

18.100C. STUDY GUIDE 3

The final exam will be Wednesday, May 24, at 1:30-4:30pm. You will have approximately 170 minutes for the exam. You should show all work, unless instructed otherwise; partial credit will be given only for work shown.

This guide contains a checklist of important skills, definitions and theorems to learn before this test. You do not need to memorize the proofs of the theorems, but you should learn the statements, understand the main ideas of the proofs, and be able to use the theorems. Also, it is very important to know and understand examples which illustrate the concepts and the theorems.

The material for this test is from Chapters 1-7 and part of 8 in Rudin, as covered in the lectures and in the homework sets. Please consult the first two study guides for Chapters 1-6. Some important topics in Chapters 7 and 8:

1. *Convergence of sequences and series of functions*: pointwise convergence: definition and examples (pages 144-146); uniform convergence: definition (7.7), Cauchy criterion (Theorem 7.8), Weierstrass' M-test (Theorem 7.10), uniform convergence and continuity (Theorems 7.11, 7.12), integration (Theorem 7.16 and Corollary), differentiation (Theorem 7.17).
2. *Equicontinuity*: definition and examples (7.19-7.22), convergence on countable sets (Theorem 7.23), equicontinuity and uniform convergence (theorems 7.24, 7.25).
3. *Stone-Weierstrass Theorem*: approximation by polynomials (Theorem 7.26), the completeness of $C(X)$ (7.14, 7.15), Stone's generalization (Theorems 7.32, 7.33, exercise 21/169).
4. *Analytic functions*: power series, radius of convergence, examples; power series and uniform convergence, continuity, differentiability (Theorem 8.1 and Corollary), convergence at the endpoints of the interval of convergence (Theorem 8.2), inversion in the order of summation (Theorem 8.3), Taylor series expansions (Theorem 8.4); exponential, logarithms, trigonometric functions (definition, properties, power series representations); the algebraic completeness of \mathbb{C} (Theorem 8.8).