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Summary: An Earth-attached and thus rotating reference frame is almost always used for the analysis of geophysical flows. The equation of motion transformed into a steadily rotating reference frame includes two terms that involve the rotation vector; a centrifugal term and a Coriolis term. In the special case of an Earth-attached reference frame, the centrifugal term is exactly canceled by gravitational mass attraction and drops out of the equation of motion. When we solve for the acceleration seen from an Earth-attached frame, the Coriolis term is interpreted as a force. The rotating frame perspective gives up the properties of global momentum conservation and invariance to Galilean transformation. Nevertheless, it leads to a greatly simplified analysis of geophysical flows since only the comparatively small relative velocity, i.e., winds and currents, need be considered.

The Coriolis force has a simple mathematical form, $-2\boldsymbol{\Omega} \times \mathbf{V}'M$, where $\boldsymbol{\Omega}$ is Earth's rotation vector, \mathbf{V}' is the velocity observed from the rotating frame and M is the particle mass. The Coriolis force is perpendicular to the velocity and can do no work. It tends to cause a deflection of velocity, and gives rise to two important modes of motion: (1) If the Coriolis force is the only force acting on a moving particle, then the velocity vector of the particle will be continually deflected and rotate clockwise in the northern hemisphere and anticlockwise in the southern hemisphere. These so-called inertial oscillations are a first approximation of the upper ocean currents that are generated by a transient wind event. (2) If the Coriolis force is balanced by a steady force, say a pressure gradient, then the resulting wind or current is also steady and is perpendicular to the force. An approximate geostrophic momentum balance of this kind is the defining characteristic of the large scale, extra-tropical circulation of the atmosphere and oceans.

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