2.57 Nano-to-Macro Transport Processes
Fall 2004
Lecture 1

1. Overview for nano sciences
1.1 Length scale
1.2 Examples in microtechnology
1.3 Examples in nanotechnology
1.4 Nano for energy (phonon, phonon, electron; wavelength, mean free path)
1.5 Nanoscale heat transfer in devices (e.g., CMOS)
1.6 Nano and microfabrication
1.7 Transport regimes
1.8 Overview of the book chapters and chapters to be covered

2. Classical Laws related to transport
2.1 Heat transfer
2.1.1 Conduction

Fourier's law:

$q = -k\nabla T$ or $q = -k\dfrac{dT}{dx}$ in one dimension

where:

q [W/m^2] is heat flux,
k [W/m-K] is thermal conductivity.

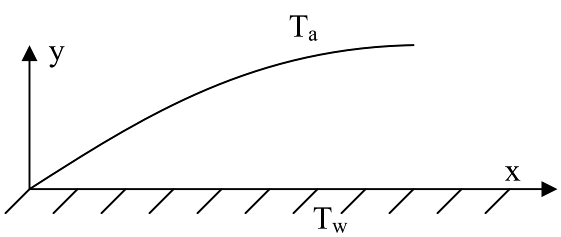
2.1.2 Convection

Newton's law of cooling:

$q = h(T_w - T_a)$

where:

h [W/m^{2}K] is heat transfer coefficient.



Nonslip boundary condition is assumed at the wall, i.e.,

$u_x(y=0) = u_y(y=0) = 0$, $T(y=0) = T_w$.

Note: this assumption is NOT accurate for small scales.

2.1.3 Radiation

Planck's law:

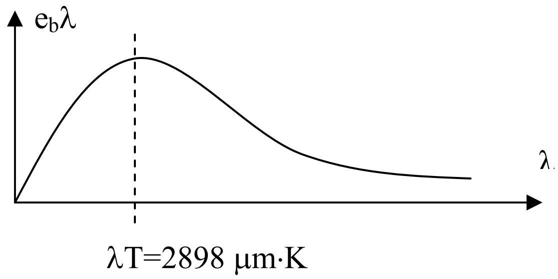
$$e_{b,\lambda} = \frac{c_1}{\lambda^5 (e^{c_2/\lambda T} - 1)}$$

where:

c_1 and c_2 are constants,

λ is wavelength, and the subscript b denotes black body.

The curve for block-body radiation is drawn as following:



Integrating the Planck's law leads to the Stefan-Boltzmann law:

$$e_b = \sigma T^4$$

where:

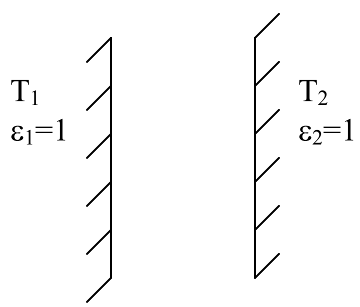
σ=5.67×10^{-8}W/m$^2K^4$.

For real surface, we define "emissivity" as

$$\varepsilon = \frac{e}{e_b}.$$

For the two planar walls shown below, the heat flux of radiation is evaluated as

$$q = \sigma(T_1^4 - T_2^4).$$



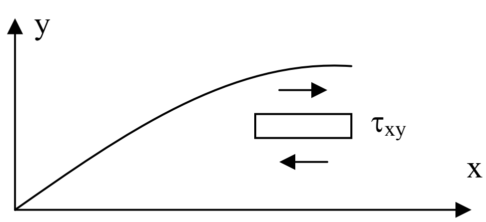
2.2 Newton shear stress law

The shear stress for the sketched one-dimensional flow is:

$$\tau_{xy} = \mu \frac{\partial u}{\partial y}$$

where:

μ [N-s/m^2] is dynamic viscosity.



2.3 Fick's diffusion law

$$j_i = -\rho D \frac{dm_i}{dx}$$

where:

D [m^2/s] is mass diffusivity,

m_i is the mass fraction for the *ith* species.

2.4 Ohm's law

$$R = \frac{V}{I}$$

or

$$J = \sigma\varepsilon = \sigma(-\frac{1}{e}\frac{d\Phi}{dx}) = -\sigma\frac{d\varphi_e}{dx}$$

where:

J [A/m^2] is electric current density,

σ [Ω^{-1}m^{-1}] is electrical conductivity,

ε [V/m] is electric field,

Φ is potential energy,

φ is electrostatic potential.

2.5 Questions

a) What are the similarities among above equations?
b) Are these laws still valid at nanoscale?

2.6 Note:

All above are constitutive equations with two unknown variables.
Another equation (e.g. mass, momentum conservation) is needed to solve problems.

3. Scaling trend

For a sphere, the volume-to-surface ratio is

$$\frac{V}{S} = \frac{4\pi r^3/3}{4\pi r^2} = \frac{r}{3}.$$

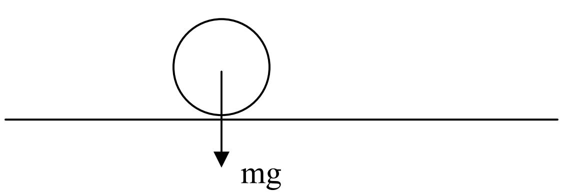
The volumetric effect decreases with the reducing length scale. Surface effect becomes dominant at smaller scales.

For a spherical fluid drop, we have

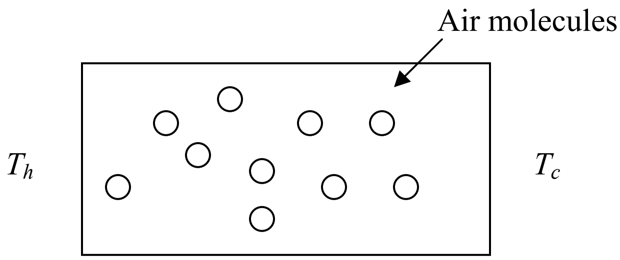
$$\gamma = \frac{\text{Gravitational force}}{\text{Surface force}} = \frac{\rho g(4\pi r^3/3)}{\sigma(2\pi r)} = \frac{2\rho g r^2}{3\sigma}.$$

Substituting $\rho = 10^3 kg/m^2$, $\sigma = 78 mN/m$ into this equation, we get

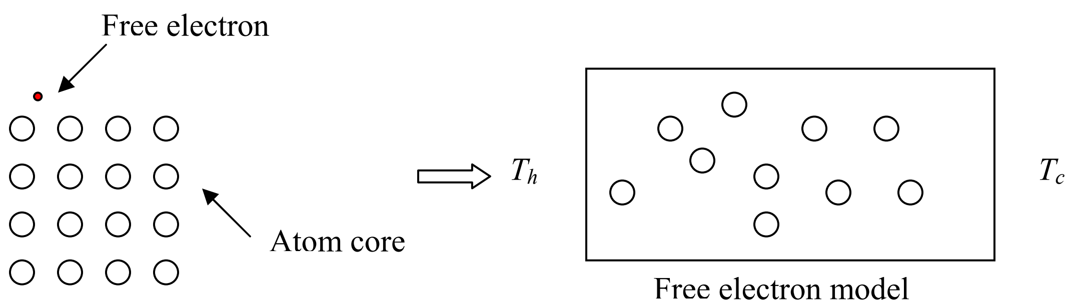
r =1 m, γ = 8.4e4; r =1 mm, γ = 8.4e-2.



4. Microscopic pictures of energy carriers
4.1 Heat
4.1.1 Heat conduction
Gases: hotter air molecules (with larger kinetic energy) randomly pass their excess energy to cooler molecules. Heat is transported to the cold side by such a process. Note the average velocity of a molecule can be as large as 500 m/s.



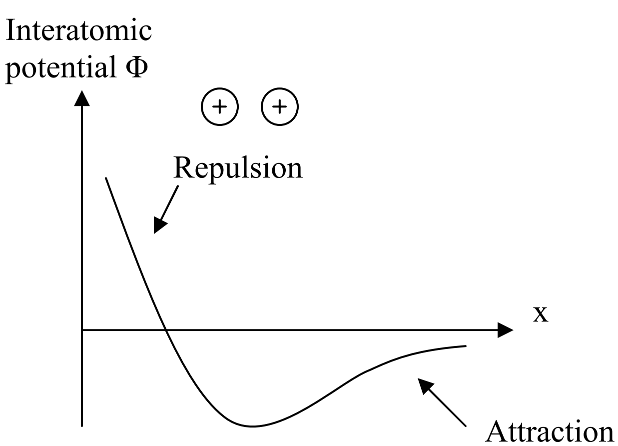
Dielectric solids: heat is conducted through the vibration of atoms. The atom cores are spaced by 2-5 Å in the lattice. Under the free electron approximation, the electrons are viewed as free electron gas.



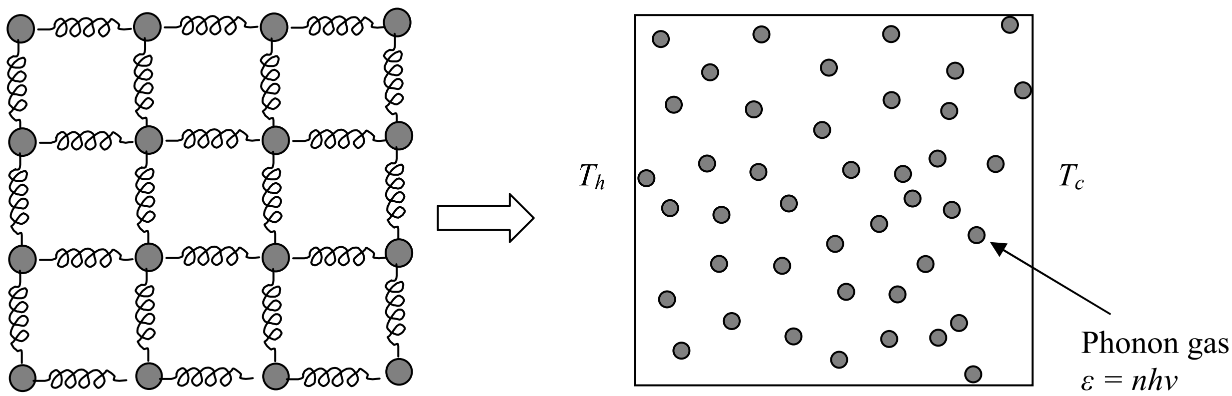
Consider two atoms with a parabolic interatomic potential. The interatomic force is

$$F = -\nabla\Phi \approx K\Delta x$$

where Δx is the displacement from the minimum potential position (equilibrium position), K is constant.



A simplified picture of the interatomic interactions in crystals can be represented by the mass-spring system. The propagation of sound in a solid is due to long wavelength lattice waves. Quantum mechanics states that the energy of each lattice wave is discrete and must be multiples of $h\nu$. Based on argument we will discuss in chapter 5, the spring system can be further simplified as a box of phonon particles.



Now, molecules, electrons, and phonons are all gases in a box. You can see similarities and why I said we can describe them in parallel.