Analysis of Multifactor Experiments

Corresponds to Chapter 13
of Tamhane and Dunlop

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Analysis of Multifactor Experiments

(See Table 13.1 on page 505 of the course textbook.)

Model and estimates

$$\mathrm{y}_{\mathrm{ijk}} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk}$$

$$\hat{\mu} = \bar{y}_{...}$$

$$\hat{\tau}_i = \bar{y}_{i..} - \bar{y}_{...}$$

$$\hat{\beta}_j = \bar{y}_{.j.} - \bar{y}_{...}$$

$$(\tau\beta)_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}$$

$$\hat{y}_{ijk} = \bar{y}_{ij.}$$

$$e_{ijk} = y_{ijk} - \hat{y}_{ijk} = y_{ijk} - \bar{y}_{ij.}$$

For any model

y = vector of observed response values

$\hat{y}$ = vector of fitted values

$\bar{y}$ = vector of grand mean

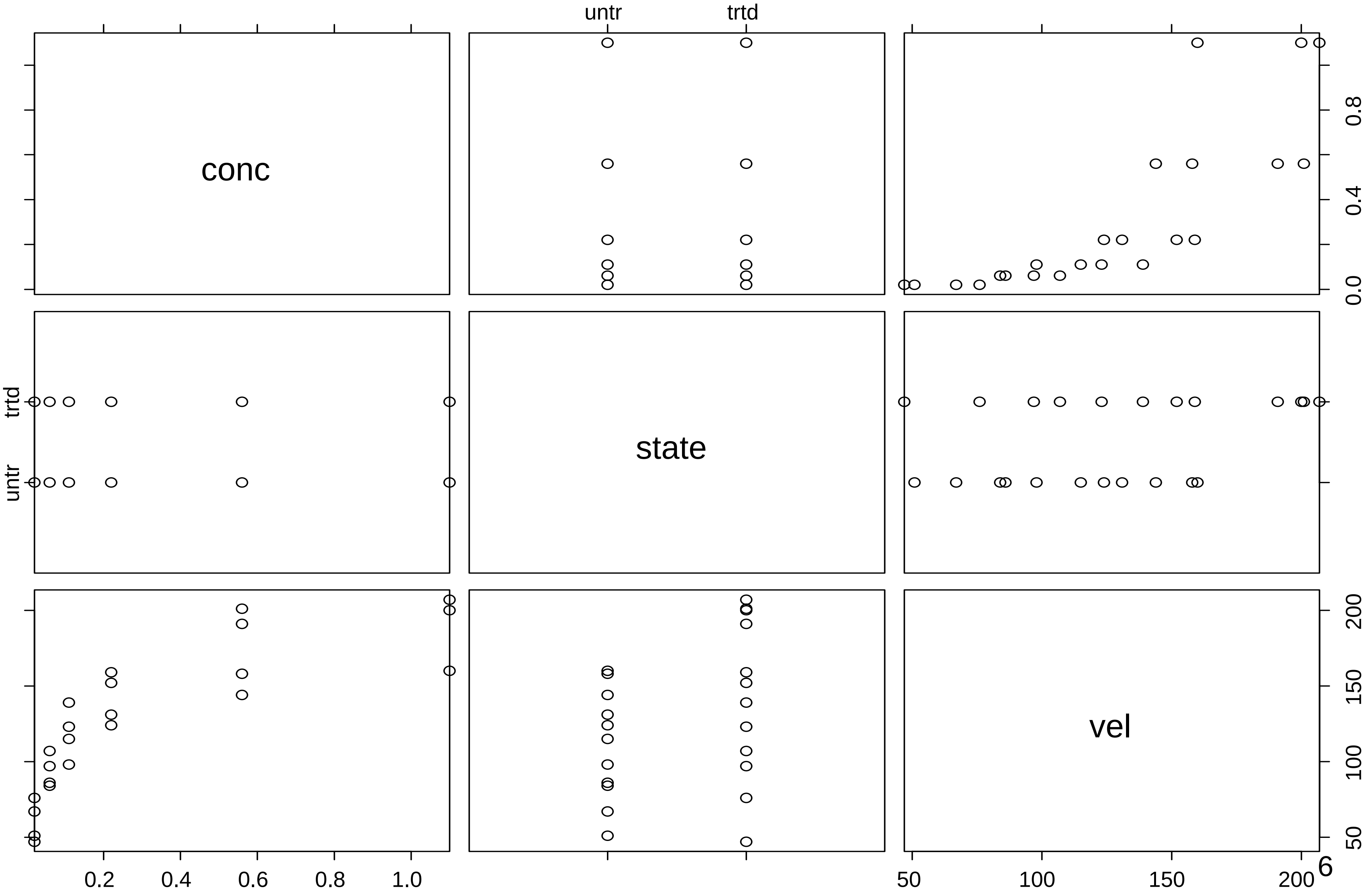
$$\text{SST} = \text{SSTotal} = (y - \bar{y})'(y - \bar{y})$$

$$\text{SSM} = \text{SSModel} = (\hat{y} - \bar{y})'(\hat{y} - \bar{y})$$

$$\text{SSE} = \text{SSError} = (y - \hat{y})'(y - \hat{y})$$

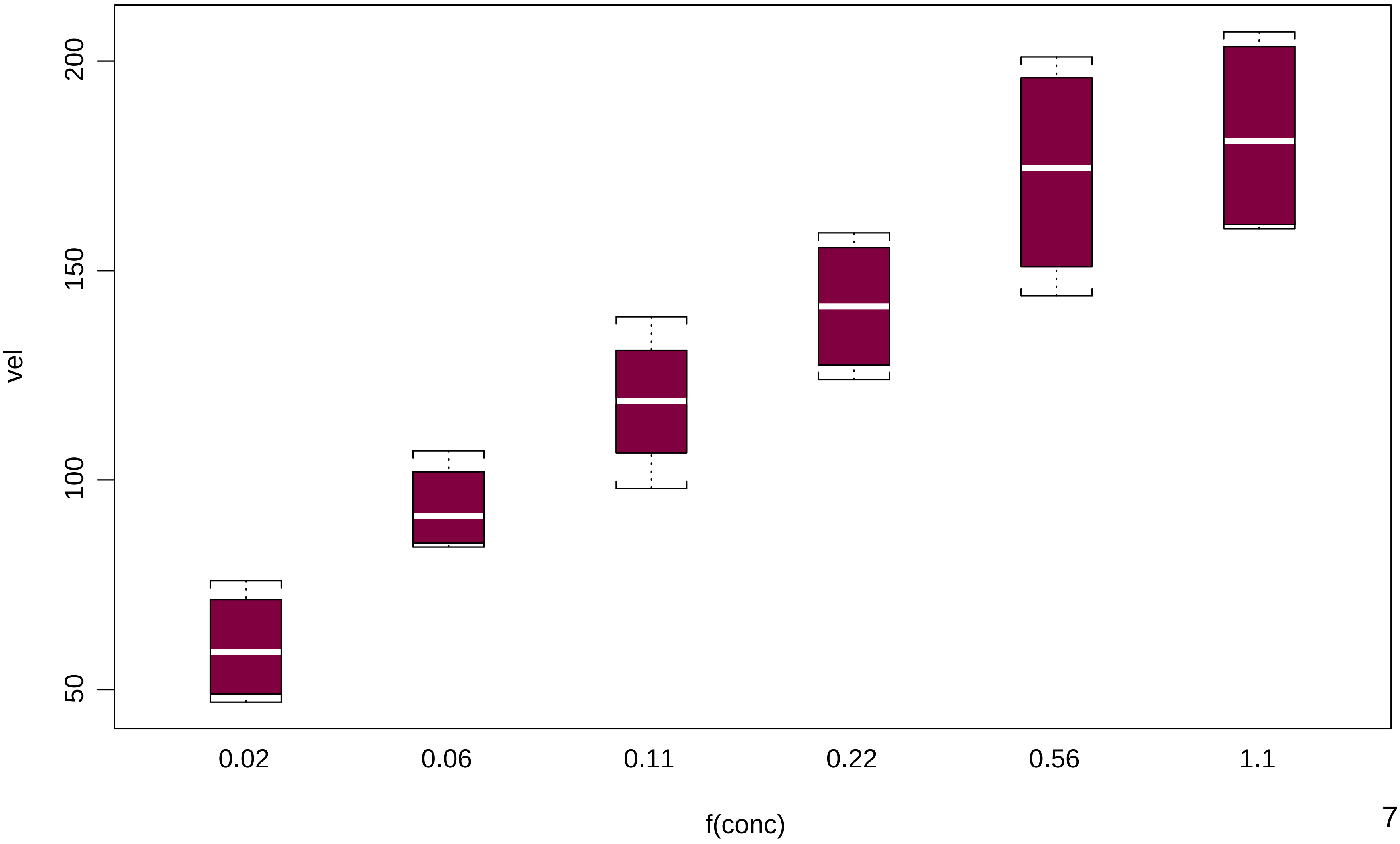
- **Biochemical Reactions of Cells Treated with Puromycin**
- **SUMMARY:**
- The “Balanced” Puromycin data frame has 24 rows representing the measurement of initial velocity of a biochemical reaction for 6 different concentrations of substrate and two different cell treatments. This data frame contains the following variables (columns):
- **ARGUMENTS:**
- **conc**
  - the concentration of the substrate.
- **vel**
  - the initial velocity of the reaction.
- **state**
  - a factor telling whether the cells involved were treated or untreated.

Scatterplot matrix for puromycin data set

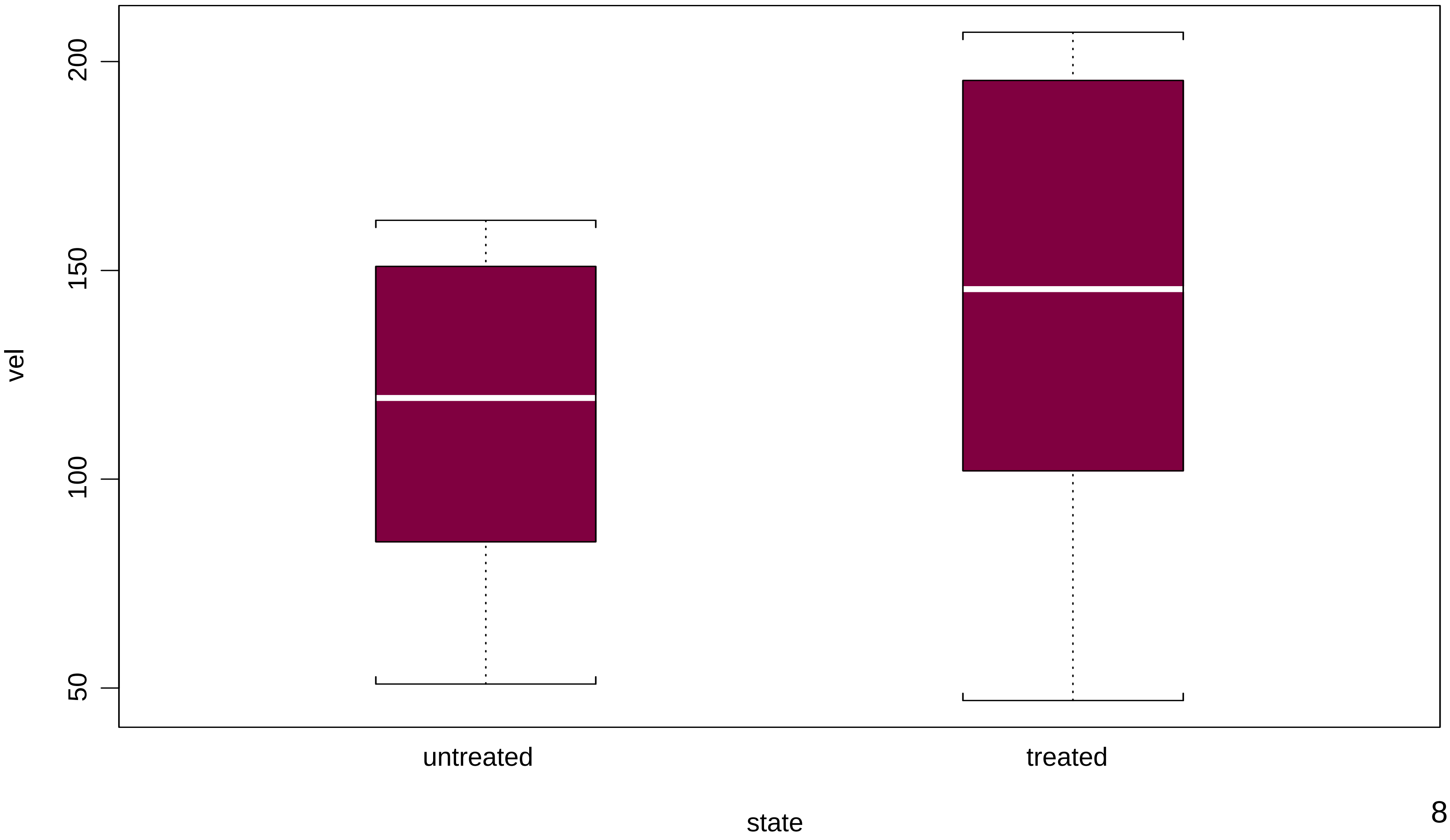


This graph was created using S-PLUS(R) Software. S-PLUS(R) is a registered trademark of Insightful Corporation.

plot.factor(conc,vel)



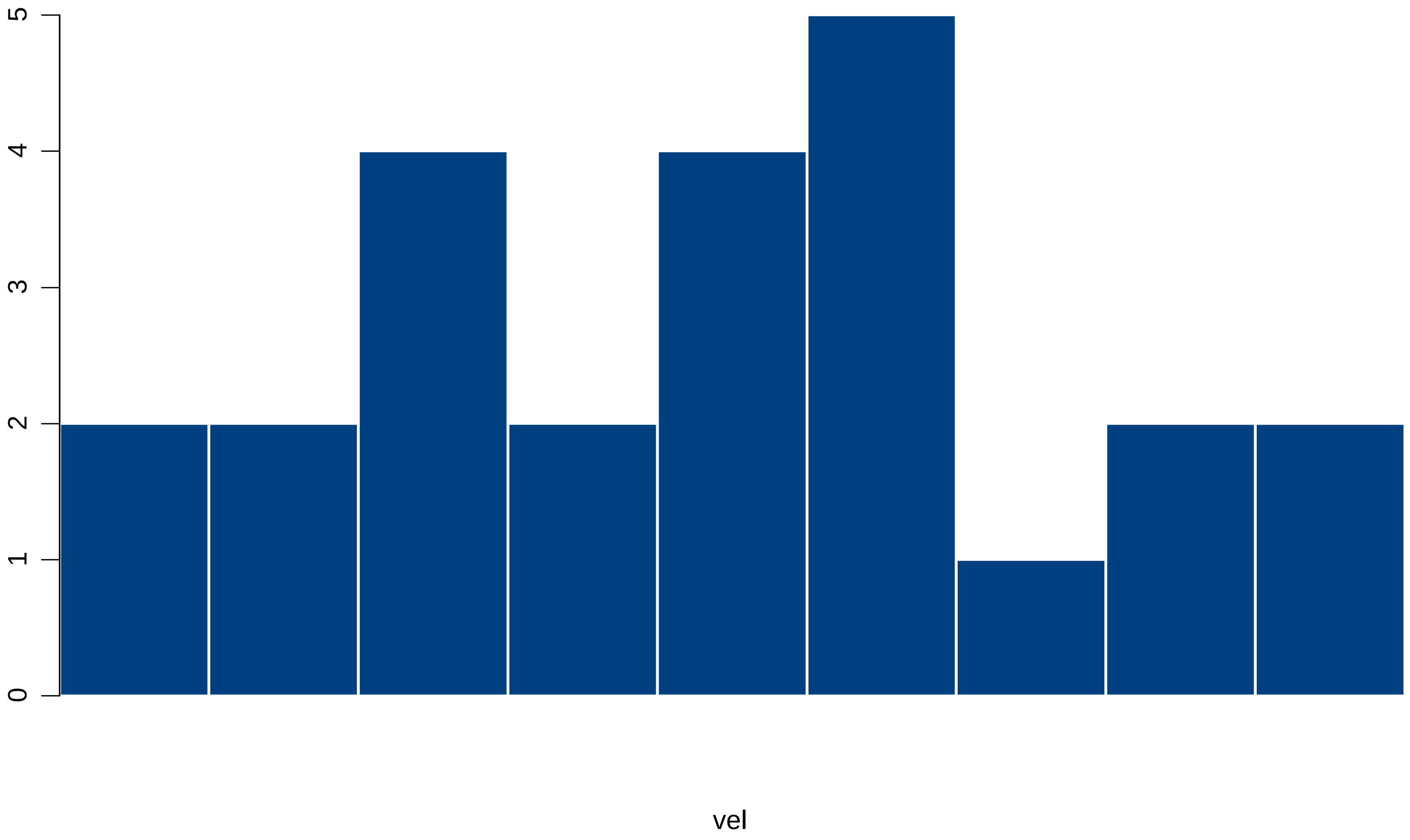
plot.factor(state,vel)



Velocity in “Balanced” puromycin data set

conc	treated		untreated	
0.02	76	47	67	51
0.06	97	107	84	86
0.11	123	139	98	115
0.22	159	152	131	124
0.56	191	201	144	158
1.10	207	200	160	162

Histogram of velocity



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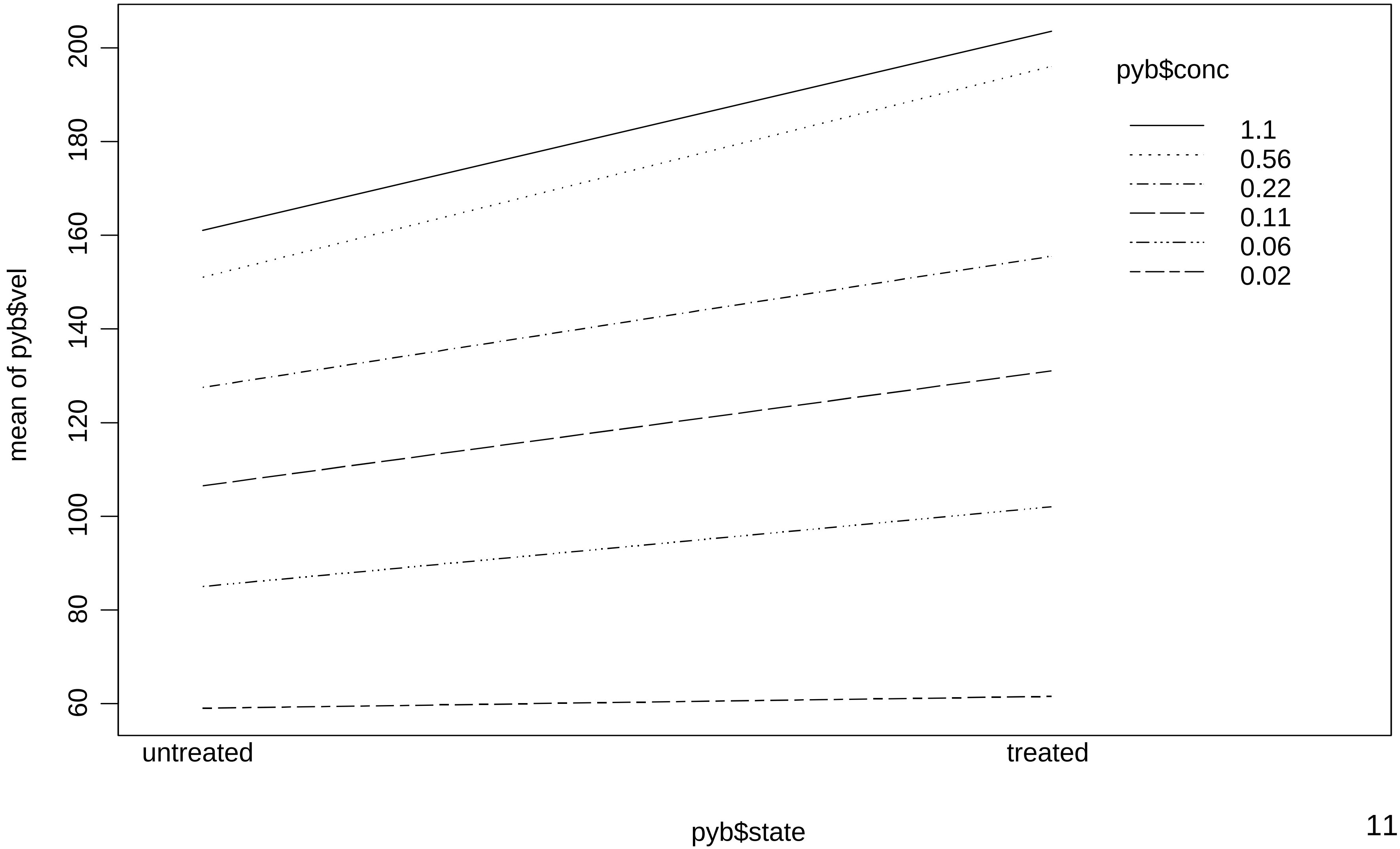
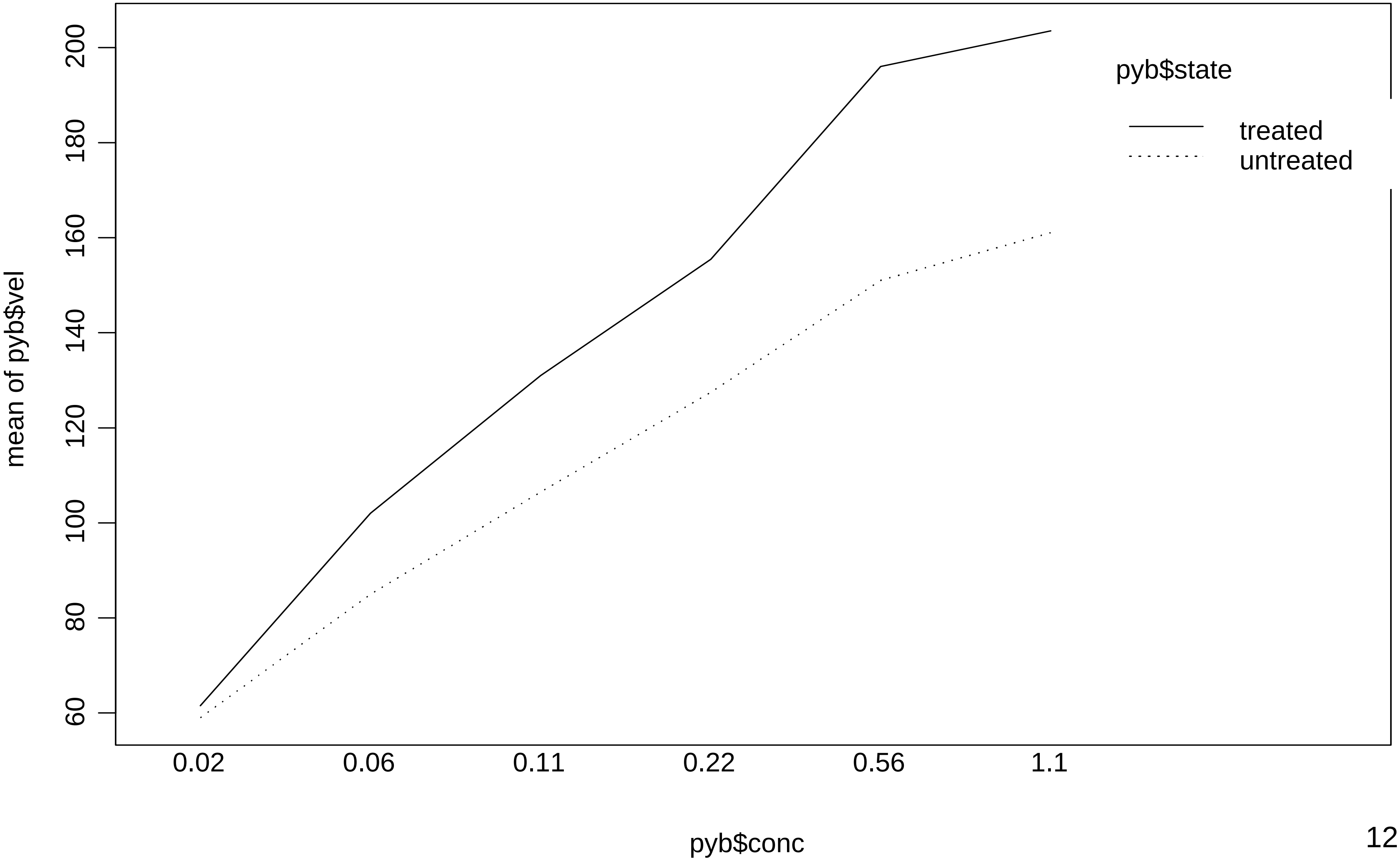
interaction.plot(pyb$state,pyb$conc,pyb$vel)



interaction.plot(pyb$conc,pyb$state,pyb$vel)



Summaries of puromycin model

```
Residuals:
   Min 1Q      Median 3Q  Max
 -14.5 -5 -4.441e-016  5 14.5

Residual standard error: 9.559 on 12 degrees of freedom
Multiple R-Squared: 0.9784
F-statistic: 49.5 on 11 and 12 degrees of freedom, the
  p-value is 2.919e-008

            Df Sum of Sq  Mean Sq  F Value    Pr(F)
     state  1   4240.04 4240.042 46.40264 0.00001871
      conc  5  44243.71 8848.742 96.83985 0.00000000
state:conc  5   1270.71  254.142  2.78130 0.06803651
 Residuals 12   1096.50   91.375
```

Observed velocity and fitted values for puromycin model with interaction

```
          Observed                 Fitted Values
conc  treated  untreated        treated     untreated
0.02  76  47     67     51       61.5  61.5    59.0  59.0
0.06  97 107     84     86      102.0 102.0    85.0  85.0
0.11 123 139     98    115      131.0 131.0   106.5 106.5
0.22 159 152    131    124      155.5 155.5   127.5 127.5
0.56 191 201    144    158      196.0 196.0   151.0 151.0
1.10 207 200    160    162      203.5 203.5   161.0 161.0
```

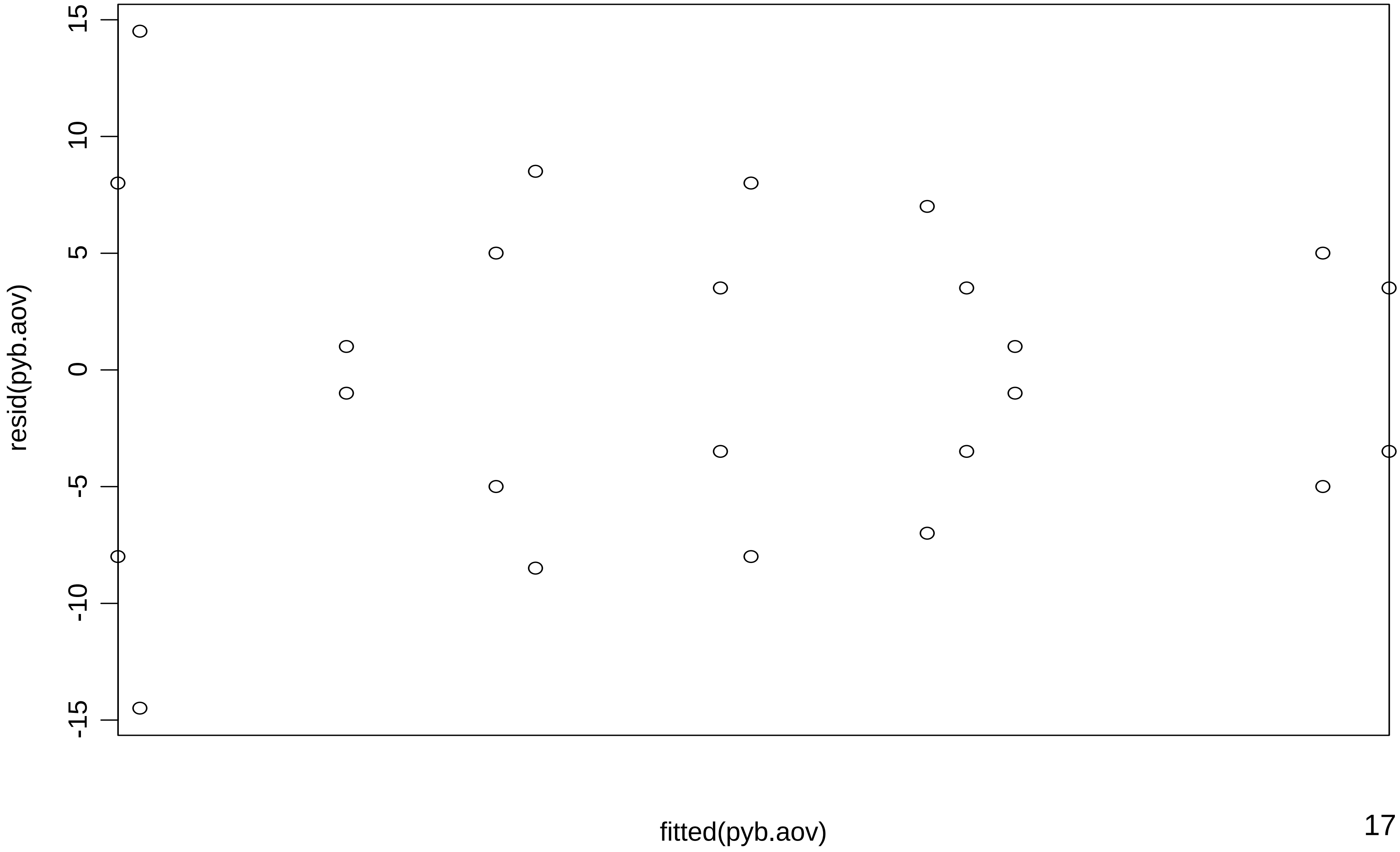
model.tables

```
Tables of means
Grand mean

 128.29

 state
 untreated treated
 115.00    141.58

 conc
   0.02   0.06   0.11   0.22   0.56    1.1
  60.25  93.50 118.75 141.50 173.50 182.25

 state:conc
Dim 1 : state
Dim 2 : conc
           0.02  0.06  0.11  0.22  0.56   1.1
untreated  59.0  85.0 106.5 127.5 151.0 161.0
  treated  61.5 102.0 131.0 155.5 196.0 203.5
```

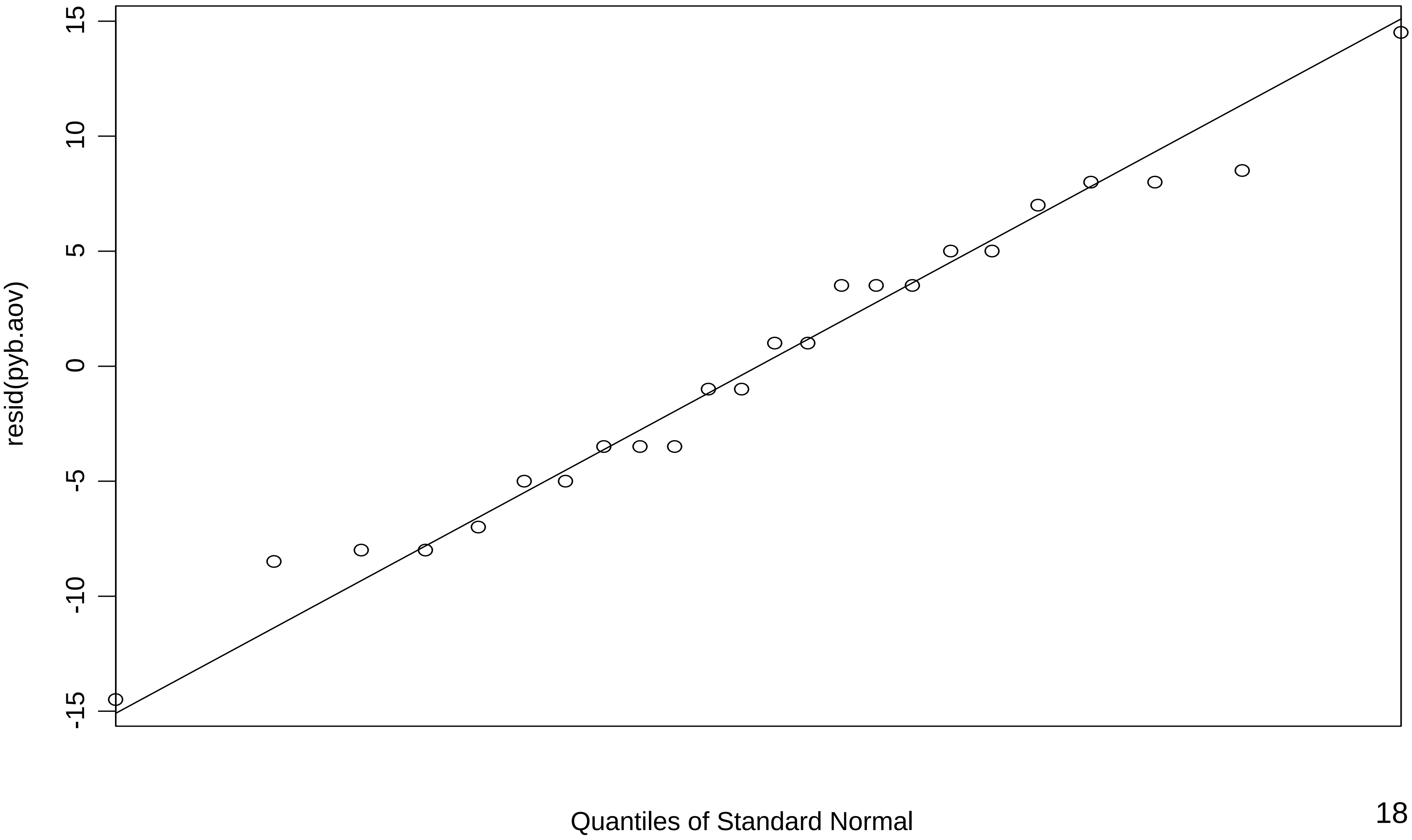
multicomp(pyb.aov,focus=“concf”)

```
95 % simultaneous confidence intervals for specified
linear combinations, by the Tukey method

critical point: 3.3595
response variable: vel

intervals excluding 0 are flagged by '****'

          Estimate Std.Error Lower Bound Upper Bound
0.02-0.06   -33.20      6.76       -56.0    -10.5000 ****
0.02-0.11   -58.50      6.76       -81.2    -35.8000 ****
0.02-0.22   -81.20      6.76      -104.0    -58.5000 ****
0.02-0.56  -113.00      6.76      -136.0    -90.5000 ****
 0.02-1.1  -122.00      6.76      -145.0    -99.3000 ****
0.06-0.11   -25.30      6.76       -48.0     -2.5400 ****
0.06-0.22   -48.00      6.76       -70.7    -25.3000 ****
0.06-0.56   -80.00      6.76      -103.0    -57.3000 ****
 0.06-1.1   -88.70      6.76      -111.0    -66.0000 ****
0.11-0.22   -22.70      6.76       -45.5     -0.0425 ****
0.11-0.56   -54.70      6.76       -77.5    -32.0000 ****
 0.11-1.1   -63.50      6.76       -86.2    -40.8000 ****
0.22-0.56   -32.00      6.76       -54.7     -9.2900 ****
 0.22-1.1   -40.70      6.76       -63.5    -18.0000 ****
 0.56-1.1    -8.75      6.76       -31.5     14.0000
```

Residual vs. fit for puromycin model



qqplot of residuals for puromycin model



Summaries of puromycin model without interaction

```
Residuals:
    Min     1Q Median    3Q   Max
 -26.54 -7.083  2.625 4.792 20.04
Residual standard error: 11.8 on 17 degrees of freedom
Multiple R-Squared: 0.9534
F-statistic: 58.03 on 6 and 17 degrees of freedom, the
  p-value is 2.18e-010

          Df Sum of Sq  Mean Sq  F Value         Pr(F)
     conc  5  44243.71 8848.742 63.54684 0.00000000021
    state  1   4240.04 4240.042 30.44967 0.00003762498
Residuals 17   2367.21  139.248
```

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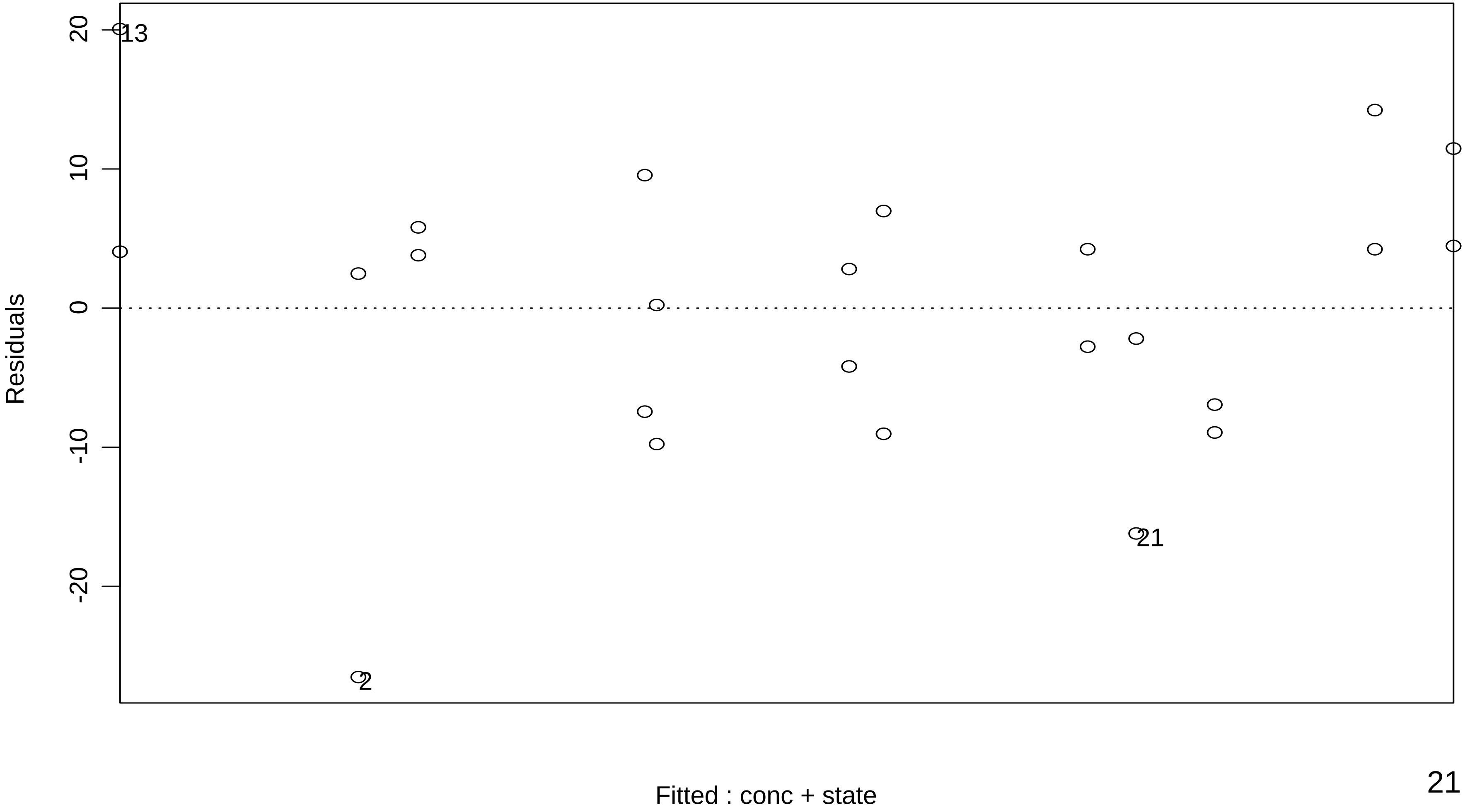
Observed velocity and fitted values for puromycin model without interaction

```
        Observed                      Fitted
conc treated    untreated     treated           untreated
0.02  76  47    67     51   73.542  73.542   46.958  46.958
0.06  97 107    84     86  106.792 106.792   80.208  80.208
0.11 123 139    98    115  132.042 132.042  105.458 105.458
0.22 159 152   131    124  154.792 154.792  128.208 128.208
0.56 191 201   144    158  186.792 186.792  160.208 160.208
1.10 207 200   160    162  195.542 195.542  168.958 168.958
```

Plot of residual vs. fit for puromycin model without interaction



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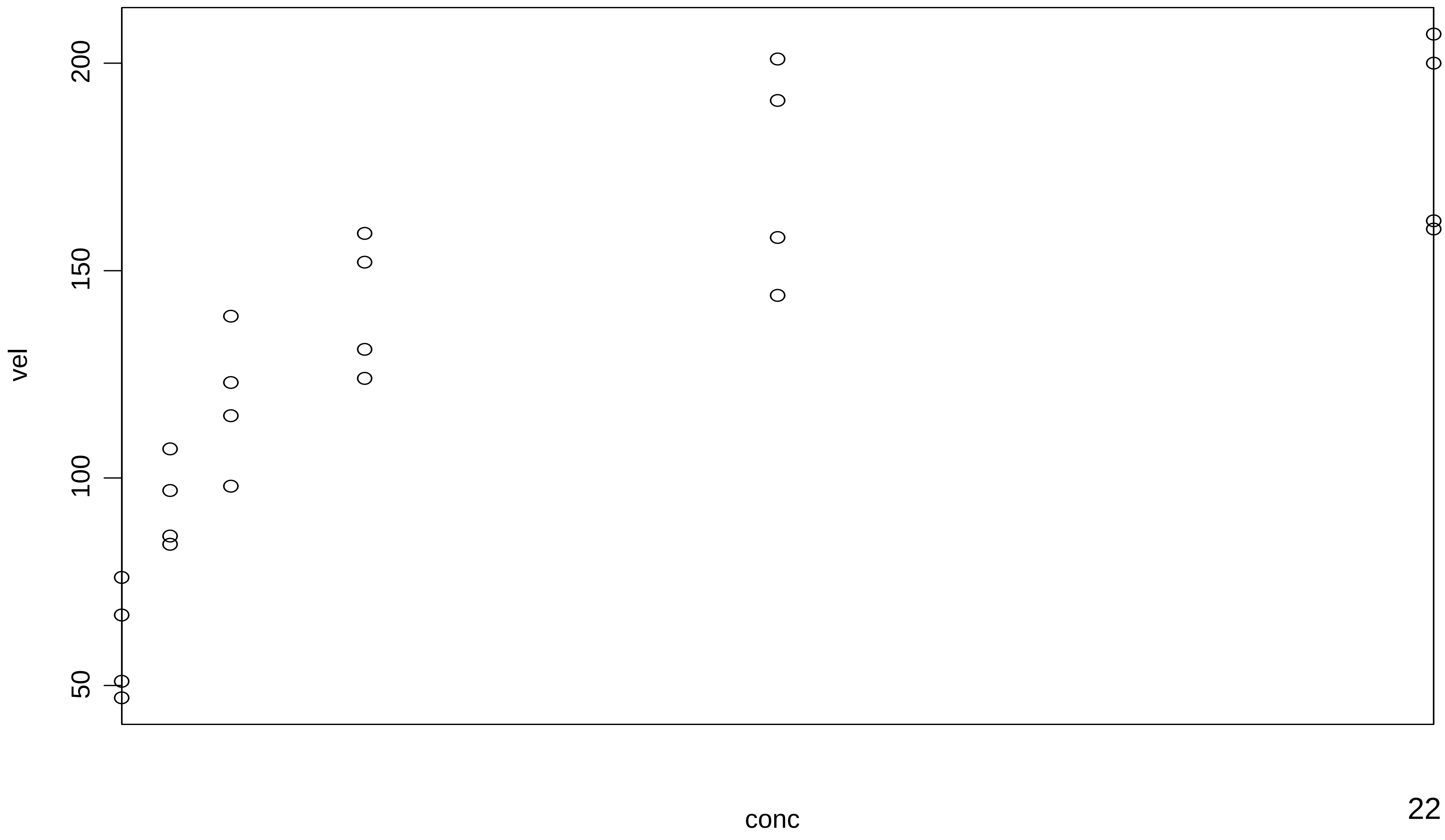
Plot of velocity vs. concentration



This graph was created using S-PLUS(R) Software. S-PLUS(R) is a registered trademark of Insightful Corporation.

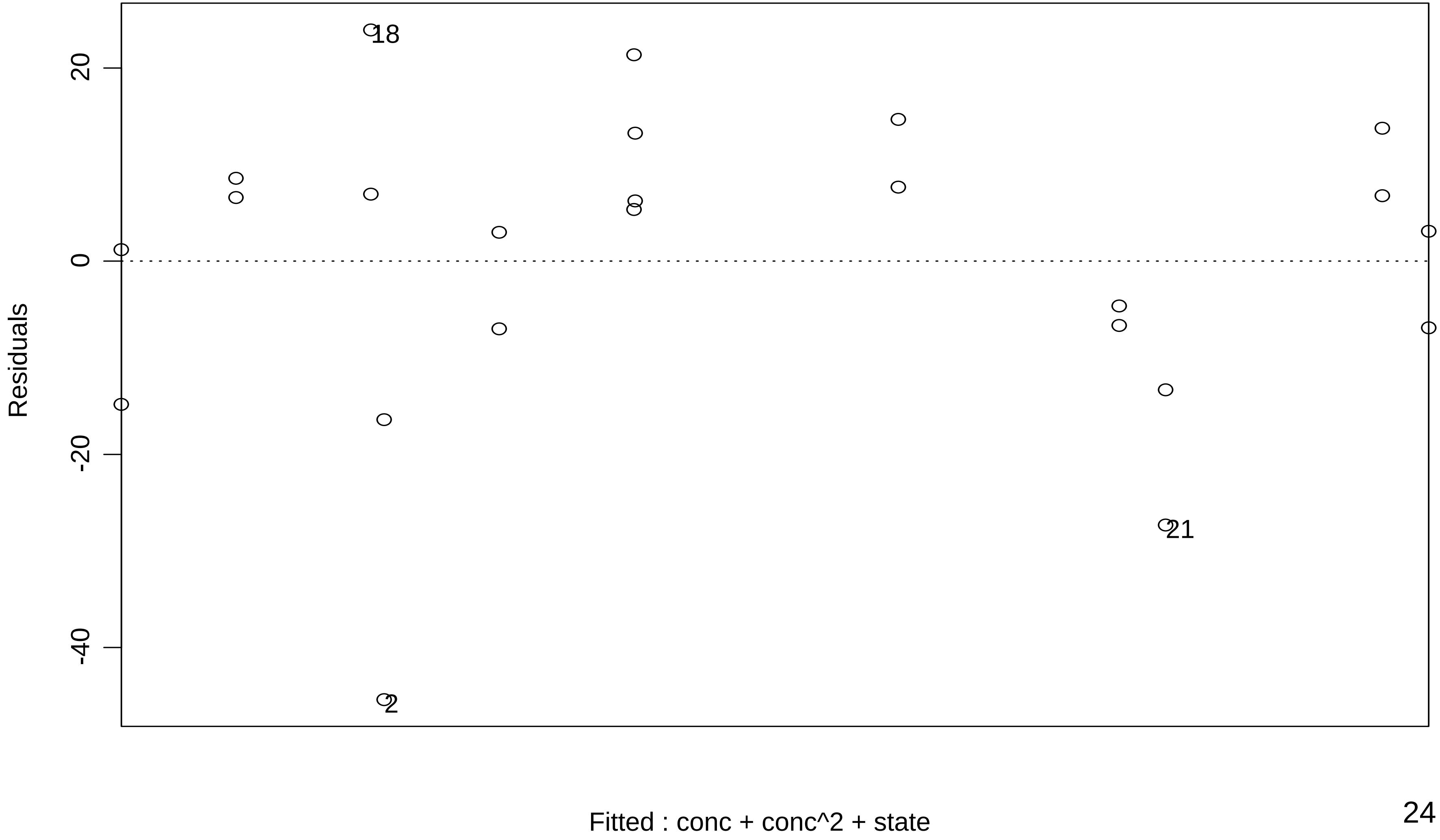
```
Call: aov(formula = vel ~ conc + conc^2 + state)
Residuals:
   Min    1Q Median    3Q   Max
 -45.4 -6.93  4.227 7.902 23.94

Coefficients:
                Value Std. Error   t value  Pr(>|t|)
(Intercept)   73.0885    6.0136    12.1539    0.0000
       conc  304.9581   37.3027     8.1752    0.0000
  I(conc^2) -188.9327   32.5953    -5.7963    0.0000
      state   13.2917    3.4172     3.8897    0.0009

Residual standard error: 16.74 on 20 degrees of freedom
Multiple R-Squared: 0.8898
F-statistic: 53.82 on 3 and 20 degrees of freedom, the p-
   value is 9.291e-010

> summary(pyb2.aov)
          Df Sum of Sq  Mean Sq  F Value        Pr(F)
     conc  1  31590.27 31590.27 112.7215 0.0000000011
I(conc^2)  1   9415.64  9415.64  33.5972 0.0000113551
    state  1   4240.04  4240.04  15.1295 0.0009104989
Residuals 20   5605.01   280.25
```

Plot of residual vs. fit for pyb2.aov



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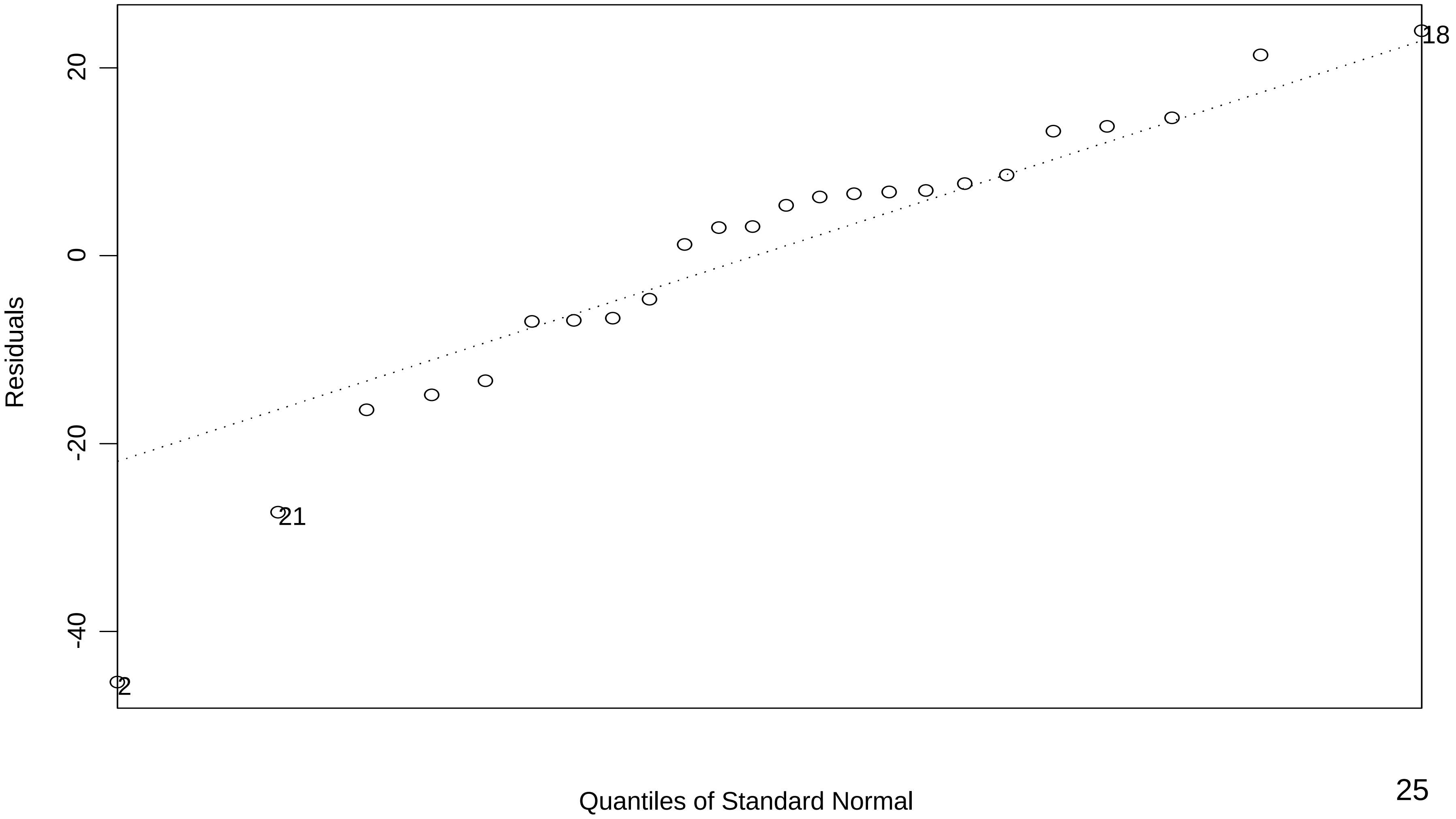
qqplot of residuals for pyb2.aov

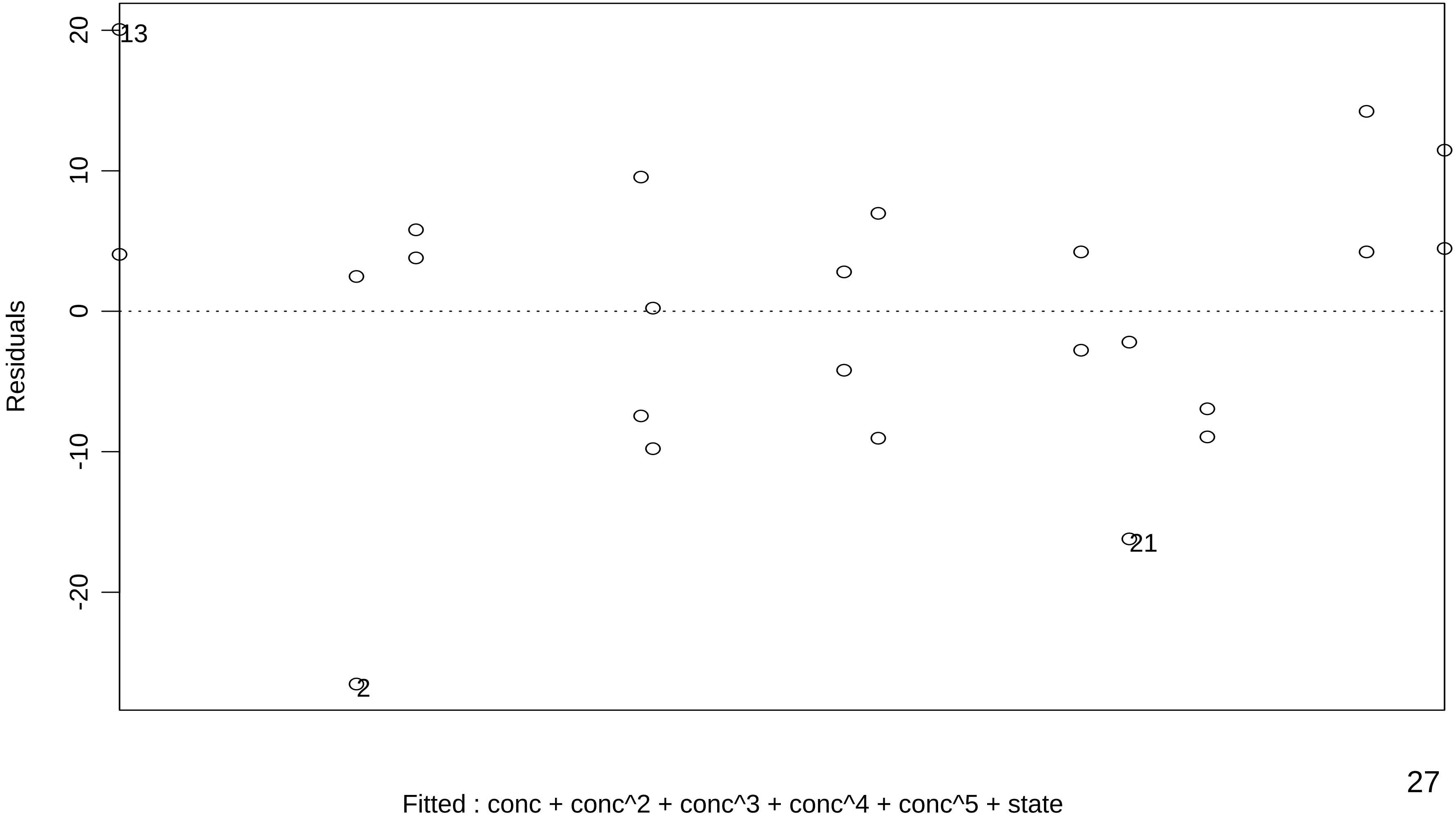


```
Call: aov(formula = vel ~ conc + conc^2 + conc^3 + conc^4
   + conc^5 + state)
Residuals:
    Min     1Q Median    3Q   Max
 -26.54 -7.083  2.625 4.792 20.04

Coefficients:
Residual standard error: 11.8 on 17 degrees of freedom
Multiple R-Squared: 0.9534
F-statistic: 58.03 on 6 and 17 degrees of freedom, the p-
   value is 2.18e-010


> summary(pyb5.aov)
          Df Sum of Sq  Mean Sq  F Value     Pr(F)
     conc  1  31590.27 31590.27 226.8641 0.0000000
I(conc^2)  1   9415.64  9415.64  67.6180 0.0000003
I(conc^3)  1   2603.71  2603.71  18.6984 0.0004604
I(conc^4)  1    631.13   631.13   4.5324 0.0481759
I(conc^5)  1      2.96     2.96   0.0213 0.8857934
    state  1   4240.04  4240.04  30.4497 0.0000376
Residuals 17   2367.21   139.25
>
```

Plot of residual vs. fit for pyb5.aov



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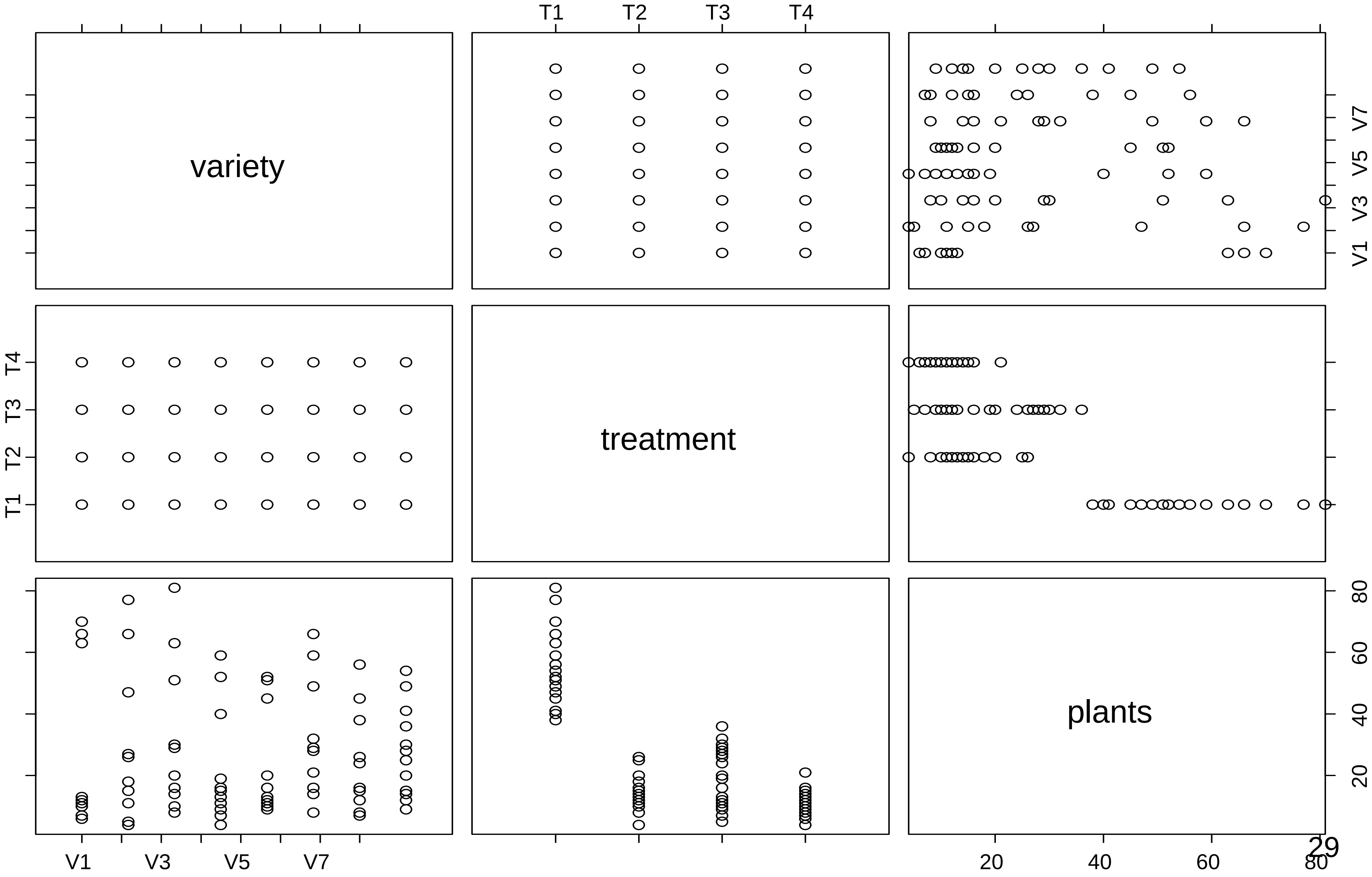
Guayule data set

- **Rate of Germination of Treated Guayule Seeds**
- **SUMMARY:**
- The guayule data frame, a design object, has 96 rows and 5 columns. The guayule is a Mexican plant from which rubber is manufactured. Batches of 100 seeds of eight varieties (variety) of guayule were given one of four treatments (treatment), and planted; the number of plants that came up in each batch (plants) was recorded.
- **ARGUMENTS:**
- **variety**
  - factor with levels V1 through V8 labeling the variety of guayule.
- **treatment**
  - factor with levels T1 through T4 labeling the treatment given to the seeds.
- **plants**
  - numeric vector givng the number seeds out of a batch of 100 that germinated.

pairs(gy)



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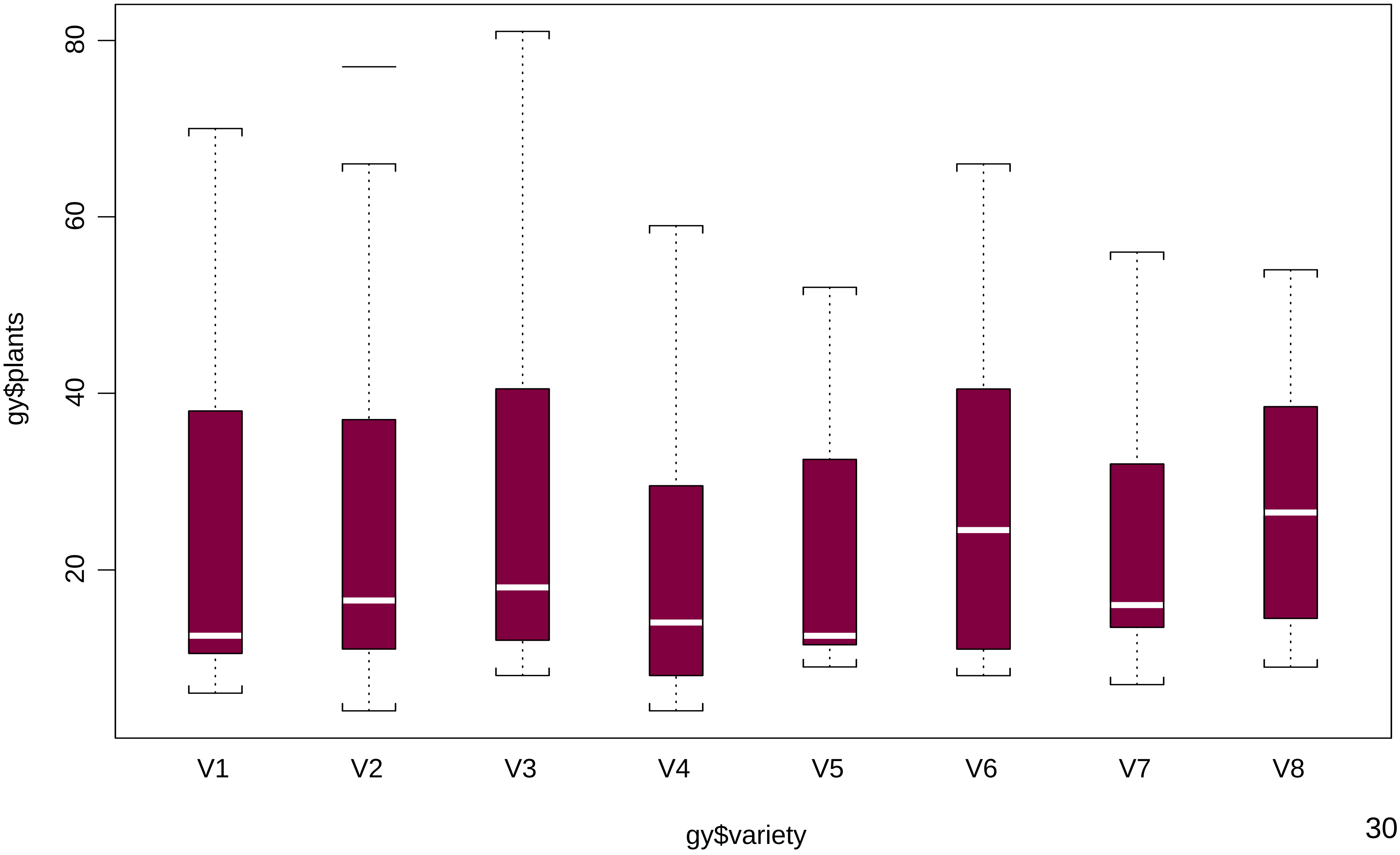
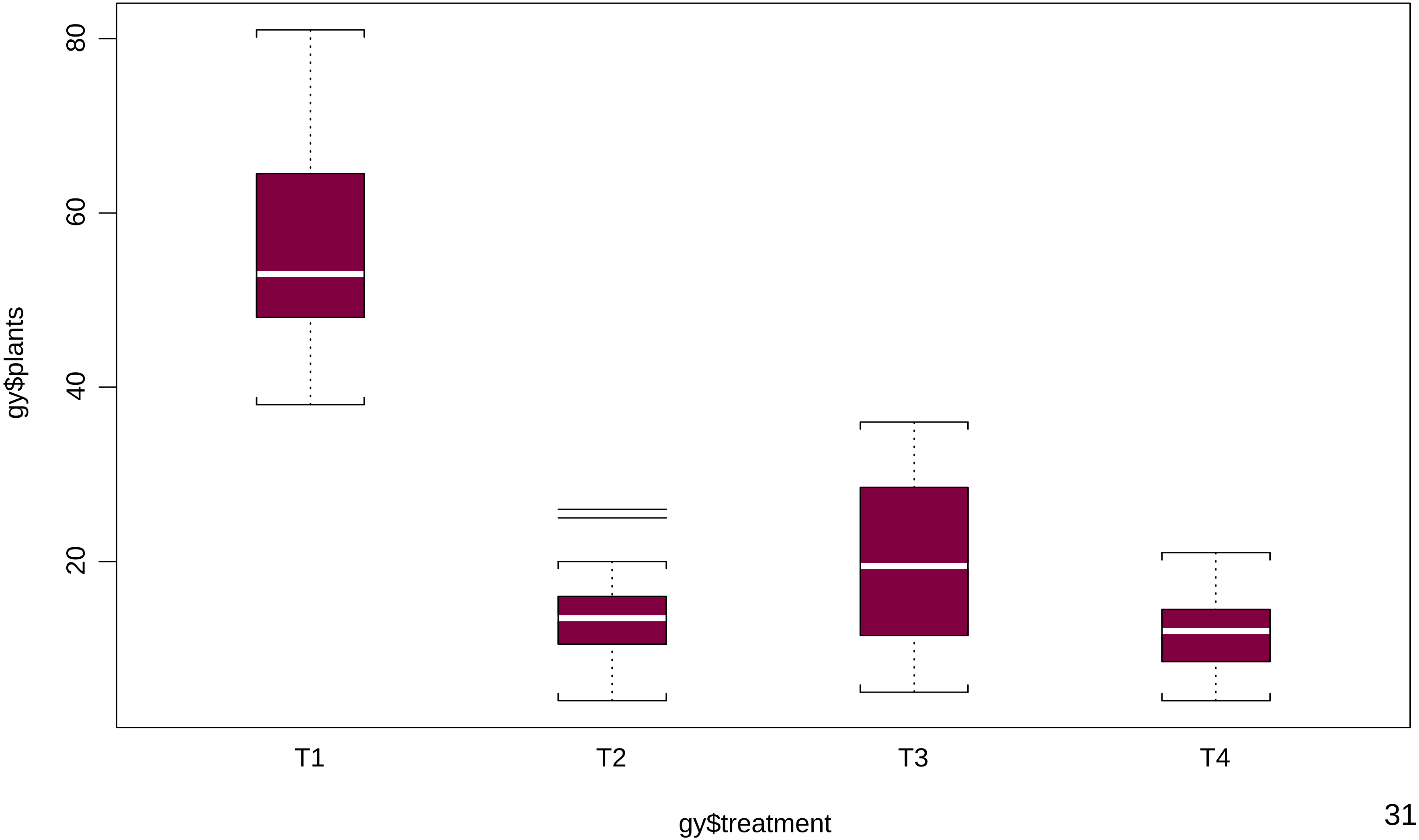
plot.factor(gy$variety,gy$plants)



plot.factor(gy$treatment,gy$plants)



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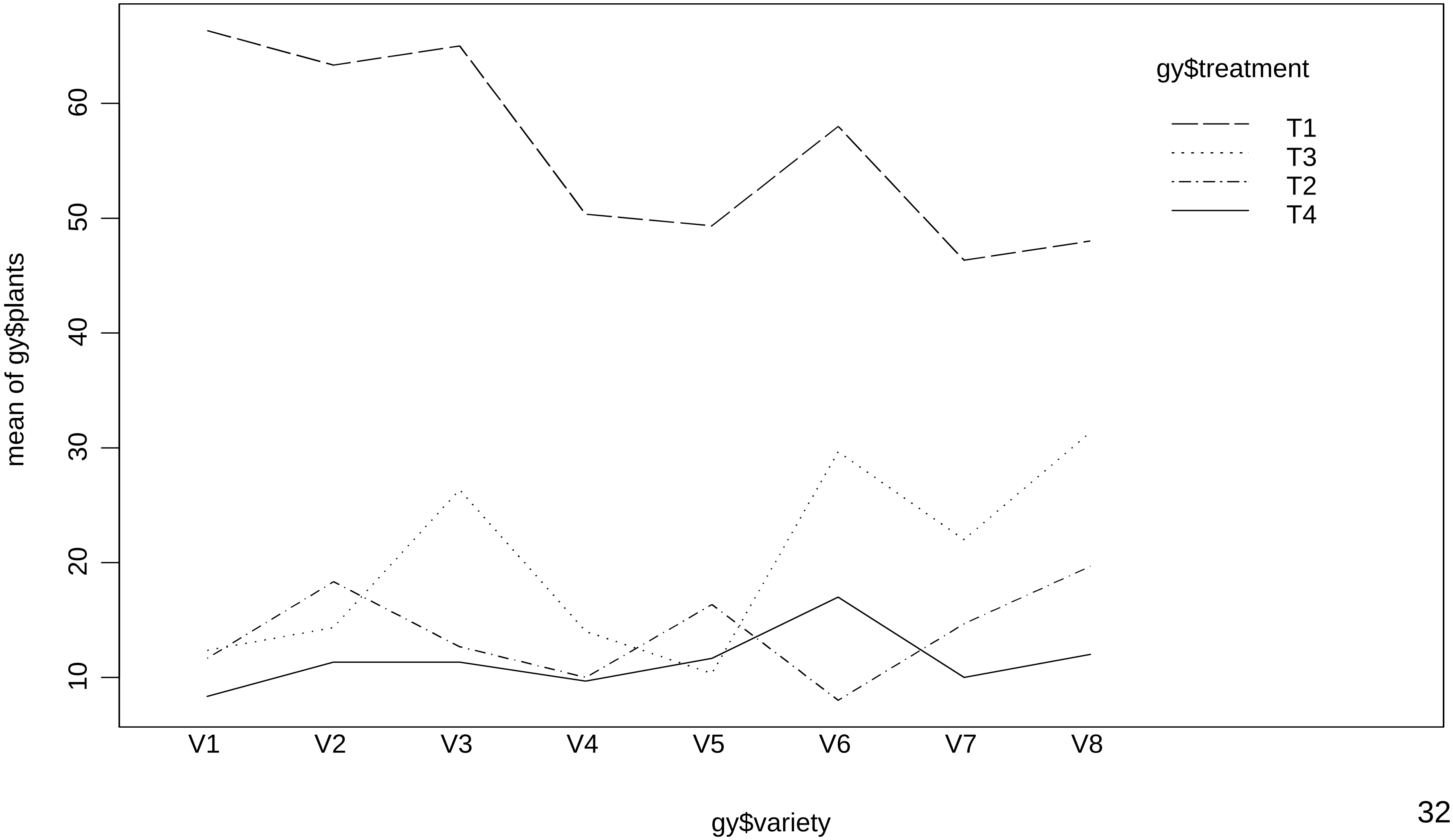
interaction.plot(gy$variety,gy$treatment,gy$plants)



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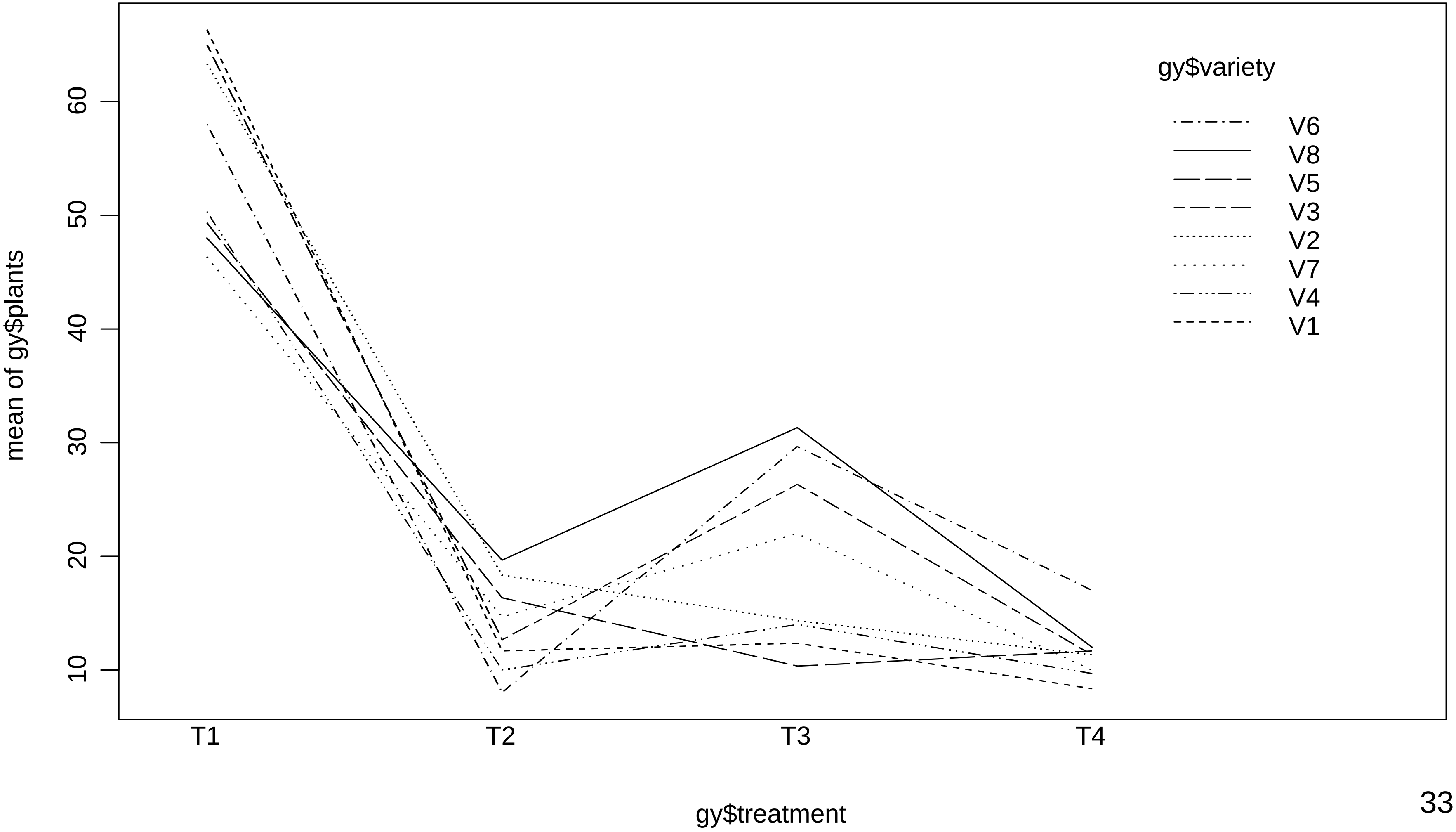
interaction.plot(gy$treatment,gy$variety,gy$plants)



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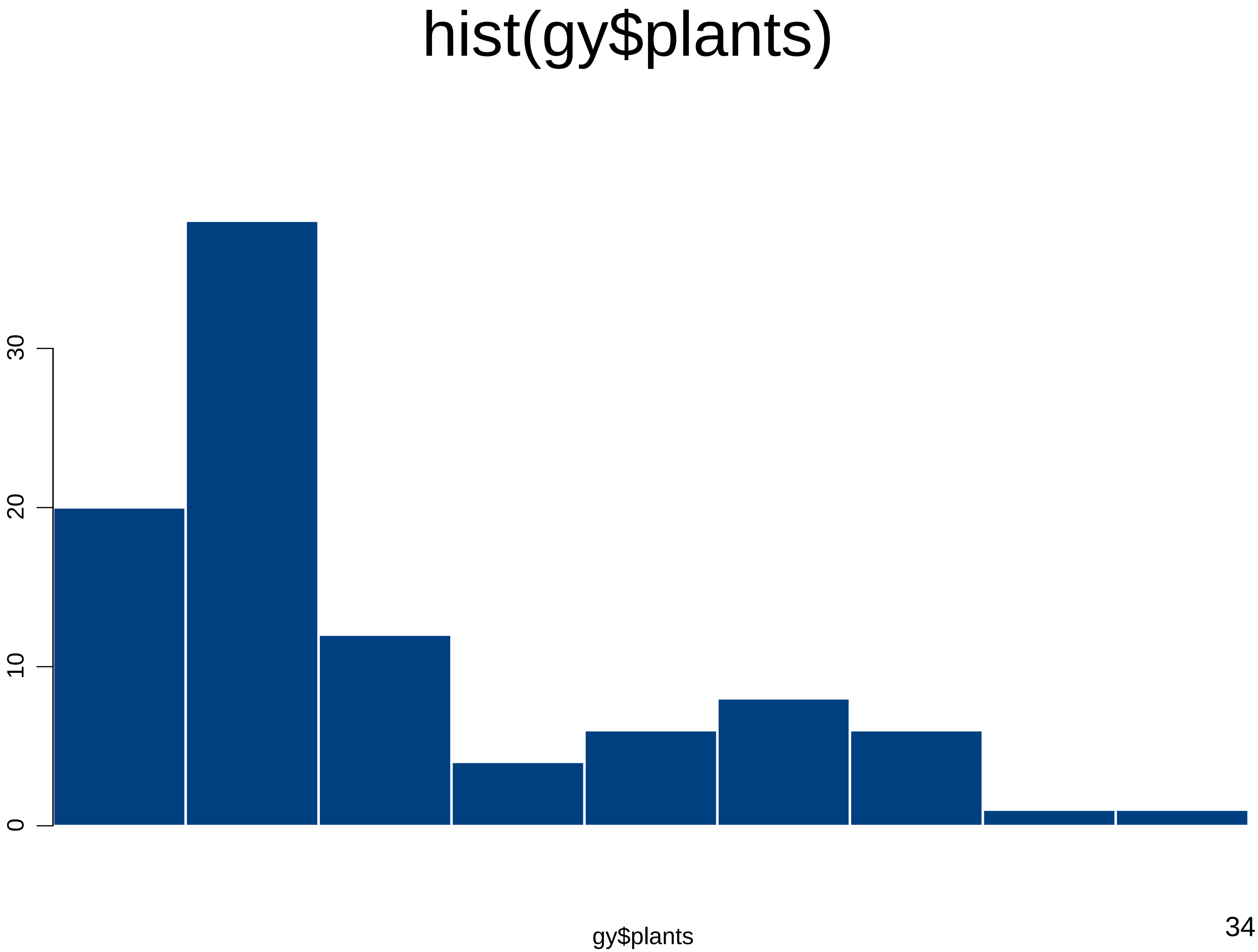
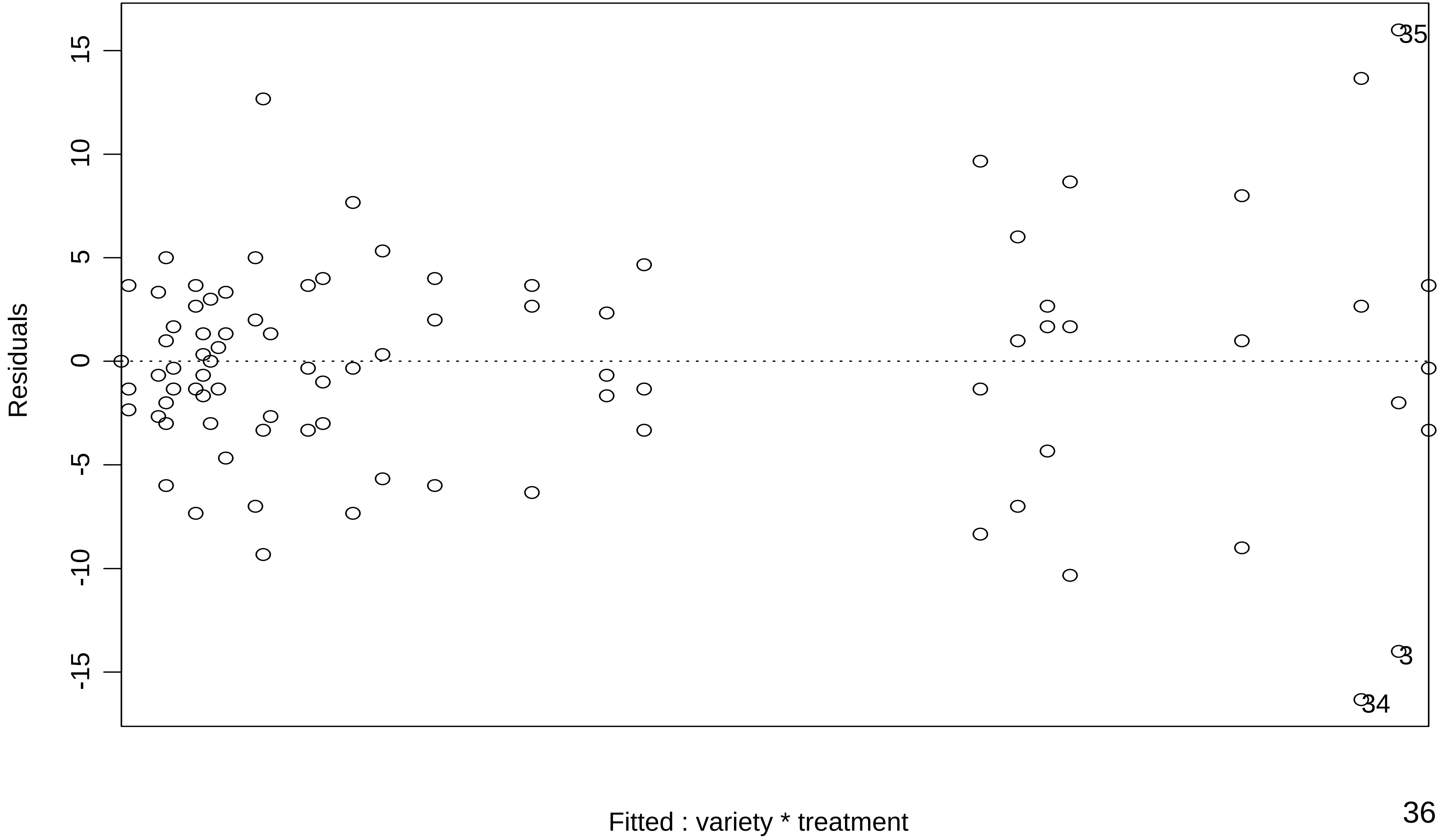

hist(gy$plants)
0
10
20
30
gy$plants

Summaries of gy.aov

```
Call: aov(formula = plants ~ variety * treatment, data = gy)
Residuals:
    Min     1Q     Median   3Q Max
 -16.33 -2.667 1.494e-015 2.75  16

Residual standard error: 6.348 on 64 degrees of freedom
Multiple R-Squared: 0.9298
F-statistic: 27.35 on 31 and 64 degrees of freedom, the p-value
   is 0
> summary(gy.aov)
                  Df Sum of Sq  Mean Sq  F Value      Pr(F)
          variety  7    763.16   109.02   2.7058 0.01604076
        treatment  3  30774.28 10258.09 254.5959 0.00000000
variety:treatment 21   2620.14   124.77   3.0966 0.00026666
        Residuals 64   2578.67    40.29
```

Plot of residual vs. fit for gy data set



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model.tables(gy.aov,type="mean")

```
Tables of means
Grand mean

 25.302

 variety
     V1     V2     V3     V4     V5     V6     V7     V8
 24.667 26.833 28.833 21.000 21.917 28.167 23.250 27.750

 treatment
     T1     T2     T3     T4
 55.833 13.917 20.042 11.417
```

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model.tables(gy.aov,type="mean")

```
 variety:treatment
Dim 1 : variety
Dim 2 : treatment
       T1     T2     T3     T4
V1 66.333 11.667 12.333  8.333
V2 63.333 18.333 14.333 11.333
V3 65.000 12.667 26.333 11.333
V4 50.333 10.000 14.000  9.667
V5 49.333 16.333 10.333 11.667
V6 58.000  8.000 29.667 17.000
V7 46.333 14.667 22.000 10.000
V8 48.000 19.667 31.333 12.000
```

```
 multicomp(gy.aov,focus="treatment")

95 % simultaneous confidence intervals for specified
linear combinations, by the Tukey method

critical point: 2.6378
response variable: plants

intervals excluding 0 are flagged by '****'

      Estimate Std.Error Lower Bound Upper Bound
T1-T2    41.90      1.83       37.10       46.80 ****
T1-T3    35.80      1.83       31.00       40.60 ****
T1-T4    44.40      1.83       39.60       49.30 ****
T2-T3    -6.12      1.83      -11.00       -1.29 ****
T2-T4     2.50      1.83       -2.33        7.33
T3-T4     8.62      1.83        3.79       13.50 ****
```

Guayule ANOVA with variety random

```
> gyr.tab
                  Df Sum of Sq  Mean Sq  F Value     Pr(F)
        treatment  3  30774.28 10258.09 82.21711 0.0000000
          variety  7    763.16   109.02  0.87380 0.5428964
treatment:variety 21   2620.14   124.77  3.09663 0.0002667
        Residuals 64   2578.67    40.29
```

Random if:

- Not interested in those particular factor levels (e.g. batches)
- Levels of factor are randomly chosen from a larger population of factor levels (e.g. 10 universities selected from all universities in country).
- Want to generalize to a larger population of factor levels.

EMS for 2-factor models

(See Table 24.5 on page 981 of Neter et al. *Applied Linear Statistical Models.*)

Nested vs. Crossed Design

(See Figure 28.1 in Neter et al. *Applied Linear Statistical Models.*)

Nested Fixed Factors

(See Table 28.3 on page 1129 of Neter et al. *Applied Linear Statistical Models.*)

Nested Mixed Factors

(See Table 28.5 on page 1133 of Neter et al. *Applied Linear Statistical Models.*)

Cross-Nested Models

(See Table 28.11 on page 1151 of Neter et al. *Applied Linear Statistical Models.*)

Images of book covers:

Patrick O’Brian, *The Commodore.*

Patrick O’Brian, *The Fortune of War*.

Nested Factors

- **Speed of Firing Naval Guns**
- **SUMMARY:**
- The gun data frame, a design object, has 36 rows representing runs of a team of 3 men loading and firing naval guns attempting to get off as many rounds per minute as possible. The three predictor variables (columns) specify the team and the physique of the men on it and the loading method used; the outcome variable is the rounds fired per minute.
- **ARGUMENTS:**
- **Method**
  - factor giving one of two methods for loading rounds into Naval guns. Levels are M1 and M2 .
- **Physique**
  - an ordered factor giving the physique of the men: S for slight, A for average, and H for heavy.
- **Team**
  - factor with levels T1 , T2 or T3 . In fact there are nine teams, three of each physique, i.e. a slight T1 , an average T1 , and a heavy T1 , etc.
- **Rounds**
  - numeric vector giving the number of rounds per minute fired by a team.

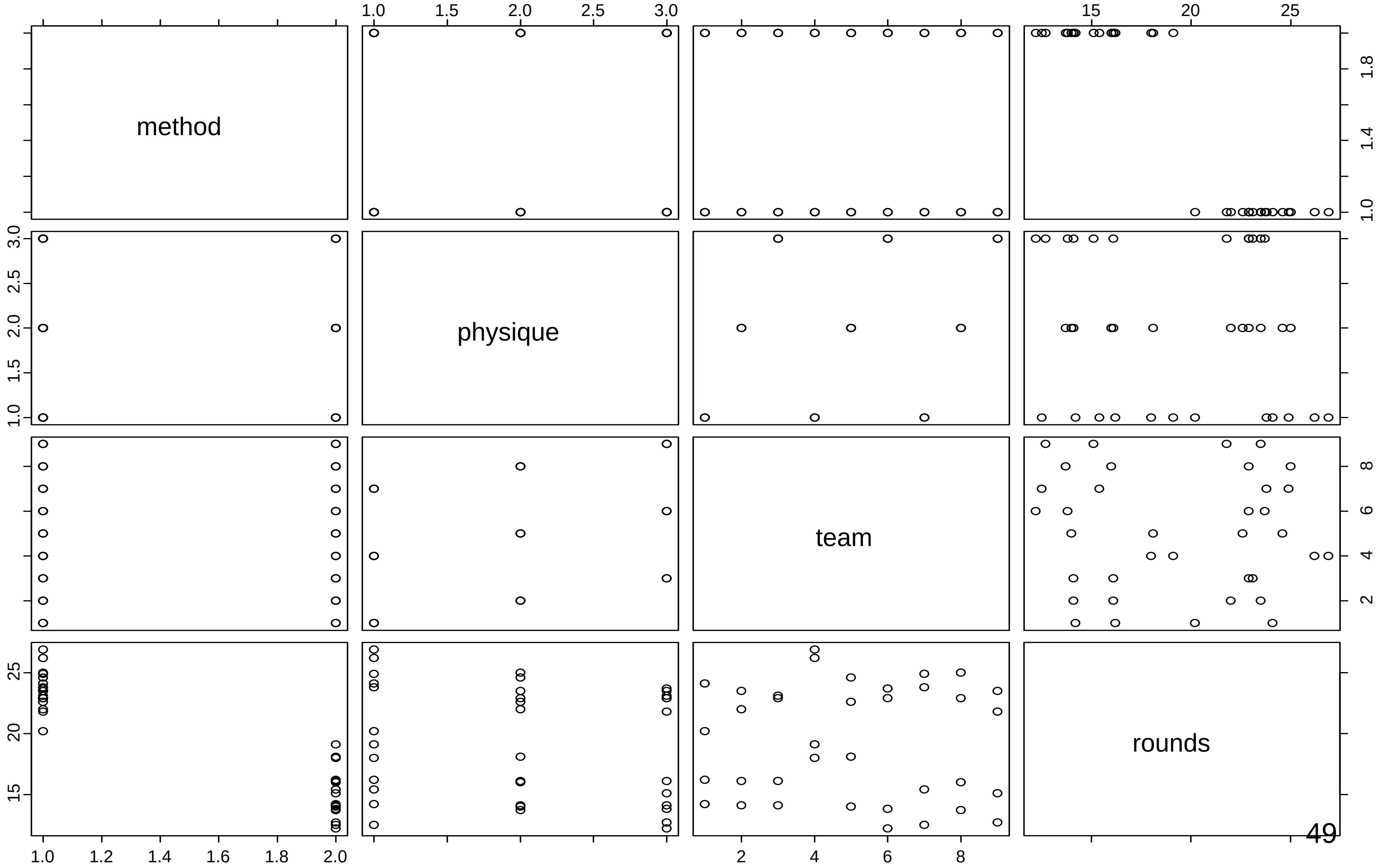
```
gun
  Method Physique Team Rounds
 1     M1        S   T1   20.2
 2     M2        S   T1   14.2
 3     M1        A   T1   22.0
 4     M2        A   T1   14.1
 5     M1        H   T1   23.1
 6     M2        H   T1   14.1
 7     M1        S   T2   26.2
 8     M2        S   T2   18.0
 9     M1        A   T2   22.6
10     M2        A   T2   14.0
11     M1        H   T2   22.9
12     M2        H   T2   12.2
13     M1        S   T3   23.8
14     M2        S   T3   12.5
15     M1        A   T3   22.9
16     M2        A   T3   13.7
17     M1        H   T3   21.8
18     M2        H   T3   12.7
19     M1        S   T1   24.1
20     M2        S   T1   16.2
```

```
 gun
   Method Physique Team Rounds
 1     M1        S   T1   20.2
 2     M2        S   T1   14.2
 3     M1        A   T2   22.0
 4     M2        A   T2   14.1
 5     M1        H   T3   23.1
 6     M2        H   T3   14.1
 7     M1        S   T4   26.2
 8     M2        S   T4   18.0
 9     M1        A   T5   22.6
10     M2        A   T5   14.0
11     M1        H   T6   22.9
12     M2        H   T6   12.2
13     M1        S   T7   23.8
14     M2        S   T7   12.5
15     M1        A   T8   22.9
16     M2        A   T8   13.7
17     M1        H   T9   21.8
18     M2        H   T9   12.7
19     M1        S   T1   24.1
20     M2        S   T1   16.2
```

Speed of firing of naval guns

	Slight	Average	Heavy
Method 1	T1: 20.2, 24.1 T4: 26.2, 26.9 T7: 23.8, 24.9	T2: 22.0, 23.5 T5: 22.6, 24.6 T8: 22.9, 25.0	T3: 23.1, 22.9 T6: 22.9, 23.7 T9: 21.8, 23.5
Method 2	T1: 14.2, 16.2 T4: 18.0, 19.1 T7: 12.5, 15.4	T2: 14.1, 16.1 T5: 14.0, 18.1 T8: 13.7, 16.0	T3: 14.1, 16.1 T6: 12.2, 13.8 T9: 12.7, 15.1

pairs(gun2)



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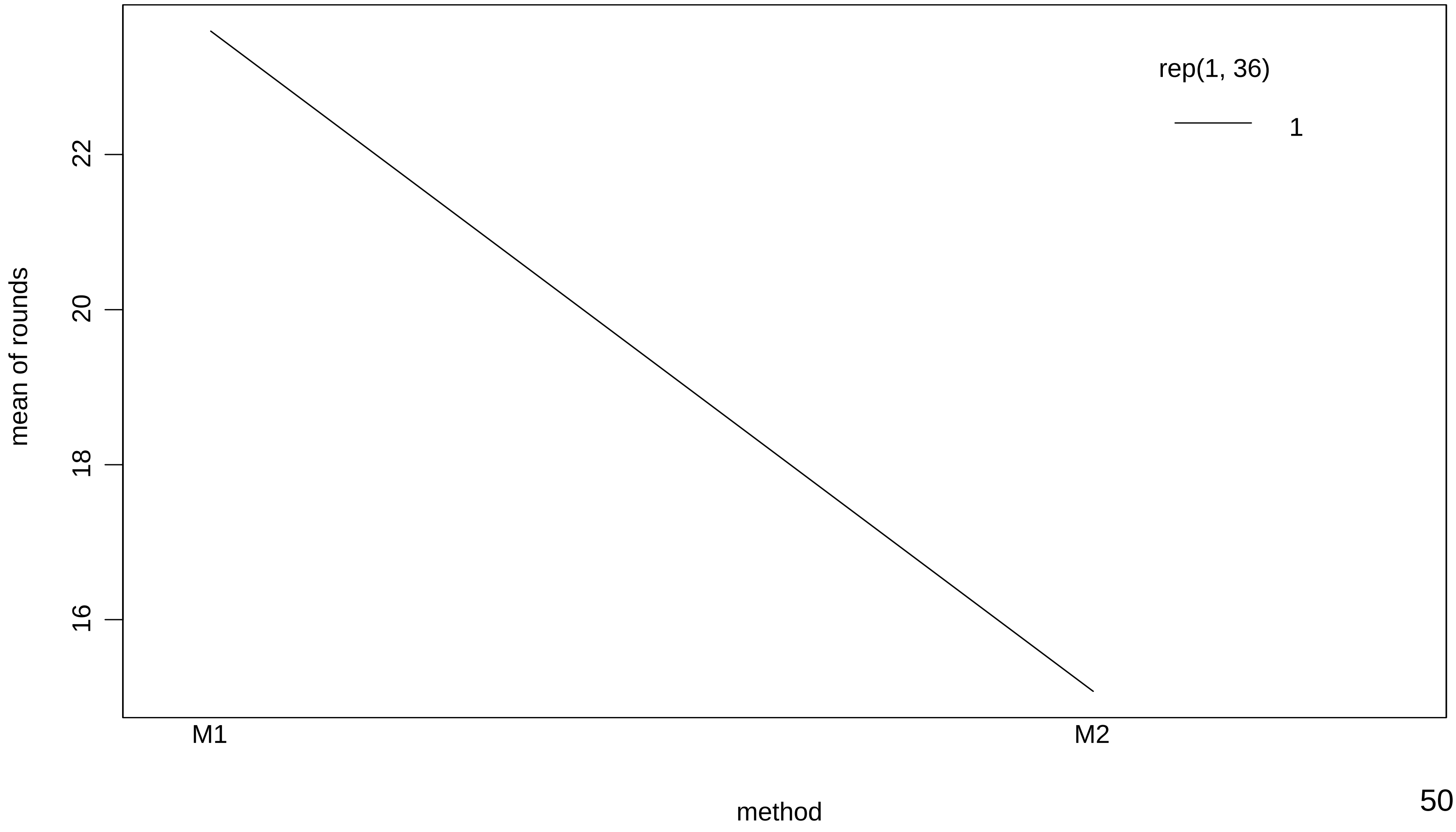
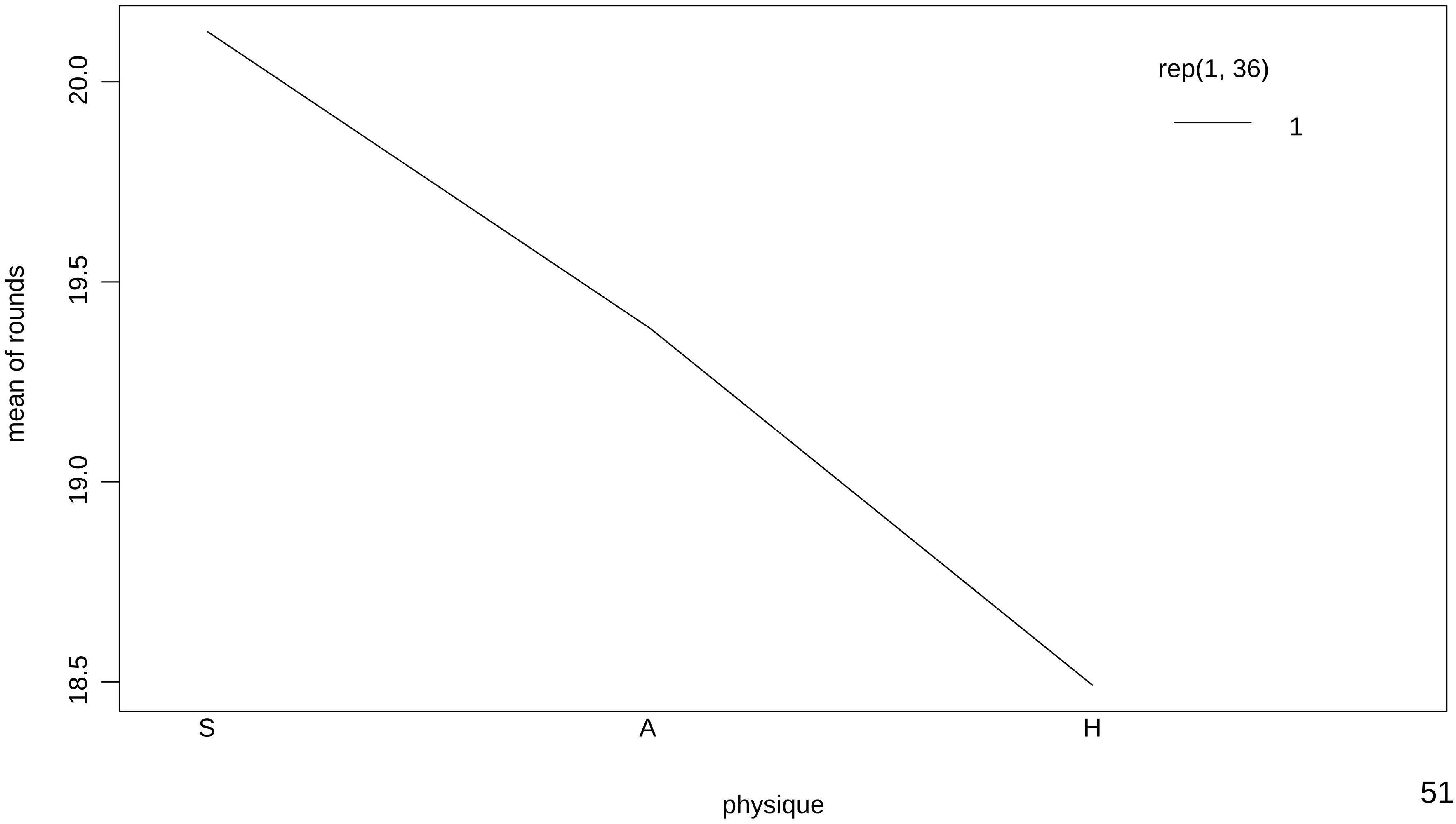
Method Effect



Physique Effect



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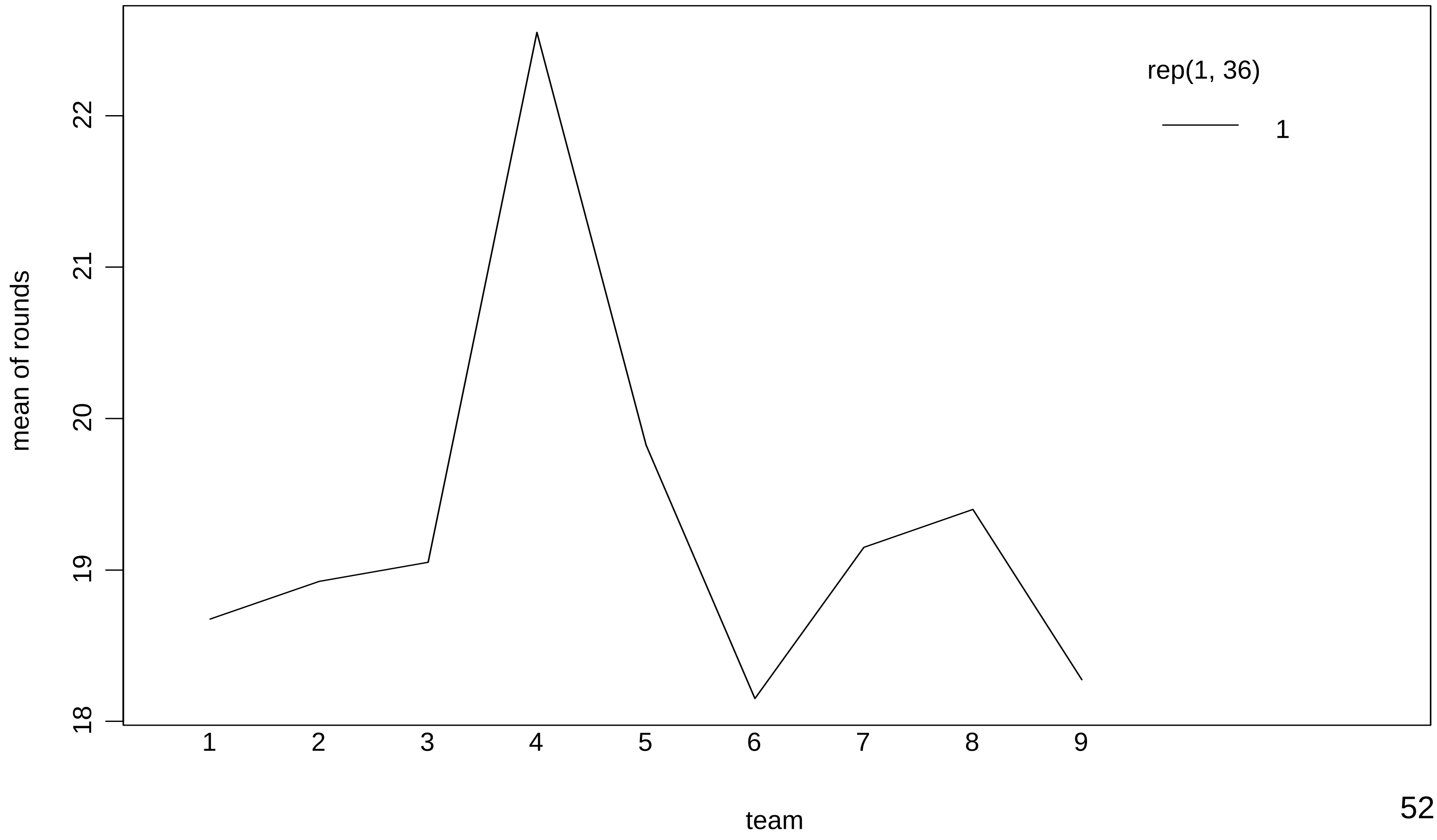
Team Effect



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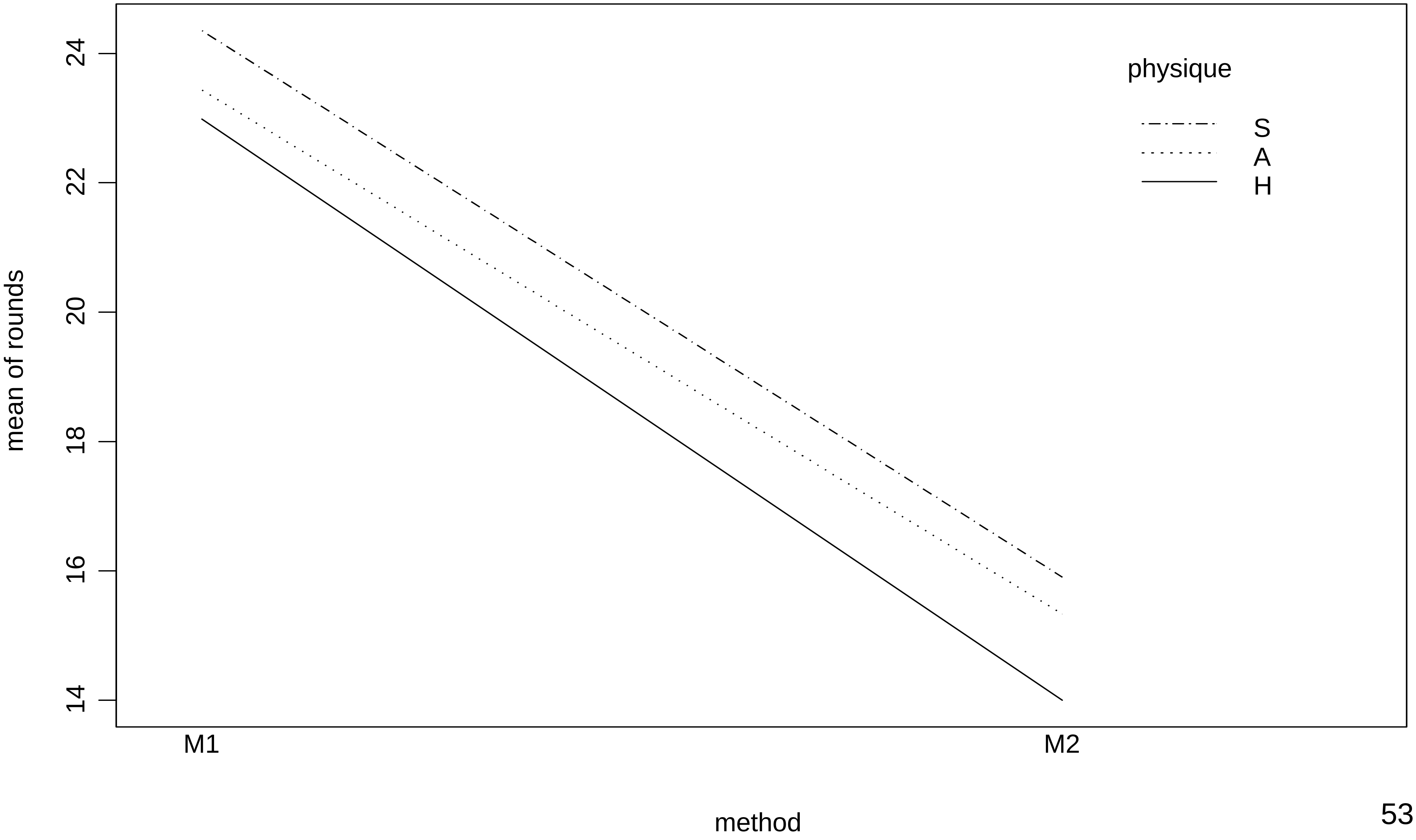
Method-Physique Interaction



ANOVA tables for firing of naval guns example (with teams numbered 1-9)

```
> summary(aov(rounds~phys*meth*team))
                     Df Sum of Sq  Mean Sq  F Value     Pr(F)
                phys  2   16.0517   8.0258   3.4736 0.0529995
                meth  1  651.9511 651.9511 282.1621 0.0000000
                team  6   39.2583   6.5431   2.8318 0.0403140
           phys:meth  2    1.1872   0.5936   0.2569 0.7762240
           meth:team  6   10.7217   1.7869   0.7734 0.6009376
           Residuals 18   41.5900   2.3106

> summary(aov(rounds~phys*meth*team%in%phys))
                     Df Sum of Sq  Mean Sq  F Value     Pr(F)
                phys  2   16.0517   8.0258   3.4736 0.0529995
                meth  1  651.9511 651.9511 282.1621 0.0000000
           phys:meth  2    1.1872   0.5936   0.2569 0.7762240
      team %in% phys  6   39.2583   6.5431   2.8318 0.0403140
meth:(team %in% phys)  6   10.7217   1.7869   0.7734 0.6009376
           Residuals 18   41.5900   2.3106
```

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```
> model.tables(gunaov,type="mean")
Tables of means
Grand mean

 19.333

 Method
     M1     M2
 23.589 15.078

 Physique
      S      A      H
 20.125 19.383 18.492

 Team %in% Physique
Dim 1 : Physique
Dim 2 : Team
      T1     T2     T3
S 18.675 22.550 19.150
A 18.925 19.825 19.400
H 19.050 18.150 18.275
```

```
Tables of means
Grand mean

 19.333

 method
        M1     M2
    23.589 15.078
rep 18.000 18.000

 physique
         S      A      H
    20.125 19.383 18.492
rep 12.000 12.000 12.000

 team %in% physique
Dim 1 : physique
Dim 2 : team
         1      2      3      4      5      6      7      8      9
  S 18.675               22.550               19.150
rep  4.000  0.000  0.000  4.000  0.000  0.000  4.000  0.000  0.000
  A        18.925               19.825               19.400
rep  0.000  4.000  0.000  0.000  4.000  0.000  0.000  4.000  0.000
  H               19.050               18.150               18.275
rep  0.000  0.000  4.000  0.000  0.000  4.000  0.000  0.000  4.000
```

Summaries of firing of naval guns example (without interaction)

```
Call: aov(formula = Rounds ~ Method + Physique/Team, data = gun)

Residuals:
    Min      1Q      Median      3Q   Max
 -2.731 -0.7368 2.498e-016 0.9972 2.531

Residual standard error: 1.434 on 26 degrees of freedom
Multiple R-Squared: 0.9297
F-statistic: 38.19 on 9 and 26 degrees of freedom, the p-value is
   9.602e-013

> summary(gunaov)
                   Df Sum of Sq  Mean Sq  F Value      Pr(F)
            Method  1  651.9511 651.9511 316.8426 0.00000000
          Physique  2   16.0517   8.0258   3.9005 0.03300457
Team %in% Physique  6   39.2583   6.5431   3.1799 0.01782181
         Residuals 26   53.4989   2.0576
```

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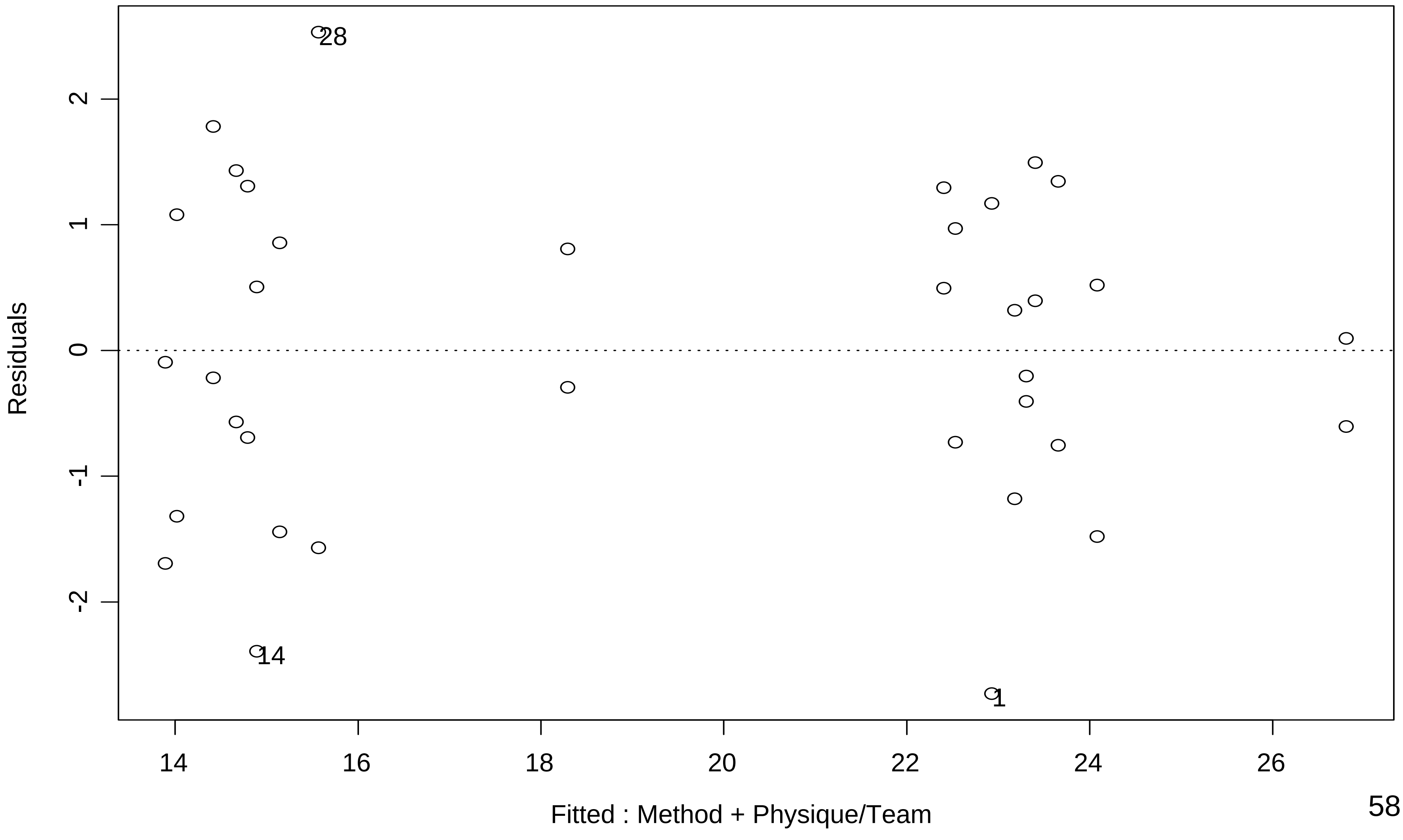
Plot of residual vs fit for gun.aov



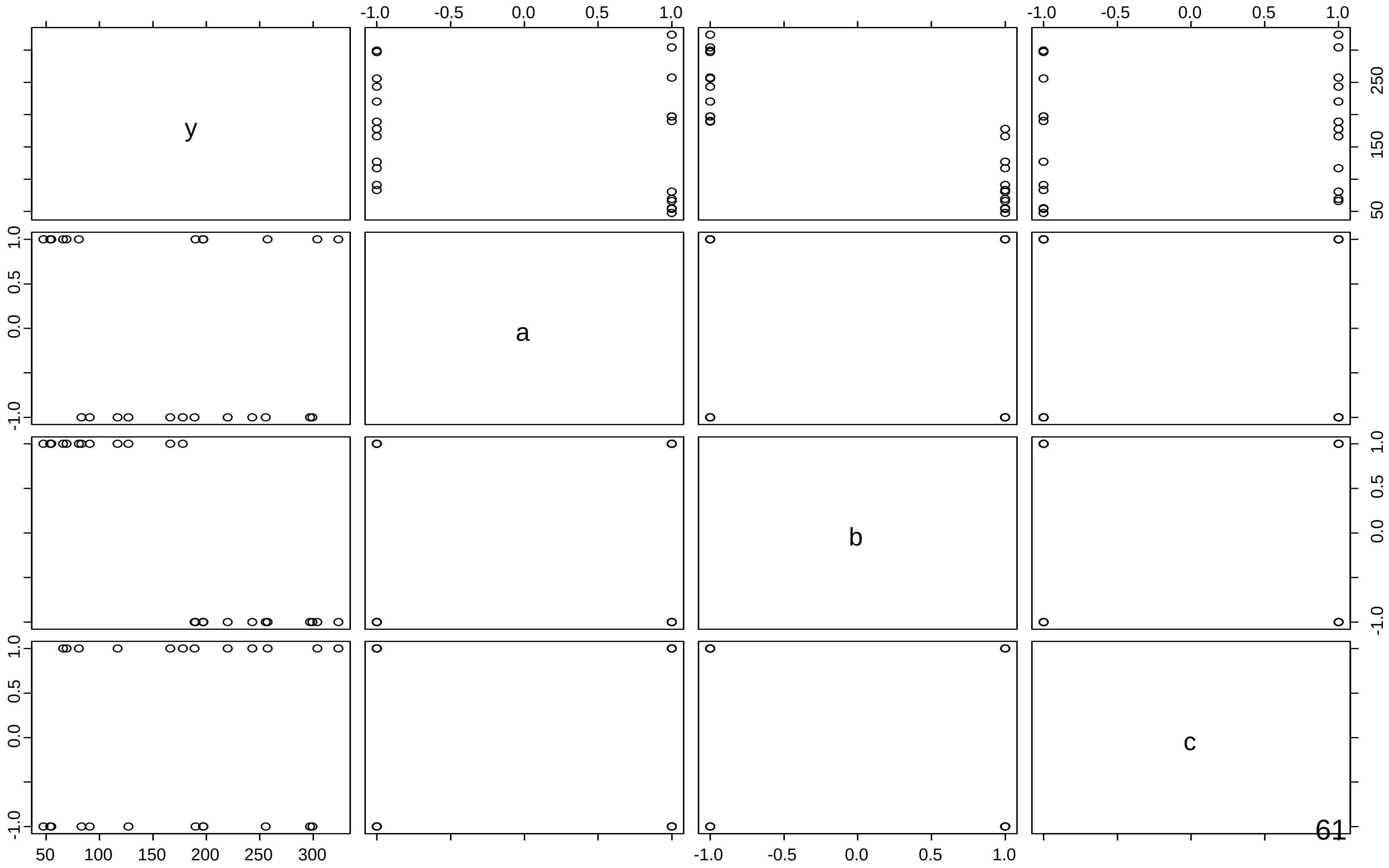
2^k Factorial Designs

- Exploratory experimental studies.
- Multifactor experiment in which each factor studied at two levels.
- Used to screen large number of factors to identify the most important.
- Sometimes 2 levels naturally occur e.g. present or absent, smoker or non-smoker
- k factors => 2^k treatment combinations

2^k Factorial Design Example

Example: 13.19, page 553 of the course textbook.

pairs(nw.df)



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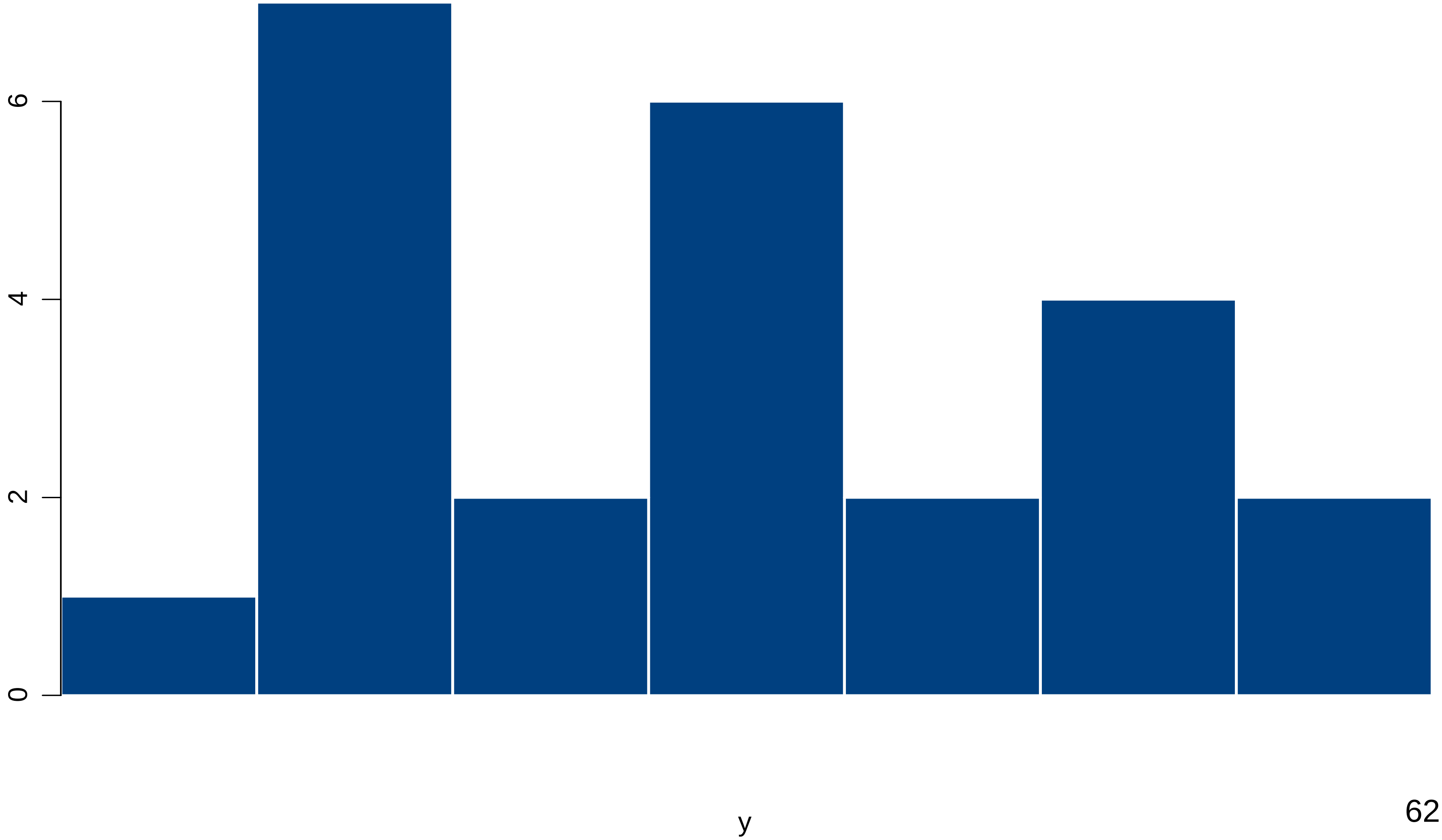
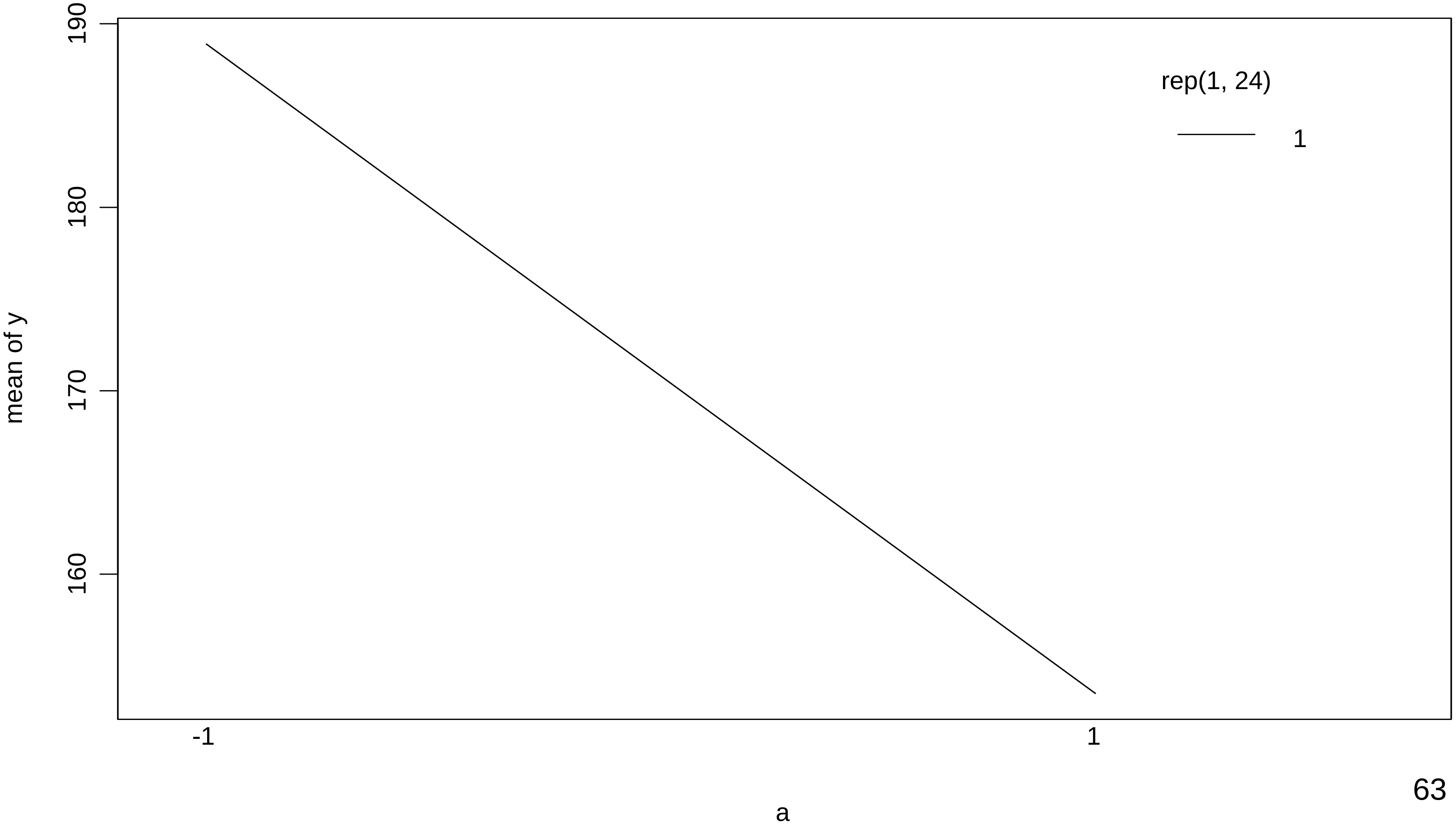
hist(y)



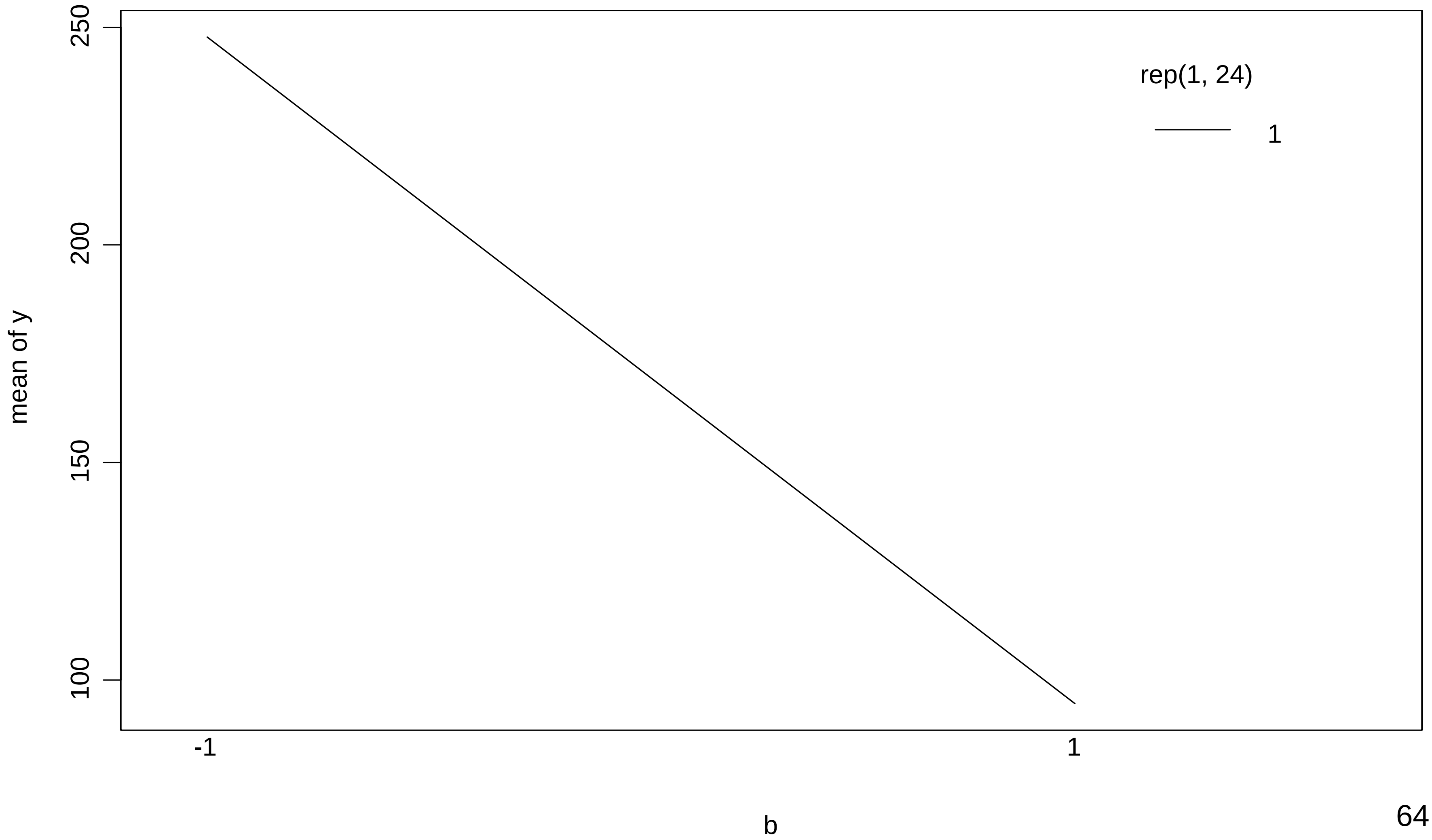
This graph was created using S-PLUS(R) Software. S-PLUS(R) is a registered trademark of Insightful Corporation.

Effect of a



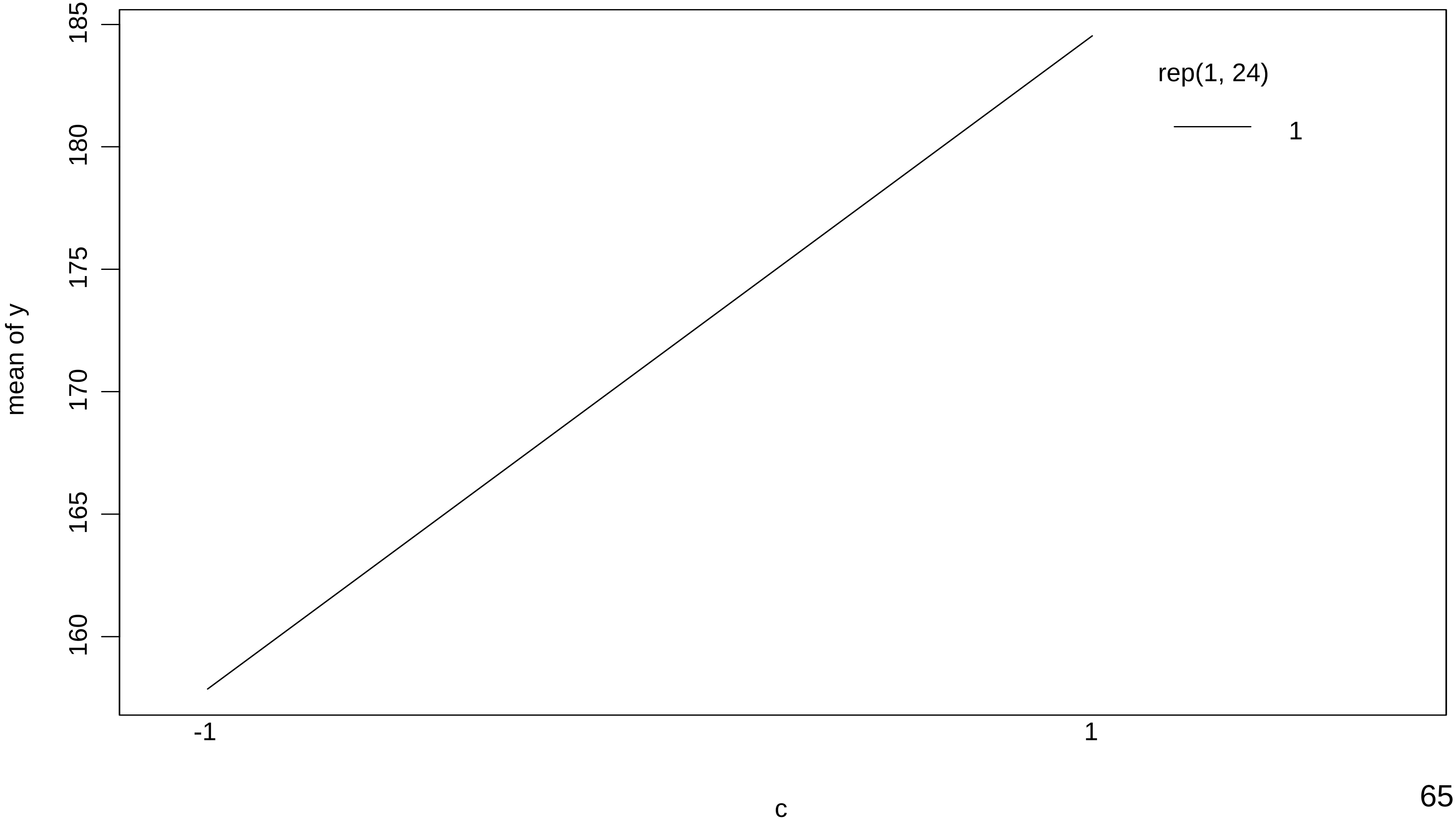
This graph was created using S-PLUS(R) Software. S-PLUS(R) is a registered trademark of Insightful Corporation.

Effect of b

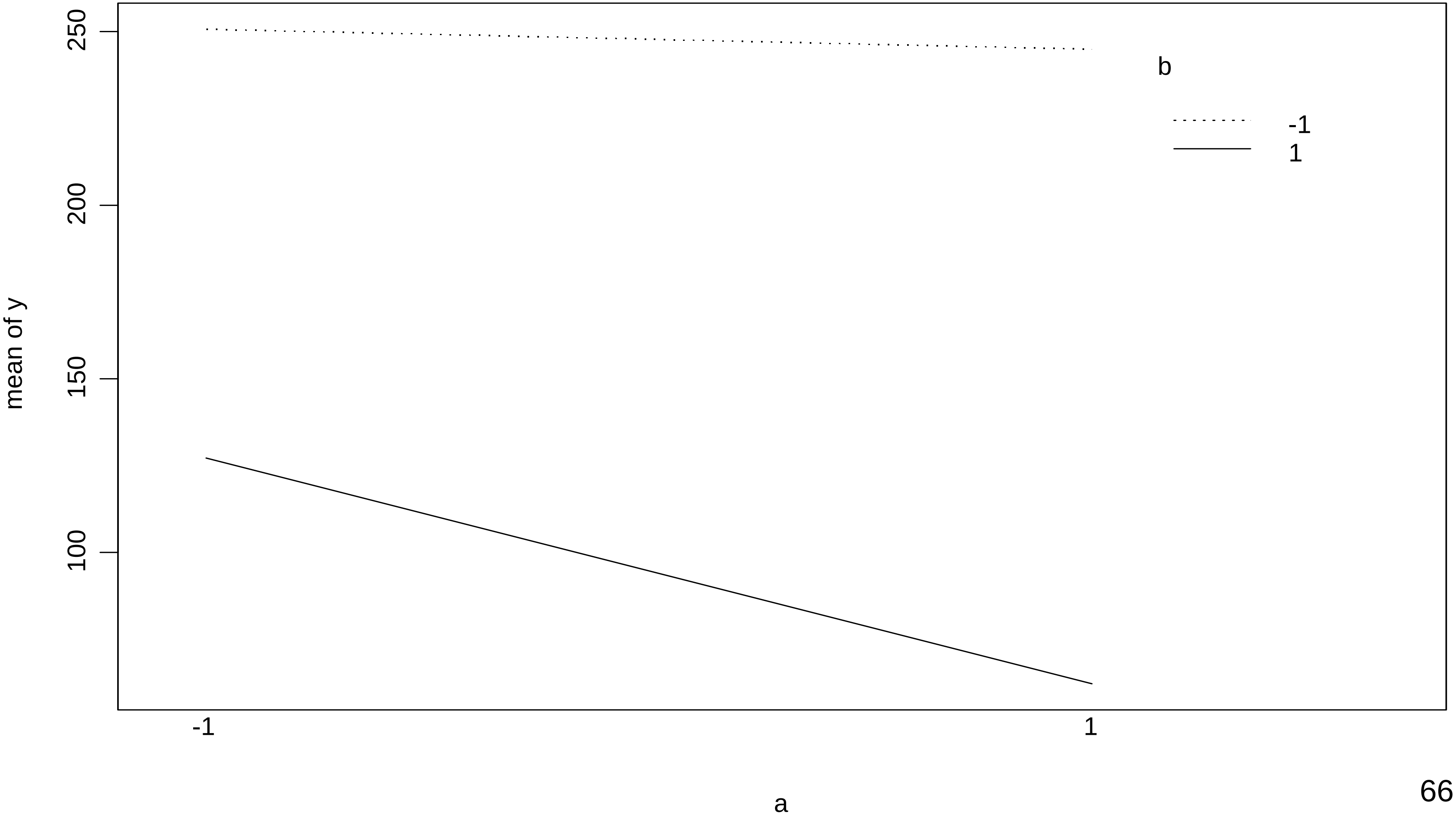


This graph was created using S-PLUS(R) Software. S-PLUS(R) is a registered trademark of Insightful Corporation.

Effect of c



interaction.plot(a,b,y)



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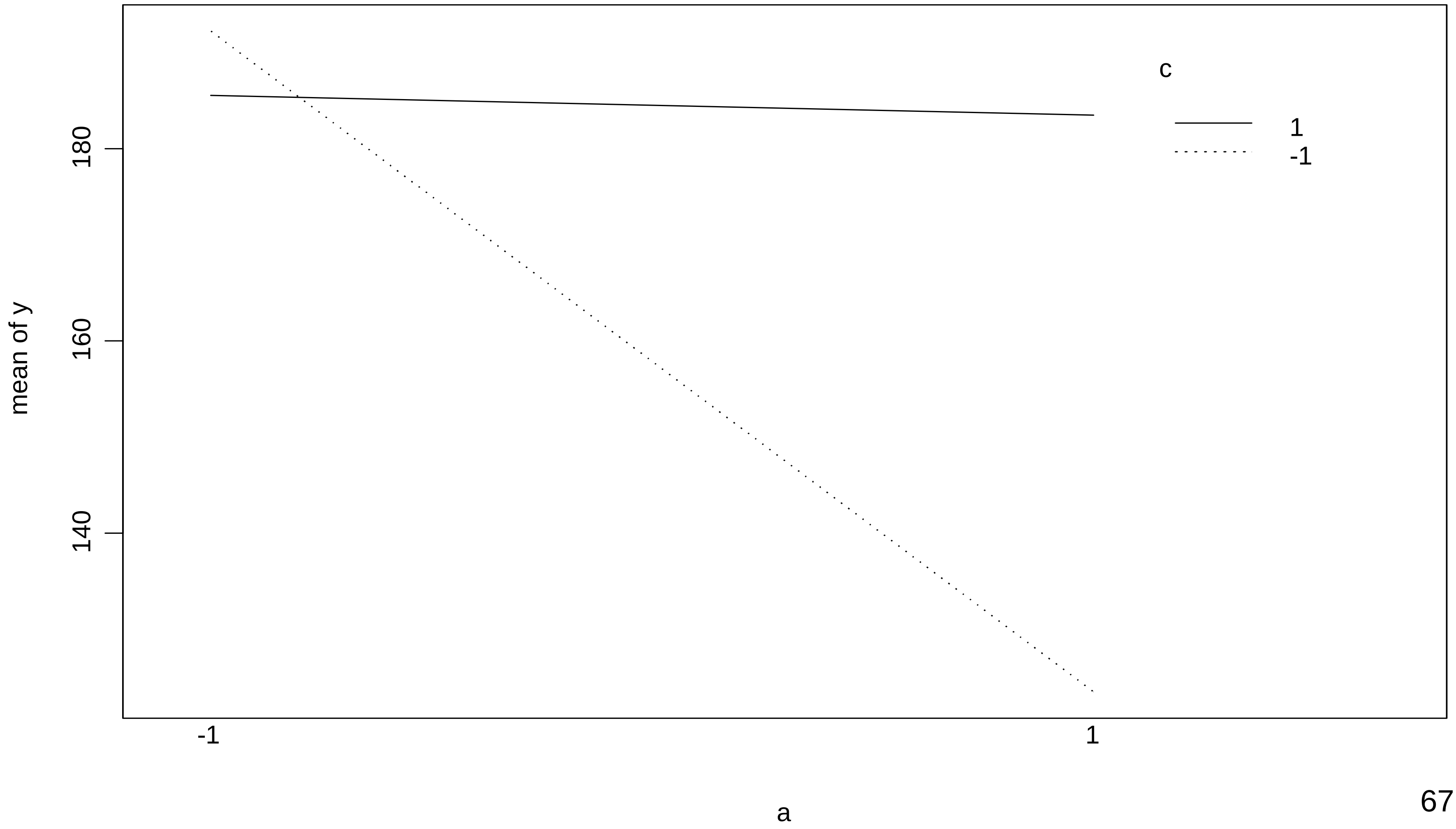
interaction.plot(a,c,y)



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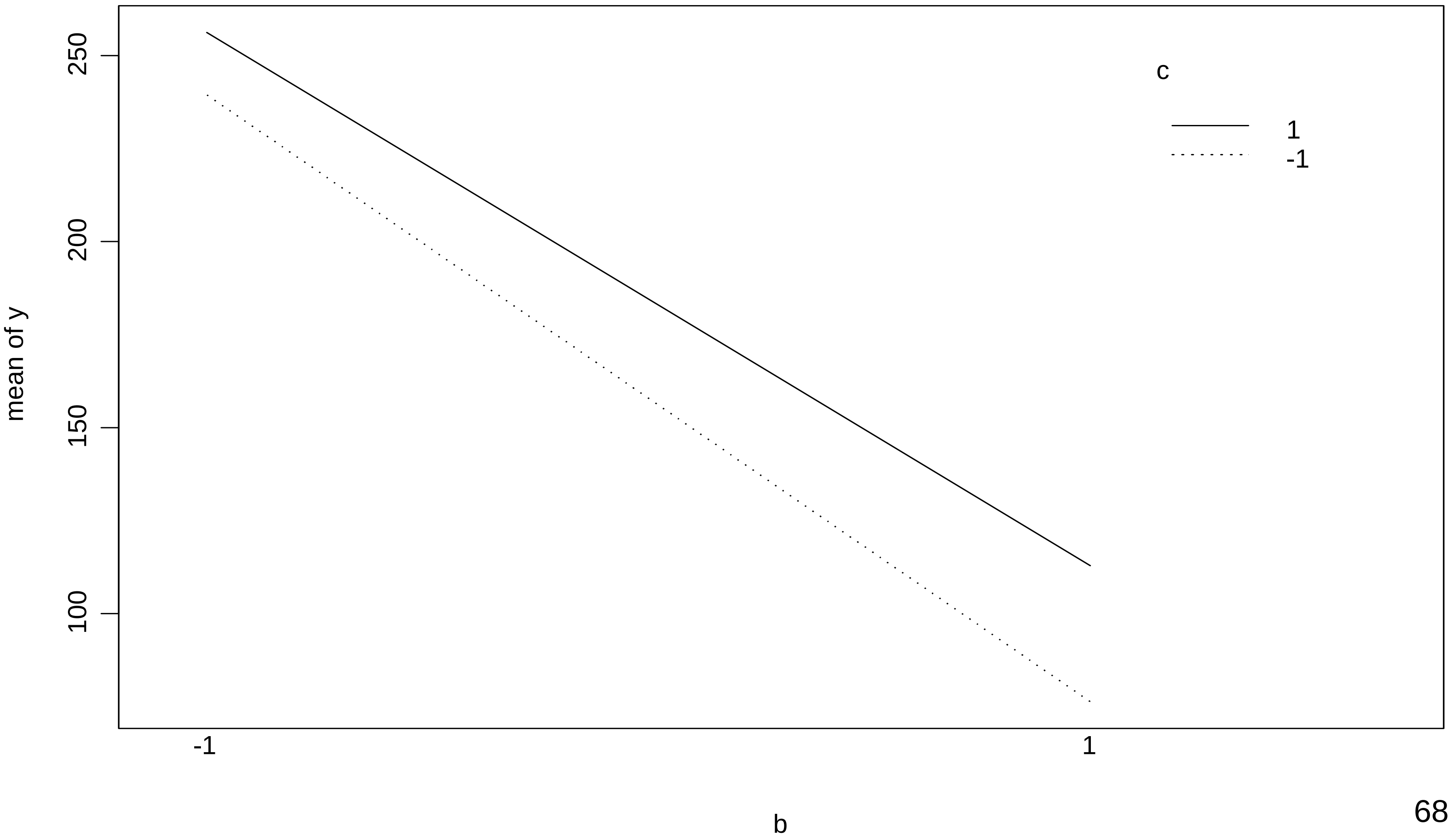
interaction.plot(b,c,y)



This graph was created using S-PLUS(R) Software. S-PLUS(R) is a registered trademark of Insightful Corporation.

summary.lm(nw.aov)

```
Call: aov(formula = y ~ a * b * c, data = nw.df)
Residuals:
    Min     1Q Median    3Q   Max
 -37.67 -6.861  2.388 12.67 28.67

Coefficients:
                Value Std. Error   t value  Pr(>|t|)
(Intercept)  171.1942    4.6675    36.6780    0.0000
          a  -17.6942    4.6675    -3.7909    0.0016
          b  -76.5833    4.6675   -16.4078    0.0000
          c   13.3333    4.6675     2.8566    0.0114
        a:b  -14.8050    4.6675    -3.1719    0.0059
        a:c   16.6667    4.6675     3.5708    0.0026
        b:c    4.9442    4.6675     1.0593    0.3052
      a:b:c  -25.0558    4.6675    -5.3682    0.0001

Residual standard error: 22.87 on 16 degrees of freedom
Multiple R-Squared: 0.9556
F-statistic: 49.21 on 7 and 16 degrees of freedom, the p-value is
   1.209e-009
```

Effect (of going from low to high level) is 2*regression coefficient

```
model.matrix(nw.aov)
  (Intercept)  a  b  c a:b a:c b:c a:b:c
 1          1 -1 -1 -1   1   1   1    -1
 2          1  1 -1 -1  -1  -1   1     1
 3          1 -1  1 -1  -1   1  -1     1
 4          1 -1 -1  1   1  -1  -1     1
 5          1  1  1 -1   1  -1  -1    -1
 6          1  1 -1  1  -1   1  -1    -1
 7          1 -1  1  1  -1  -1   1    -1
 8          1  1  1  1   1   1   1     1
 9          1 -1 -1 -1   1   1   1    -1
10          1  1 -1 -1  -1  -1   1     1
11          1 -1  1 -1  -1   1  -1     1
12          1 -1 -1  1   1  -1  -1     1
13          1  1  1 -1   1  -1  -1    -1
14          1  1 -1  1  -1   1  -1    -1
15          1 -1  1  1  -1  -1   1    -1
16          1  1  1  1   1   1   1     1
17          1 -1 -1 -1   1   1   1    -1
18          1  1 -1 -1  -1  -1   1     1
19          1 -1  1 -1  -1   1  -1     1
20          1 -1 -1  1   1  -1  -1     1
21          1  1  1 -1   1  -1  -1    -1
22          1  1 -1  1  -1   1  -1    -1
23          1 -1  1  1  -1  -1   1    -1
24          1  1  1  1   1   1   1     1
```

X’X Matrix

```
t(X)%*%X
            (Intercept)  a  b  c a:b a:c b:c a:b:c
(Intercept)          24  0  0  0   0   0   0     0
          a           0 24  0  0   0   0   0     0
          b           0  0 24  0   0   0   0     0
          c           0  0  0 24   0   0   0     0
        a:b           0  0  0  0  24   0   0     0
        a:c           0  0  0  0   0  24   0     0
        b:c           0  0  0  0   0   0  24     0
      a:b:c           0  0  0  0   0   0   0    24
```

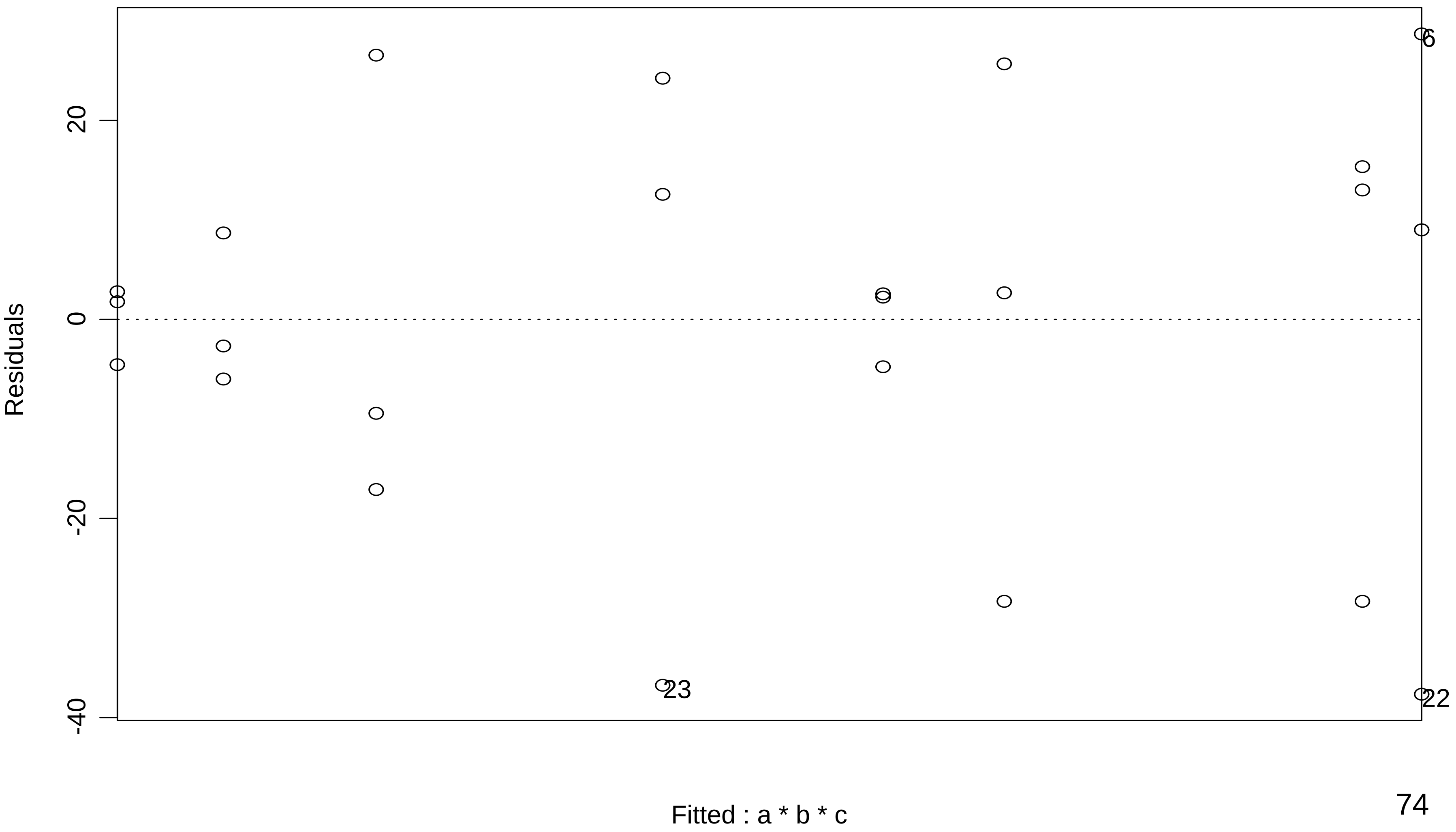
$n*(X'X)^{-1} X'$

```
> solve(t(X)%*%X)%*%t(X)*24
              1  2  3  4  5  6  7 8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
(Intercept)  1  1  1  1  1  1  1 1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1  1
          a -1  1 -1 -1  1  1 -1 1 -1  1 -1 -1  1  1 -1  1 -1  1 -1 -1  1  1 -1  1
          b -1 -1  1 -1  1 -1  1 1 -1 -1  1 -1  1 -1  1  1 -1 -1  1 -1  1 -1  1  1
          c -1 -1 -1  1 -1  1  1 1 -1 -1 -1  1 -1  1  1  1 -1 -1 -1  1 -1  1  1  1
        a:b  1 -1 -1  1  1 -1 -1 1  1 -1 -1  1  1 -1 -1  1  1 -1 -1  1  1 -1 -1  1
        a:c  1 -1  1 -1 -1  1 -1 1  1 -1  1 -1 -1  1 -1  1  1 -1  1 -1 -1  1 -1  1
        b:c  1  1 -1 -1 -1 -1  1 1  1  1 -1 -1 -1 -1  1  1  1  1 -1 -1 -1 -1  1  1
      a:b:c -1  1  1  1 -1 -1 -1 1 -1  1  1  1 -1 -1 -1  1 -1  1  1  1 -1 -1 -1  1
```

summary(nw.aov)

```
> summary(nw.aov)
          Df Sum of Sq  Mean Sq  F Value     Pr(F)
        a  1    7514.0   7514.0  14.3712 0.0016031
        b  1  140760.2 140760.2 269.2166 0.0000000
        c  1    4266.7   4266.7   8.1604 0.0114229
      a:b  1    5260.5   5260.5  10.0612 0.0059164
      a:c  1    6666.7   6666.7  12.7506 0.0025519
      b:c  1     586.7    586.7   1.1221 0.3052037
    a:b:c  1   15067.1  15067.1  28.8171 0.0000628
Residuals 16    8365.6    522.9
```

Plot of residual vs. fit for nw.aov



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