18.100C. STUDY GUIDE FOR MIDTERM 2

Exam 2 will be in lecture on Tuesday, April 25. You will have approximately 80 minutes for the exam. You should show all work, unless instructed otherwise; partial credit will be given only for work shown.

This guide contains a checklist of important skills, definitions and theorems to learn before this test. You do not need to memorize the proofs of the theorems, but you should learn the statements, understand the main ideas of the proofs, and be able to use the theorems. Also, it is very important to know and understand examples which illustrate the concepts and the theorems.

The material for this test is from Chapters 4,5,6 in Rudin, as covered in the lectures and in the homework sets. Some important topics:

1. *Limits of functions:* definition and characterization (4.1, 4.2), uniqueness of limit, limits and operations with functions (4.4), infinite limits and limits at infinity (page 98).
2. *Continuous functions:* definition and characterization (4.5, 4.6), continuity and operations of functions (4.7, 4.9-4.11, 4.17), the characterization of continuity with open/closed sets (4.8), continuity and compactness (4.14, 4.15), extrema of continuous functions on compact sets (4.16), uniform continuity (definition 4.18, theorem 4.19), counterexamples (4.20, 4.21), continuity and connectedness (theorem 4.22), the intermediate value property (4.23, 4.27 (d)), types of discontinuity (4.25, 4.26 and examples).
3. *Monotonic functions:* definition (4.28), one-sided limits (4.29), monotonic functions can have only simple discontinuities, and at most countable many (corollary 4.28, theorem 4.30, example 4.31).
4. *Derivatives of real functions:* definition (5.1), differentiable implies continuous (theorem 5.2), differentiation and operations with functions (5.3, ex 2/114), the chain rule (5.5).
5. *The mean value theorem:* local extrema (definition 5.7, theorem 5.8), the generalized mean value theorem (5.9) and particular case (5.10), the first derivative and monotonicity (5.11), the second derivative and convexity (ex 14/115), derivatives have the intermediate value property (5.12).
6. *Applications of the derivative:* l'Hôpital rule (5.13), Taylor's theorem (5.15).
7. *The Riemann-Stieltjes integral:* definition (6.1,6.2), upper/lower sums and refinements (6.4,6.5), criterion for integrability (6.6), classes of functions which are integrable (continuous 6.8, monotonic 6.9, set of discontinuity of measure zero 6.10 and class notes), integrability and composition of functions (theorem 6.11), properties of the integral (6.12, 6.13), extreme cases for the Riemann-Stieltjes integral: series (6.15, 6.16), Riemann integral (6.17), change of variable (6.19).