18.100C. STUDY GUIDE FOR MIDTERM 1

Exam 1 will be in lecture on Tuesday, March 14. You will have approximately 80 minutes for the exam. You should show all work, unless instructed otherwise; partial credit will be given only for work shown.

This guide contains a checklist of important skills, definitions and theorems to learn before this test. You do not need to memorize the proofs of the theorems, but you should learn the statements, understand the main ideas of the proofs, and be able to use the theorems. Also, it is very important to know and understand examples which illustrate the concepts and the theorems.

The material for this test is from Chapters 1,2,3 in Rudin, as covered in the lectures and in the homework sets. Some important topics:

1. *Ordered sets, fields*; supremum and infinimum, the least upper bound property.
2. *Countable, and uncontable sets*: definitions; infinite subsets of countable sets are countable (Thm 2.8); a countable union of countable sets is countable (Thm 2.12 and Corollary); cartesian products of countable sets are countable (Thm 2.13); Cantor's diagonal process; examples of countable or uncountable sets; the power set (Pset 2a).
3. *Metric spaces*: definitions (2.15), examples, the Euclidean metric spaces, ℓ^2, ℓ^∞ (Pset 2b).
4. *Topology (in metric spaces)*: definitions (open, closed, bounded, dense, perfect, convex, connected, separated sets), examples; basic properties of open/closed sets (intersections and unions, Thm 2.24); limit points.
5. *Compact sets*: definitions (open covers, compact), examples; Thm 2.33 (the notion of compact is intrinsic); compact subsets of metric spaces are closed (Thm 2.34); closed subsets of compact sets are compact (Thm 2.35 and Corollary); the intersection of a decreasing chain of compact sets is nonempty (Thm 2.36 and Corollary, also Pset 2b); the Heine-Borel theorem in $\mathbb{R}^k$ (Thm 2.41, Psets 2b and 3); the Bolzano-Weierstrass theorem (Thm. 2.42).
6. *Perfect sets*: the nonempty perfect sets are uncountable (Thm 2.43 and Corollary); the Cantor set: definition, properties; other examples (Pset 2b).
7. *Sequences and convergence*: definition (3.1), examples (3.1 and 3.20); uniqueness of the limit, boundedness (Thm 3.2), operations (Thm 3.3); subsequences, subsequential limits (3.5, 3.6, 3.7., 3.16, 3.17 for limsup and liminf), examples; convergence of bounded monotonic sequences (3.13, 3.14).
8. *Cauchy sequences*: definition (3.8, 3.9); every Cauchy sequence is convergent (Thm 3.11); complete metric spaces (3.12); all compact metric spaces, and the Euclidean spaces are complete (Thm 3.11).
9. *Series and convergence*: Cauchy criterion (Thm 3.22); Thm 3.23; the Comparison (3.25), Root (3.33), and Ratio (3.34) tests; examples (p-series, geometric, Cauchy's test "with 2^k"); absolute and conditional convergence.