14.471 Public Economics I
Fall 2007

14.471: Problem Set #2

Due: Wednesday, October 17 in class

1. This problem asks you to consider the taxation of various labor supply activities. Suppose utility is given by

$$U^j(G^c(c_1, c_2, \ldots, c_N), G^Y(Y_1, Y_2, \ldots, Y_M))$$

where $(c_1, c_2, \ldots, c_N)$ denotes a vector of consumption goods and $(Y_1, Y_2, \ldots, Y_M)$ a vector of labor supply activities. Technology is linear with unit prices (ignoring government consumption)

$$\sum_j \left(\sum_{i=1}^N c_i^j - \sum_{i=1}^M Y_i^j \right) \leq 0$$

where $j = 1, 2, \ldots, J$ is the household index.

(a) Suppose the government can confront agents with any budget set B. Prove that it is most efficient to use

$$\sum_{i=1}^N c_i^j \leq T\left(\sum_{i=1}^M Y_i^j \right)$$

for some tax function $T(\cdot)$ over total income $\sum_{i=1}^M Y_i^j$.

(b) Suppose now that taxation is restricted to imposing linear taxes on c_i and y_i and a lump sum tax (independent of j). Under what conditions (if any) is it efficient to just set a tax on total income (with no tax on consumption goods)?

(c) Interpret your results in terms of a household consisting of a husband and wife supplying labor services Y_h and Y_w, respectively. Can this framework accomodate a high labor supply elasticity for Y_w and low for Y_h? Is "gender-based taxation" efficient?

2. This problem asks you to evaluate the Pareto Efficiency of a tax schedule. Assume the income elasticitity of labor supply is zero. Let ε_w^* denote the compensated elasticity of labor supply with respect to the real wage. Let the distribution of income generated by the current tax system be Pareto $h(Y) = k(Y)^{-k-1}\underline{Y}^k$ for $Y \geq \underline{Y}$ and $k > 0$.

(a) Suppose there is a linear "flat" tax:

$$T(Y) = T + \tau Y.$$

with marginal tax τ and intercept T. Suppose ε_w^* does not vary across individuals (at all

income levels). Note that this would be true if the utility function is $U(c, Y, \theta) \equiv c - \theta Y^{\alpha}$. Starting from the general test for Pareto efficiency derive an inequality for τ, ε_w^* and k. Consider some empirically plausible values.

(b) How would an elasticity ε_w^* that varies across individuals, and is higher for individuals with higher income affect your analysis? (Gruber-Saez paper may be useful to think of plausible numbers). How would progressivity of the tax schedule (convexity of $T(Y)$) affect the analysis?

3. This question asks you to solve the Mirrlees (1971) model numerically. In particular, following Saez (Section 5, 2001, RevStud) adopt the utility specification

$$\log\left(c - \frac{l^2}{2}\right)$$

where $l = Y/\theta$ and θ is the skill level.

(a) Find the skill distribution so that the resulting distribution of income, when individuals face a flat tax $T(Y) = .3Y$ is Pareto with $h(Y) = k\left(Y\right)^{-k-1}\underline{Y}^{k}$ for $Y \geq \underline{Y}$ and $k = 4$. Set the lower bound $\underline{Y}$ so that scale of the distribution of income is relatively close to the empirical one in the U.S.

(b) Solve the optimum numerically ignoring the monotonicity condition. To do so, truncate your distribution at the top x percentile, for some small x. Compare your results to Saez's.

4. You are invited to a small tropical island nation to become a tax policy consultant, with particular responsibility for analyzing tax-induced distortions in labor supply. Sensing the opportunity for an interesting vacation, you accept. The republic's finance minister faxes you a copy of the current tax schedule, defined over total daily (labor plus non-labor) income, which is:

$$T(y) = 0 \quad \text{if} \quad y < 60$$

$$T(y) = .5 * (y - 60) \quad \text{if} \quad y \geq 60.$$

After arriving in the republic, you discover it is smaller than you thought. The Complete Population Survey contains only four observations. Undaunted, you proceed with your proposal to estimate a linear hours-of-work model for the republic's population:

$$h = \alpha + \beta y_v + \gamma w,$$

where h denotes daily hours, y_v is virtual income, and w is the household's after-tax wage

rate. The data you receive are shown below:

Household	Non-labor income	Hours worked	Total Pretax Income
1	0	15.15	45.45
2	0	10.00	60.00
3	10	8.25	92.50
4	20	7.25	92.50

(a) Use these data to estimate (α, β, γ). You should be able to solve for the parameters exactly (i.e., with no error terms in the hours equation) using three data points. (Hint: Remember to check whether each household is on a linear segment of the budget set, or at a 'kink point.')

(b) Compute the total amount of revenue currently collected by the tax system, and find the lump sum tax (equal across all households) that would be needed to raise the same amount of revenue. For household 1, find the equivalent variation of shifting to this tax.

5. You are invited to visit a small island nation and to help in analyzing a proposed tax reform. The economy consists of two types of individuals, with each type accounting for half of the population. Jello is the numeraire consumption good in the economy. The first type of individual has a wage of 1 jello packet per unit of labor supplied, and the second has a wage of 2 jello packets per unit of labor. Individuals of each type have an endowment of one unit of labor. Each individual also has a lump sum income of one packet of jello.

Individuals of both types have preferences over three goods: leisure (1-L), jello (J), and coconuts (C). These preferences are described by a utility function:

$$U = \log J + \log C + \log(1 - L).$$

There is a domestic production technology that turns one package of jello into one coconut. The process can be reversed.

The current tax system is as follows. Taxable income, which is measured in units of jello, is defined as labor earnings plus lump sum income. The marginal tax rate on the first 1.5 jellos of income is 25 percent, while the marginal tax rate is 50 percent on all income above this level.

(a) You begin your analysis of the tax system by trying to determine the optimal labor supply behavior of the two types of individuals in this economy. Find the labor supply, consumption of jello, and consumption of coconuts for individuals with a wage of 1, and for those with a wage of 2.

(b) What is per capita tax revenue from the current tax code?

(c) The domestic coconut lobby has proposed a tax deduction for coconut purchases. If all expenditures on coconuts were excluded from taxable income, describe the new tax schedule. How would per capita revenue change if this policy were adopted? Explain the factors that contribute to any revenue change that you identify.

(d) The coconut lobby has evaluated the potential revenue cost of the proposal in (c) by multiplying outlays on coconuts under the current (no coconut deduction) tax system by estimates of marginal tax rates for consumers of types 1 and 2. (This is known as 'static revenue estimation.') Do your calculations offer any insights on the validity or limitations of this approach?