12.086 / 12.586 Modeling Environmental Complexity
Fall 2008

Lecture notes for 12.086/12.586, Modeling Environmental Complexity

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1 Introduction

1.1 What is *environmental complexity?*

The basic laws of physics are simple.

- Newton's second law, $F = ma$, tells us how apples fall, tennis balls bounce, and molecules collide.

- Maxwell's equations describe interactions between electric currents and magnetic fields.

- There are modifications at high velocities (relativity) and small sizes (quantum mechanics).

These laws are simply described by differential equations and have tremendous predictive power.

Now look out the window. You see

- People, grass, trees.

- Rocks, birds, squirrels, clouds, the river.

- Further out: oceans and fish; mountains and glaciers.

- Further in: vast microbial communities (up to a billion/cc)

Why are they there? How did they get there? Why do they look like they do?

These things are all obvious to the eye, and such a familiar part of our everyday experience that they are virtually unnoticed. Yet answering these questions using the laws of physics can appear impossible. Why?

These phenomena all exhibit *structure with variability.*

If the structure were perfectly ordered, like a crystal, we could understand it.

Conversely, if the structure were perfectly disordered, like an ideal gas, we could also understand it.

The structure of the natural world, on the other hand, usually lies somewhere in between.

We refer to this type of intermediate variability as *complexity.*

1.2   Why study complexity?

Complex systems often exhibit variability over a wide range of scales. Consider again the Earth, from large to small scales:

- Continents and oceans.
- Mountains, rivers.
- Lakes, glaciers.
- Rocks and soils.

Studies of complexity *seek the origin of such variability.*

In some cases, we are able to specify a set of qualitative criteria—*universality classes*—that allow for quantitative predictions of "emergent behavior" in a wide variety of systems.

This general goal has however been achieved only in relatively narrow cases (e.g., the transition to chaos) or with respect to well-defined models (e.g., self-organized criticality).

Rather than seeking general laws applicable all complex systems, we instead assume that *all complex systems are different.* Earthquakes are neither avalanches nor stock market crashes; river networks are neither evolutionary trees nor leaf venation patterns.

We nevertheless suggest that studies of individual, exemplary, complex systems, can offer insight applicable to other complex systems.

1.3 Complexity can emerge from simple interactions

Here's one such insight:

Nature can produce complex structures from simple interactions.

We illustrate this idea with a model of a fluid. We like fluids because much of the complexity of our natural environment derives from fluid motions: the weather, the climate, rivers, oceans. At some scales the flow can appear chaotic (e.g., turbulent bursts of wind), while at others it can appear organized (e.g., the jet stream, hurricanes, tornadoes).

To model a fluid, we must describe how the flow at a particular point in space affects the flow at other points. We build a model from three ideas:

- *Locality.* Fluid particles influence only their immediate neighbors.

- *Conservation.* Particles, and their momentum, are conserved.

- *Symmetry.* There is no preferred direction for flow (isotropy).

Rather than assuming a continuum and deriving partial differential equations, we instead seek the simplest possible model from which fluid motions emerge.

We include only the following ingredients:

- Identical particles hop at unit speed from site to site on a regular lattice.

- Collisions conserve particle number and momentum.

Here's one time step in the model's evolution.
(a) initial condition; (b) hop; (c) collide.
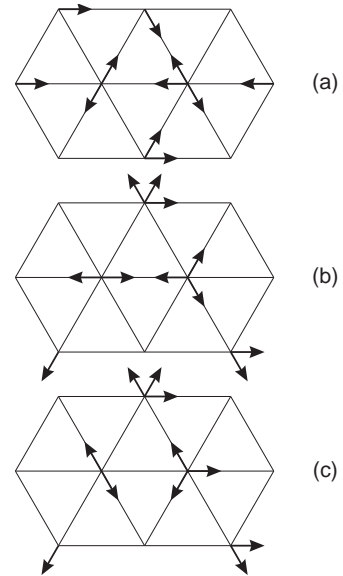
Now increase the size of the system, letting many particles evolve over many time steps. Then average the particle motions in space and time, to obtain a "smooth" picture of the flow.

Remarkably, these coarse-grained flows are virtually identical to the motion of real fluids, including all the beautiful structure of turbulent motions.

What have we learned?

Simple caricatures of the real world can produce beautifully complex behavior.

Why does it work?

- Formally: because our three requirements of fluid motion are fulfilled.

- Informally: because many particles interact over long times. (Thus one could say we obtain "quality from quantity" [1].)

- More subtly: because there is a separation of length and time scales between the microscopic motion of particles and the macroscopic motion of fluids.

1.4 Objectives

The objective of this course is to teach lessons such as those which we just learned with our discrete fluid.

To do so, we will consider a mix of fundamental models (like the discrete fluid) along with studies of specific systems that occur in our natural environment.

The natural systems are chosen because a) they provide a context in which to learn interesting lessons; and b) they are interesting in their own right.

Such systems include

- Landscapes: river networks, drainage basins, the shape of topography.

- Transport through disordered (porous) media.

- Ecology: community structure, food webs, diversity, stability.

- Biogeochemical cycles: origin, rates, and structure

- Evolution and the fossil record.

In other words, we emphasize what you see out the window: why the natural (physical and biological) environment looks like it does, how it is maintained, and (to a lesser extent) how it got there.

In these studies we will encounter several fundamental models and concepts. These include

- Random walks and non-Gaussian fluctuations.

- Percolation theory: critical phenomena, scaling, universality, fractals, renormalization.

- Models of growing surfaces, networks, and aggregates.

- Self-organized criticality.

- Chaos.

Lessons to be learned include

- How complexity can derive from simplicty.

- How to derive (macroscopic) statistical descriptions of systems from their (microscopic) dynamics.

- Why fractals and scale-invariance are ubiquitous.

- How to use computer simulations to reveal fundamental phenomena.

- How to construct a "minimal" model of a complex system that

    - matches the right level of description to the phenomena of interest; and

    - answers a question worth asking.

1.5   Requirements

- Class participation.

- Project-oriented problem sets.

- Final project, to be submitted on paper and in an oral presentation.

- No exams.

References

[1] Bak, P. *How Nature Works: the Science of Self-Organized Criticality* (Springer-Verlag, New York, 1997).

[2] Goldenfeld, N. & Kadanoff, L. P. Simple lessons from complexity. *Science* **284**, 87–89 (1999).

[3] Rothman, D. H. & Zaleski, S. *Lattice-gas Cellular Automata: Simple Models of Complex Hydrodynamics* (Cambridge University Press, Cambridge, 1997).