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2.57 Midterm Exam No. 1

November 2, 2004

* Mass of one proton : 1.67×10^{-27} kg. You should remember other constants needed.

1. Answer the following ten short questions. Briefly explain your answer (**60 Points**).

(1) Estimate the average velocity of helium atoms in helium gas maintained at 300 K.

(2) Air molecules at 1 atm have a mean free path approximately 70 nm. Estimate the mean free path at 1 torr, i.e., 1/760 of 1 atm.

(3) Light is incident from a medium of refractive index 2 towards an interface with vacuum. What is the reflectivity if the angle of incidence is 65°.

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(4) Silicon crystals are fcc structures with two atoms as the basis. The lattice constant of the conventional cubic unit cell is 5.4 Å. What is the phonon specific heat of silicon per unit volume at high temperatures? What is the specific heat per unit mass at the same temperatures?

(5) A quantum dot has a cubic shape of 20x20x20 $Å^3$. The potential barriers surrounding the quantum dot is infinitely high. The electron effective mass equals that of the free electron. At what wavelength of light does the quantum absorb due to transition from the lowest energy level to the next lowest energy level.

(6) A electron with an energy of 0.5 eV encounters a potential barrier of height 1 eV. The electron effective mass equals that of free electrons. Estimate the ratio of the electron transmissivity values for two different barrier widths: 1 Å and 10 Å.

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2 (**20 Points**). The electron number density per unit volume in the conduction band of a crystal is n. The electrons in the band obey the following dispersion relation:

$$E = E_c + \alpha\hbar k + \frac{(\hbar k)^2}{2m^*}$$

where E_c is the edge of the conduction band, and k is the magnitude of the wavevector:

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

and

$$k_x = \pm\frac{2\pi i}{L},\ k_y = \pm\frac{2\pi j}{L},\ k_z = \pm\frac{2\pi n}{L} \qquad (i, j, n = \pm 1, \pm 2, \ldots.)$$

Answer the following questions:

(a) Derive an analytical expression for the density of states per energy interval, D(E).
(b) Derive an integral equation that determines the electron chemical potential, μ.
(c) Derive an integral expression that determines the electron total energy per unit volume.

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3 (**20 Points + 10 bonus points (d)**) A diatomic molecule is made of two different atoms with mass m_1 and m_2, as shown in the figure below. The spring constant between the two atoms is K. Assume that the motion of the atoms is restricted to the line joining the two atoms. Answer the following questions:

(a) Based on the Newtonian mechanics, show that the fundamental frequency of the relative vibration of the two atoms is

$$\nu = \frac{1}{2\pi}\sqrt{K\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$$

(b) If quantum effect is considered, what are the allowed energy levels of the diatomic molecule due to the relative vibration.

(c) Based on classical mechanics, derive an express for the potential energy stored by the molecule.

(d) (bonus) The kinetic energy of the molecule is given by the expression $\frac{1}{2}m_1\left(\frac{dx_1}{dt}\right)^2+\frac{1}{2}m_2\left(\frac{dx_2}{dt}\right)^2$. This includes the kinetic energy due to the translational motion of the center of mass of the molecule as well as the kinetic energy due to the relative vibrational motion of the molecule. Derive expressions for the kinetic energy due to (a) translation motion of the center of mass, and (b) relative vibrational motion between the atoms.