Name: ____________________

2.57 Midterm Exam No. 2

Fall, 2004

Take Home Exam

Distributed: Friday, November 19, 7:00 pm
Due: November 24, 2:30 pm

Four questions, each counts for 25 points

Rules: (1) You are required to finish these problems independently, without consultation to other fellow students in the class or anyone else. Replies to specific questions will be shared with other students by email. You can check whatever references you can find. You are on your own honor. (2) State all the assumptions you made in solving each problem.

1. An fcc crystal has a lattice constant b and one atom at each lattice point. The phonon dispersions for transverse and longitudinal phonons are degenerate and are given by the following relation,

$$\omega = 2\sqrt{\frac{K}{m}}\left|\sin\frac{ka}{2}\right|$$

where a is the equivalent lattice constant of an isotropic crystal that has the same number of phonon modes as the fcc crystal, and

$$k^2 = k_x^2 + k_y^2 + k_z^2 \quad (k_x, k_y, k_z = \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, ...)$$

The phonon relaxation time τ in the crystal is a constant. Answer the following questions:

(a) Relate the equivalent lattice constant "a" to the lattice constant of the fcc crystal "b".

(b) Derive an integral expression for the phonon specific heat (per unit volume) of the crystal as a function of temperature, using phonon frequency as the integration variable.

(c) Derive an integral expression for the thermal conductivity of the crystal as a function of temperature, using phonon frequency as the integration variable.

Name: ____________________

2. Consider the reflection of an electron wave at an interface of two media at an oblique incidence angle as shown in the following figures. The energy barrier height is U_o. The electron energy dispersion in the two media form the interfaces can be approximated by

$$E_1 = \frac{1}{2m_1}\left(k_{1x}^2 + k_{1y}^2 + k_{1z}^2\right)$$

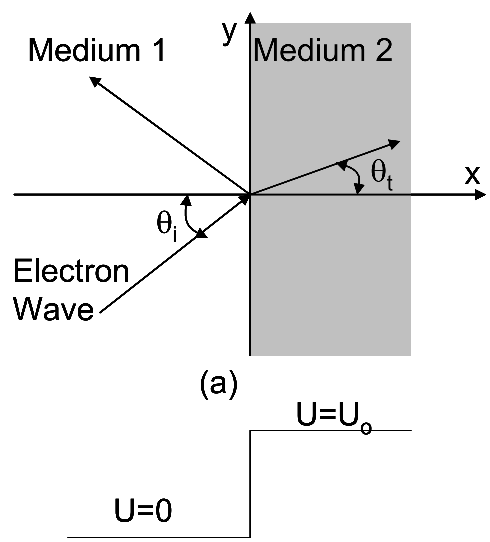
$$E_2 = U_o + \frac{1}{2m_2}\left(k_{2x}^2 + k_{2y}^2 + k_{2z}^2\right)$$

where m_1 and m_2 are the effective masses of the two media. Assuming that the electron energy E is larger than U_o, answer the following questions:

(a) Derive a relation between the angle of incidence θ_i and the angle of refraction θ_t.

(b) Derive an expression for the electron transmissivity, reflectivity as a function of the angle of incidence and electron energy.

(c) Does a critical angle exist above which all incident electrons are reflected? And if yes, under what condition that this critical angle exists and what is this critical angle.



(b)

Name: ____________________

3. Consider one-dimensional heat conduction in a gas along the x-direction. There exists no convection in the gas. Both the density (n) and temperature (T) vary along the x-direction. The gas molecular relaxation time τ is a constant.

(a) Show that the mass flux (J_m) and the heat flux (J_q) along the x-direction can be expressed as

$$J_m = L_{11}\frac{dn}{dx} + L_{12}\frac{dT}{dx}$$

$$J_q = L_{21}\frac{dn}{dx} + L_{22}\frac{dT}{dx}$$

(b) Derive analytical expressions for these coefficients in terms of the local temperature, density, molecular weight, and relaxation time.

(c) Derive a relationship between L_{12} and L_{21}.

(d) For J_m=0, derive an expression for the thermal conductivity of the gas in terms of L_{11}, L_{12}, L_{21}, and L_{22}.

Name: ____________________

4. Consider thermal radiation heat transfer through a square rod as shown in the following figure. The two sides of the rod are maintained at temperature T_1 and T_2. The photon dispersion relationship inside the rod are given by

$$\omega^2 = \frac{c_o^2}{n^2}\left[\left(\frac{j\pi}{a}\right)^2 + \left(\frac{\ell\pi}{a}\right)^2 + k_z^2\right] \qquad (j, \ell = 1, 2, \ldots; \text{ and } k_z = \pm\frac{2\pi}{D}, \pm\frac{4\pi}{D}, \pm\frac{6\pi}{D})$$

where n is the refractive index and c_o is the speed of light in vacuum. Assume that radiation heat transfer between the two reservoirs occurs only through the rod. The two reservoirs have the same refractive index values as the rod, and the photon transmissivity through the rod equals one.

(a) Show that when the temperature difference between T_1 and T_2 is small, the radiative heat transfer between the two reservoirs can be expressed as,

$$Q = K(T_1 - T_2)$$

where K is the thermal conductance for radiative heat transfer.

(b) Derive an expression for the thermal conductance K.