

16 May 2003

Summary of discussion of example 11.5:

```
> x
  3  2  1  0 -1 -2 -3

Y
11 18 18 10  7 -2 -13

> X<-cbind(1,x)
> X

  1  3
  1  2
  1  1
  1  0
  1 -1
  1 -2
  1 -3

n<-nrow(X) # (n=7)

p<-ncol(X) # (p=2)

> XtX<-t(X)%*%X
> XtX-
  7  0
  0 28

> XtXi<-solve(XtX)
> XtXi

  0.1428571 0.0000000
  0.0000000 0.03571429

> Xty<-t(X)%*%y
> Xty
  49
 123

> beta<-XtXi%*%Xty
> beta

  7.000000
  4.392857

> yhat<-X%*%beta
> yhat

20.178571
15.785714
11.392857
 7.000000
 2.607143
-1.785714
-6.178571

> res<-y-yhat
> res

-9.1785714
 2.2142857
 6.6071429
 3.0000000
 4.3928571
-0.2142857
-6.8214286
```

```

SSE<-sum(res^2)
SSE
207.6786

> MSE<-SSE/(n-p)
> MSE
41.53571

> SEbeta<-sqrt(MSE*diag(XtXi))
> SEbeta
2.435913 1.217957

> tvalue<-beta/SEbeta
> tvalue
  2.873666
  3.606743

> pvalue<-2*(1-pt(tvalue,n-p))
> pvalue
0.03484499 0.01543324

> cbind(beta,SEbeta,tvalue,pvalue)
      SEbeta      pvalue
7.000000 2.435913 2.873666 0.03484499
x 4.392857 1.217957 3.606743 0.01543324

>SST<-sum((y-mean(y))^2)
>SST
748

> SSR<-sum((yhat-mean(y))^2)
> SSR
540.3214

>MSR<-SSR/(p-1)
>MSR
540.3214

> summary(lm(y~x))

Call: lm(formula = y ~ x)
Residuals:
    1     2     3     4     5     6     7
-9.179  2.214  6.607  3  4.393 -0.2143 -6.821

Coefficients:
            Value Std. Error t value Pr(>|t|)
(Intercept)  7.0000  2.4359     2.8737  0.0348
            x  4.3929  1.2180     3.6067  0.0154

Residual standard error: 6.445 on 5 degrees of freedom #=sqrt(MSE)
Multiple R-Squared: 0.7224 # (=SSR/SST)
F-statistic: 13.01 on 1 and 5 degrees of freedom, the p-value is 0.01543
#(F statistic=MSR/MSE)

Correlation of Coefficients:
(Intercept)
0

> summary.aov(lm(y~x))
      Df Sum of Sq Mean Sq F Value    Pr(F)
x      1  540.3214  540.3214  13.0086 0.01543324
Residuals  5  207.6786  41.5357

```

Looking at the plot below, perhaps we should have fit a model with an  $x^2$  term.

If you have time, try to generate these tables yourself.

```
> summary.aov(lm(y~x+x^2))
      Df Sum of Sq Mean Sq F Value    Pr(F)
x      1  540.3214  540.3214  99.75165 0.000564715
I(x^2) 1  186.0119  186.0119  34.34066 0.004232515
Residuals 4    21.6667    5.4167
```

```
> summary(lm(y~x+x^2))
```

Call: lm(formula = y ~ x + x^2)

Residuals:

```
      1      2      3      4      5      6      7
-1.738  2.214  2.143 -2.952 -0.07143 -0.2143  0.619
```

Coefficients:

```
              Value Std. Error t value Pr(>|t|)
(Intercept)  12.9524   1.3437    9.6393  0.0006
x              4.3929   0.4398    9.9876  0.0006
I(x^2)       -1.4881   0.2539   -5.8601  0.0042
```

Residual standard error: 2.327 on 4 degrees of freedom

Multiple R-Squared: 0.971

F-statistic: 67.05 on 2 and 4 degrees of freedom, the p-value is 0.000839

Correlation of Coefficients:

```
      (Intercept)      x
x      0.0000
I(x^2) -0.7559      0.0000
```

