$, the FOCs w.r.t. c_t and k_{t+1} for $t > 0$ yield

$$W_c(c_t, l_t, \mu_t) + u_{cc}(c_t, l_t)k_{t+1}(\mu_t - \mu_{t+1}) = \lambda_t$$

and

$$u_c(c_t, l_t)(\mu_t - \mu_{t+1}) + \lambda_t - \beta\lambda_{t+1}(F_k(k_{t+1}, l_{t+1}) - (1-\delta)) = 0.$$

At the steady state, quantities and hence multipliers must be constant, so that the conditions reduce to

$$W_c(c, l, \mu) = \lambda$$

and

$$1 + F_k(k, l) - \delta = 1/\beta.$$

The consumers' Euler equation at the steady state implies

$$1 + (1-\kappa)(r - \delta) = 1/\beta.$$

Together with $r = F_k(k, l)$, this establishes $\kappa = 0$.

4 Lucas's Supply Side Calculations

See the MATLAB code available in the assignments section for the numerical computation, results and figures.