LAB 8: DARCY FLOW AND STOKES FLOW

The viscosity of a fluid describes its resistance to deformation. Water has a very low viscosity: the force of gravity causes it to flow immediately. Honey has a higher viscosity. The viscosities of Earth materials largely control mantle convection and therefore volcanic activity, heat loss from the interior, and plate tectonics; how volcanoes erupt and how far lava travels; and the shapes of landforms created through weathering, to give a few examples.

To give you a feeling for both what viscosity values are and how they are measured in the lab, we will measure honey viscosity using the Darcy velocity of honey sinking in graduated cylinders filled with glass marbles and hex nuts, and compare these viscosity calculations with others calculated from measurements using Stokes law.

Darcy's law, described empirically by Henry Philibert Gaspard Darcy in the early 1800s, describes fluid flow through a porous medium. Flow of this kind is relevant in a number of geological settings, including oil in subsurface reservoirs and groundwater flow.

Stokes law, derived by George Gabriel Stokes around 1850, describes the velocity of a sphere sinking or rising through a viscous fluid under the influence of gravity.

The ratio of inertial to viscous forces is commonly used to scale fluid flow, and is called the Reynolds number, given as:

$$\mathrm{Re} = \frac{\rho v L}{\eta}, \qquad 1.$$

where ρ is the density of the fluid, v is the fluid velocity, L is a characteristic length scale, and η is fluid viscosity. The solid flow of the mantle has very low inertia compared to viscosity, so its Reynolds number is vanishingly small. Stokes law pertains well to mantle flow. In the mantle the flow is entirely laminar and very far from turbulence. Darcy's law also pertains to low Reynolds number flows.

Viscosity in SI is measured in units of $Pa \cdot s$, or Pascal-seconds, which are $\frac{kg}{m \cdot \mathrm{sec}}$. If a fluid with a viscosity of one Pa·s is placed between two plates, and one plate is moved laterally with a shear stress of one pascal for the time interval of one second, the plate moves a distance equal to the thickness of the fluid layer. The cgs unit for viscosity is the poise, which equals $0.1 * Pa \cdot s$. Thus ten poise make a pascal-second.

A number of forms of Darcy's law, describing fluid flow through a porous medium, are in common use. The general form is given as

$$v_D = \frac{-k}{\eta}\left(\nabla p + \rho g \nabla z\right), \qquad 2.$$

in which v_D is the Darcy velocity of the fluid, k is permeability (the ability of the medium to transmit fluids), η is fluid viscosity, p is the pressure field, ρ is the density of the fluid (in the case of buoyant magma rise, use $\Delta\rho$, the difference in density between the magma and its country rock), and g is gravitational acceleration.

The permeability k of a porous medium is expressed in terms of grain diameter d and porosity ϕ (as a fraction) as

$$k = \frac{d^2\phi}{270} \quad m^2. \qquad 3.$$

The most common groundwater problems involve pressure head, the pressure driving flow from reservoir of fluid into the porous medium, such as a lake lying above a permeable strata of sandstone. In our case there is no forcing pressure, and the percolation is strictly vertical, so equation 2 simplifies to

$$v_D = \frac{k\rho g}{\eta} \quad \frac{m}{s}, \qquad 4.$$

where η is fluid viscosity, and. While the Darcy velocity has units of velocity, it is an average velocity of the system and not the velocity of the fluid with respect to stationary matrix, since the matrix takes up area. To convert Darcy velocity to average pore fluid velocity, divide by the porosity:

$$v_f = \frac{v_D}{\phi} \quad \frac{m}{s}. \qquad 5.$$

Measure the grain size d used in the graduated cylinders. The porosity ϕ for close-packed spheres is known for the case of regular lattice packing: Gauss proved that the highest average density that can be achieved by a regular lattice arrangement is

$$\frac{\pi}{3\sqrt{2}} \approx 0.74, \qquad 6.$$

so the porosity is ~0.26. The shape and diameter of these graduated cylinders, however, do not allow the spheres to be regularly packed. The problem of density in irregular packing is an open mathematical problem: the Kepler conjecture states that Gauss's solution is also the highest density that can be reached by any spherical packing, but it remains unproven.

In our case, I measured the porosity of the objects in the graduated cylinder by measuring the volume of water that fills the spaces. Both the glass marbles and the hex nuts have 50% porosity. The density of honey varies with temperature, water content, and some secondary parameters, but averages about 1,450 kg/m^3.

Viscosity can be also measured using the Stokes flow law (recall from the problem set about plagioclase flotation or sinking in magma). The original formulation of Stokes law simply described the force needed to drive a sphere through a quiescent, continuous, viscous fluid:

$$F = 6\pi r v_S, \qquad 7.$$

where a sphere of radius r is moving with velocity v_S. This law can be rewritten as

$$v_S = \frac{2(\rho_{solid} - \rho_{liquid})gr^2}{9\eta} \ \frac{m}{s}. \qquad 8.$$

to describe how a sphere moves under the influence of gravity. The density of the steel ball bearing with radius r is 7,800 kg/m^3. Thiq equation 8 can be solved for viscosity:

$$\eta = \frac{2(\rho_{solid} - \rho_{liquid})gr^2}{9v_S} \ Pas. \qquad 9.$$

Our spheres are not falling through a continuous fluid, though, and nor do they have an infinite distance to fall. Stokes velocity can be tempered for edge effects (critical in all fluid tank experiments). Experimentalists have found that the best correction for edge effects is called the Faxon correction, as follows:

$$\eta = \left[\frac{2gr^2(\rho_{solid} - \rho_{liquid})}{9v_s\left(1 + \frac{3.3r}{h_c}\right)}\right]\left[1 - 2.104\left(\frac{r}{r_c}\right) + 2/09\left(\frac{r}{r_c}\right)^3 - 0.95\left(\frac{r}{r_c}\right)^5\right] \text{poise}, \qquad 10.$$

where rc is the radius of the container, and hc is the distance the sphere falls. ***Note: this equation is written for cgs units, NOT SI units.***

By measuring the fall of the sphere using a ruler and a stopwatch, this equation can be used to calculate the viscosity of the fluid.

There will be a series of experiments measuring honey viscosity using Darcy velocity, eqs. 4 and 5.

1. Make a table for data. *First column*: Radius of the particles. *Second column*: Temperature of the honey. *Fourth column*: Distance between the marks on the graduated cylinder. *Fifth column*: Time the honey takes to sink between the markings on the graduated cylinder. *Later columns*: Calculations.
2. Record the temperature of the honey.
3. Pour the honey steadily into the cylinder, but do not build up a pool at the top of the porous medium.
4. Time the progress of the honey front over a measured distance.
5. Calculate viscosity using eqs. 4 and 5.

Other experiments will measure honey viscosity using Stokes flow, eq. 10.

1. Make a table for data. *First column*: Trial number. *Second column*: Radius of the ball bearing. *Third column*: Temperature of the honey. *Fourth column*: Distance between the marks on the graduated cylinder. *Fifth column*: Time the ball takes to sink between the markings on the graduated cylinder. *Later columns*: Calculations.
2. Measure the radii of several ball bearings.
3. Record the temperature of the honey.

4. Drop the ball into the honey and time its fall over a measured distance.
5. Calculate viscosity separately for each ball experiment using eqs. eqs. 8, 9, and 10.

Each team should hand in a *very* neat sheet showing their measurements and calculations and including a diagram of their experimental setup. Don’t forget to write everyone’s name on the sheet.