12.001 Introduction to Geology
Spring 2008

RHEOLOGICAL MODELS

Rheology is the study of how materials deform, and the word is also used to describe the behavior of a specific material, as in "the rheology of ice on Ganymede." Deformation is a change in shape in response to an applied force, which can be tension, compression, shear, bending, or torsion. A force applied to an area is called a stress. In geology deformation is usually described in terms of strain: The strain of a specific segment of the material (a "material line") is defined as the change of the length of the segment divided by the original length, as shown in Figure 1:

σ, stress (force per area), in this case,

L, original length

ΔL, change in length

strain: $\varepsilon = \Delta L/L$

strain rate: $\dot{\varepsilon} = \varepsilon /t$

FIG. 1
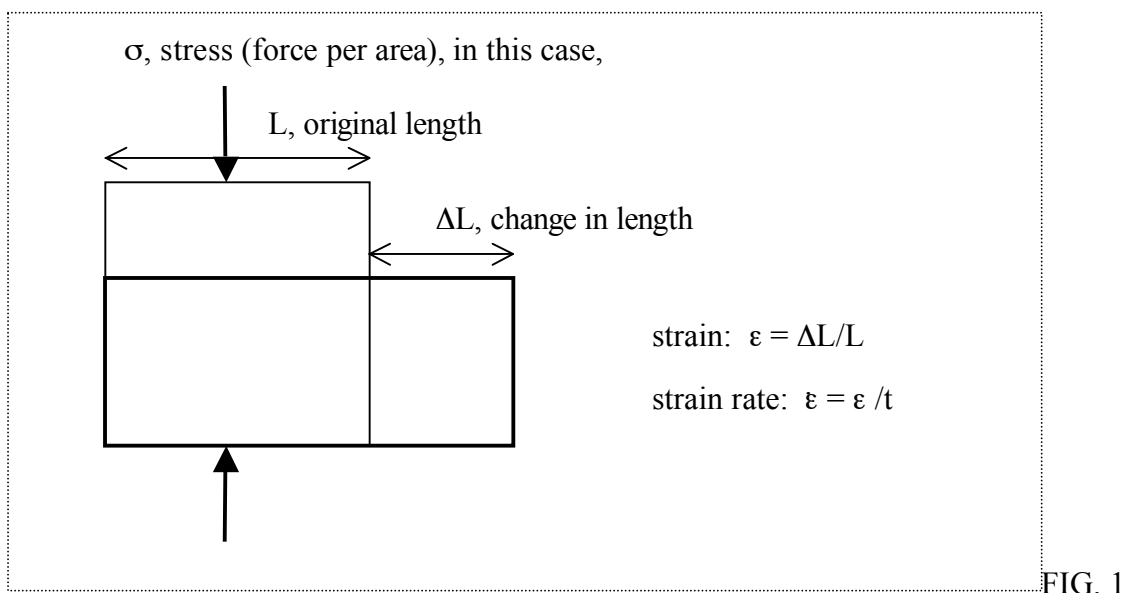
where the strain *ΔL/L* is approximately 0.5.

Rheological behavior of solids can be categorized as brittle (fracturing under stress), plastic (deforming under stress but not returning to the original form after the stress is removed), and elastic (deforming under stress and returning to the original form after stress is removed). Brittle materials, like rocks at the surface of the Earth, are in general elastic under some range of stresses, and as stress increases, they fracture. Ductile materials, in contrast, deform elastically for only a small range of stresses, and as stress increases, they behave plastically. These rheological properties are shown in Figure 2.

FIG. 2
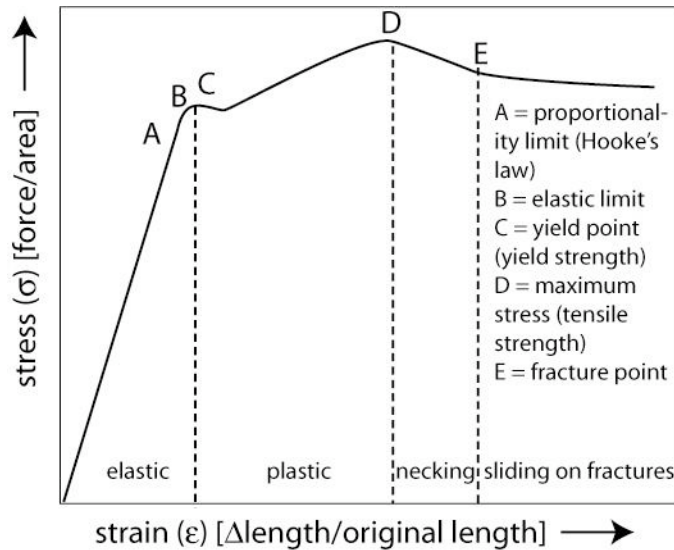
The key to calculating solid flow (ductile, plastic rheology) is viscosity, the material's resistance to deformation. Water has a very low viscosity: it deforms readily under the pull of gravity, so readily that we refer to water flowing. Air has lower viscosity still. The viscosities of honey and molasses are higher. The higher the viscosity, the more difficult the material is to deform, and the slower the flow. Obviously, the viscosities of ice and rock are much higher than those of water and molasses and so their deformation is smaller under a given stress. The viscosity of water at room temperature is about 0.001 Pas (pascal seconds), and the viscosity of honey is about 1,900 Pas. By comparison, the viscosity of window glass at room temperature is about 10^{27} Pas, the viscosity of warm rocks in the Earth's upper mantle is about 10^{19} Pas.

The viscosity of fluids can be measured in a laboratory. In one method, the liquid being measured is put in a container, and a plate is placed on its surface. The liquid sticks to the bottom of the plate and when the plate is moved, the liquid is sheared (pulled to the side). Viscosity is literally the relationship between shear stress σ and the rate of deformation $\dot{\varepsilon}$ (shear stress is pressure in the plane of a surface of the material, like pulling a spatula across the top of brownie batter):

$$\eta = \frac{\sigma}{\dot{\varepsilon}}.$$

The higher the shear stress needed to cause the liquid to deform (flow), the higher the viscosity of the liquid.

Both ice and rock, though they are solids, behave like liquids and deform plastically over long periods of time at relatively low strain rates when they are warm or under pressure. They can both flow without melting, following the same laws of motion that govern the flow of liquids, though the time scale is much longer. Fluids flow by molecules sliding past each other, but the flow of solids is more complicated. At low stresses mantle mineral deform by diffusion creep, in which vacancies in the crystal diffuse toward the crystal face with highest stress. Under high stress the individual mineral grains in the mantle may flow by dislocation creep, in which flaws in the crystals migrate across the crystals and effectively allow the crystal to deform or move slightly. This and other flow mechanisms for solids are called plastic deformations, since the crystals neither return to their original shape nor break.

A material's viscosity is often dependent upon temperature and pressure, and sometimes upon other quantities such as shear stress or grain size, or in the case of honey, sugar content. The viscosity of water is lowered by temperature and raised by pressure, but shear stress does not affect it. Materials with viscosity not dependent upon stress are called Newtonian materials. Honey has a similar viscosity relation with temperature: the hotter the honey, the lower its viscosity. Honey is 200 times less viscous at 70°C than it is at 14°C. Glass is technically a liquid even at room temperature, because its molecules are not organized into crystals. The flowing glass the glass blower works with is simply the result of high temperatures creating low viscosity. In rock-forming minerals, temperature drastically lowers viscosity, pressure raises it moderately, and shear stress lowers it. Materials with viscosity dependent upon stress are called non-Newtonian materials.

Materials that become less viscous in response to shear stress are called thixotropic. Latex house paint is a good example of a material with thixotropic shear-stress dependent viscosity. When painting it on with the brush, the brush applies shear stress to the paint, and its viscosity goes down. This allows the paint to be brushed on evenly. As soon as the shear stress is removed, the paint becomes more viscous, and resists dripping. This is a material property that the paint companies purposefully give the paint, to make it perform better. Ketchup is also frequently thixotropic.
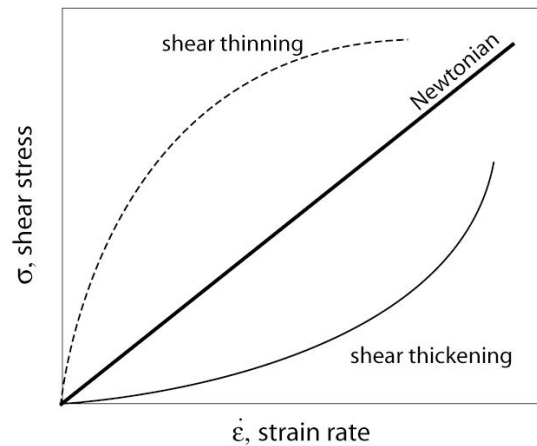
FIG. 3

Some strange materials, called dilatent materials, actually obtain higher viscosity when placed under shear stress. The most common example of a dilatent material is a mixture of cornstarch and water. This mixture can be poured like a fluid and will flow slowly when left alone, but when pressed or thrown it immediately becomes hard, stops flowing, and cracks in a brittle manner. The three possible responses of strain rate to stress are shown in Figure 3.

Another category of material is called Bingham plastic. These do not deform until a certain threshold strear stress is applied. Toothpaste is a Bingham plastic: It will retain its shape until a certain stress is applied, at which point it will flow.

Though dependence upon stress is important in many geological settings, temperature is generally the most important independent variable in viscosity. The dependence of viscosity η upon temperature varies from material to material, and researchers have used various models to fit to the viscosities they measure. Here are some of the more common models:

Arrehenius model

$$\eta = \eta_0 e^{\left(-\frac{Q}{RT}\right)} ,$$

where η_0 is a reference viscosity (the value of viscosity at a specified temperature) in Pa s, Q is the activation energy of the material in J/kg, R is the universal gas constant (8.314472 J/molK), and T is temperature in C or K.

Power-law model

$$\eta = A(T - T_r)^B \,,$$

where A and B are constants and T_r is a reference temperature.

Vogel-Tammann-Fulcher model

$$\eta = Ae^{\left(-\frac{B}{(T-T_r)}\right)} \,,$$

where A and B are constants and T_r is a reference temperature.

Williams-Landel_Ferry model

$$\eta = \eta_0 e^{\left(-\frac{C_1(T-T_r)}{C_2+(T-T_r)}\right)} \,,$$

where η_0 is a reference viscosity, C_1 and C_2 are constants and T_r is a reference temperature.

The viscosity of honey is often described using a power low. An equation for Stokes flow (recall, the flow of a viscous fluid around a sphere that has a velocity determined by gravity) allows the measurement of viscosity of some fluids in the laboratory:

$$v = \frac{2R^2 g \Delta \rho}{9\eta} \,.$$

The fluid can be placed in a graduated cylinder and a metal sphere of radius R dropped into it. The velocity v of the sphere can be measured, and if its density compared to that of the fluid $\Delta\rho$ is known, then the fluid viscosity η can be calculated. Researchers at the University of Maine published on the web some results for honey that they obtained in this way (Figure 4).

Viscosity Data

◆ Steffe's Data
■ Our Data
— Power (Steffe's Data)
— Power (Our Data)
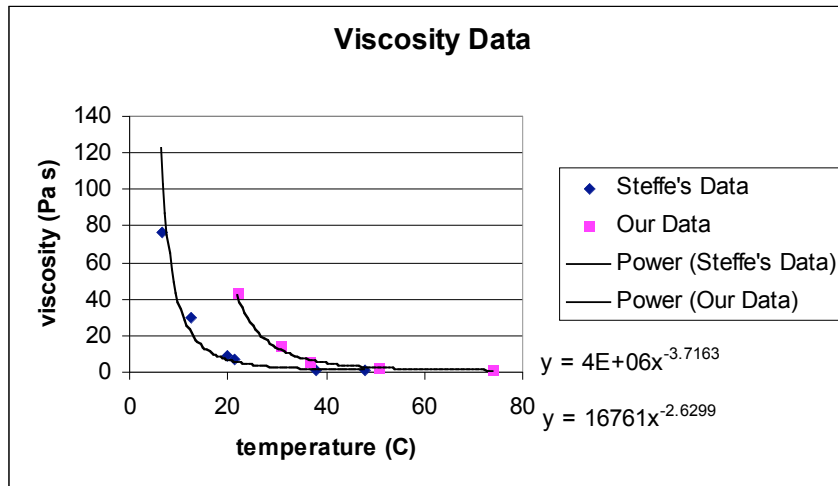
$y = 4E+06x^{-3.7163}$

$y = 16761x^{-2.6299}$

FIG. 4: Experimental data for honey from Erwin A. Melis and Lucy Brown at the University of Maine at Orono.

Temperature is by far the most important control on viscosity. Inside the Earth's upper mantle, where temperatures vary from about 1,100°C to 1,400°C, the solid rocks are as much as ten or twenty orders of magnitude less viscous than they are at room temperature. They are still solid, crystalline materials, but given enough time, they can flow like a thick liquid. The mantle flows for a number of reasons. Heating in the planet's interior makes warmer pieces of mantle move upward buoyantly, and parts that have cooled near the surface are denser and sink. The mantle flows in response to these and other forces at the rate of about two to ten centimeters per year.

In my numerical models of mantle flow I use the following viscosity laws:

$$\eta = \eta_0 e^{\left(\frac{Q}{RT_i}\left(\frac{T_i}{T}-1\right)\right)}$$, or the simpler $\eta = \eta_0 e^{\left(\frac{Q}{RT_i}\left(\frac{T_i-T}{\Delta T_i}\right)\right)}$,

where η_0 is a reference viscosity, Q is the activation energy of the material in J/mol (for olivine, typically 250,000 to 500,000 J/mol), R is the universal gas constant (8.314472 J/molK), T_i is the initial fluid temperature, ΔT_i is the difference between the initial fluid temperature and the temperature at the top boundary of the model domain. For temperature- and pressure-dependent viscosity, this law is used:

$$\eta = \eta_0 e^{\left(\frac{Q*+V*z}{T+T_0} - \frac{Q*+V*z_0}{1+T_0}\right)}$$,

where $Q*$ is a non-dimensional activation energy, $V*$ is a non-dimensional activation volume, z is depth, z_0 is a reference depth, and T_0 is a reference temperature. Note that virtually all numerical models and, indeed, fluid dynamical analysis is conducted in non-dimensional space: All values are divided by reference values with units such that the resulting number is unitless. This process significantly simplifies the form of equations. When material in numerical experiments is significantly sheared, it becomes important to use a non-Newtonian or stress-dependent viscosity law such as

$$\eta_{nonN.} = \eta_o \left(\frac{\sigma_o}{\sigma} \right)^2 \exp\left(\frac{Q*+V*z}{T+T_0} - \frac{Q*+V*z_0}{1+T_0} \right)$$

The efforts of a number of researchers over several decades have begun to quantify more accurately the parameters that mantle viscosity actually depends upon, which include not only temperature T, pressure P, and stress σ, but also grain size d. One model shows the relationship in the following way:

$$\eta \propto \sigma^{1-n} d^m e^{\frac{Q+PV*}{RT}},$$

where R is the gas constant, the activation energy Q may vary from 250 to 550 kJ/mol, the activation volume $V*$ may vary from ~5 to ~25, the stress exponent n varies from 1 to 3, and the grain size exponent m varies from 0 to 3 (Karato and Wu, 1993). The addition of water to vacancies or crystal defects in mantle minerals also lowers viscosity significantly even in the absence of any fluid phase or melting, but this parameter has not been added into the equation shown here. Hirth and Kohlstedt (1996) find that olivine viscosity is reduced by a factor of 100 to 200 when saturated with water.

References

Hirth G. and Kohlstedt D.L. (1996) Water in the oceanic upper mantle: Implications for rheology, melt extraction and the evolution of the lithosphere. Earth Planet. Sci. Lett. 144, 93-108.

Karato S.-I. and P. Wu (1993) Theology of the upper mantle: A synthesis. Science 260, 771-778.

Steffe, J. F. (1996) *Rheological Methods in Food Process Engineering*. Freeman Press, East Lansing MI, 82-84.