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2.57 Midterm Exam No. 1
Fall, 2002

1. Answer the following ten short questions. Briefly explain your answer (**60 Points**).

(1) Diamond has an fcc structure with a lattice constant (conventional unit cell) of 3.57 Å, calculate the density of diamond.

(2) The fundamental vibrational frequency of H_2 is 1.3×10^{14} Hz, estimate the effective spring constant between the two hydrogen atoms (note: normal hydrogen atom has only one proton and does not have a neutron).

(3) Light is emitted between the first two energy levels of a 60 Å wide one-dimensional quantum well with infinite potential barrier height, what is the wavelength of the light?

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(4) At high temperature limit, each mode of lattice vibration has an average energy of $k_bT/2$. What is the total vibrational energy per unit volume for a silicon crystal at 1000 K. Silicon has an fcc lattice with a lattice constant of 5.43 Å.

(5) Copper has an fcc lattice with a lattice constant of 3.61 Å (one Cu atom per lattice point). Each Cu atom contributes one valence electron. Calculate its Fermi level.

(6) The dispersion relation of photon inside a cubic crystal of length L and refractive index n is

$$\omega = \frac{c_o}{n}\sqrt{k_x^2 + k_y^2 + k_z^2}$$

where c_o is the speed of light in vacuum and $k_x, k_y, k_z = \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, \ldots$, and L the crystal leg Derive an expression for the photon density of states inside the crystal.

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2 (**20 Points**). We have shown two ways in class to derive the energy levels of an electron inside an infinite potential well: one based on the standing wave formation and one based on solving the Schrödinger equation. Now, I would like to ask you to use the **standing-wave** strategy to show that the energy level of a harmonic oscillator is given by

$$E = h\nu n \qquad\qquad (n=1, 2, \ldots)$$

where we have a neglected the zero-point energy. To solve this problem, you can assume that the average kinetic energy equals the potential energy of the harmonic oscillator. The potential energy of a harmonic oscillator is

$$U = \frac{1}{2}kx^2$$

Note: The derivation is simple but can be considered only as a hand waving argument. The frequency of the oscillator, ν, thus derived, is different from the solution of the Schrödinger equation.

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3 (**20 Points**) At 300 K and 1 atm, the thermal conductivity of helium is 0.152 W/mK. Estimate the mean free path of the helium atoms.