

14.41, 2002

Midterm Solutions

1. Uncertain. The method described here is referred to as *compensating differentials*. It can be used to determine the value of life. The technique requires comparing the wage differential at two jobs identical in all regards except for the risk of death. There are three significant problems with these calculations:
  - Most significantly, the method ignores the fact that there is a distribution of risk tolerance in the population. Workers sort into jobs based on their tolerance for risk. Workers who have a high risk tolerance for risk, and hence place a low value on life, sort into risky jobs. Those with low risk tolerance sort into safe jobs. When the wage difference is taken between jobs identical except for the level of risk, the compensating differential for risk is based on those with an abnormally high tolerance for risk. The resulting estimate of the value of life will be lower than the estimate for the average individual in the population – the estimate is downward biased. In your answer, you needed to state BOTH the point about a distribution of risk tolerance in the population and that this leads to a downward bias.
  - As stated above, all characteristics of the jobs being compared, other than the risk of death, must be identical. In practice, these type comparisons, referred to as hedonic analysis, are impossible. Jobs differ in numerous ways, many of which are unobservable to the economist computing the compensating differential.
  - Workers may not have complete information on the riskiness of jobs. If they lack complete information, they may not demand the appropriate compensating differential.
2. False. The Coase Theorem says that, given *property rights* and *costless bargaining*, externalities can be internalized. In this case, we have neither clear property rights nor costless bargaining. There are no clear property rights over the river that is being contaminated. The large number of homeowners and the difficulty in proving how much each homeowner is polluting (an assignment problem) would make bargaining difficult and costly.

A very interesting point was made by a single student. Property rights ARE well defined for the homes. Therefore the fisherman could pay the homeowners to not use fertilizer.
3. Uncertain. This depends crucially on which model you use. If you use the standard *exponential discounting* model, then smokers are time consistent and taxes are too high. If you use the *hyperbolic discounting* model, then smokers are *time inconsistent*. Time inconsistent smokers may value the higher taxes as a *commitment device*. Furthermore, there is likely no private market for commitment devices. In this case, the higher tax may be optimal.
4. Uncertain. There are two possible models to use here. Under the human capital model of education, education increases human capital. Wages reflect marginal productivity which is a function of the level of human capital obtained by the individual. Under this model, an increase in wages resulting from graduating

college suggests that college increases human capital. Under the signaling/screening model of education, education does not increase human capital. Education merely identifies those with high ability without increasing their marginal productivity or human capital.

5. True. Cost =  $(100 * 70) * \$6 + (100 * 30) * \$8 = \$42,000 + \$24,000 = \$66,000$ .

Benefit = \$80,000

PV = Benefit – Cost = \$14,000

Benefit > Cost, so the dam should be built (as long as there is not another project under consideration with a higher PV). Note that the per hour cost of the unemployed workers is the value of their leisure, \$6. The additional \$2 they are paid is a transfer and is not considered a cost from a societal standpoint.

### Short Essay

Under the Tiebout model, taxes act like prices. Individuals select the bundle of desired public goods via their community choice and then pay for the bundle via taxes. Prior to the court decision, there was a strong tax – benefit linkage in New Hampshire and the Tiebout model was likely a reasonable approximation. The court decision broke, or weakened, the tax – benefit linkage and thereby weakened the Tiebout sorting process. Public goods are provided efficiently under the Tiebout hypothesis. The weakening of the Tiebout sorting process likely resulted in education being provided *less efficiently* in New Hampshire.

Housing prices include the *capitalization* of the future stream of local public goods (a benefit) and required tax payments (a liability). Due to the decrease in the efficiency of education provision, housing prices are likely to fall overall. There is also a strong prediction about the movement of housing prices in the poor vs. rich communities. The rich communities now have to pay higher taxes. The poor communities now receive a stream of benefits which will increase the value of the local bundle of public goods. As a result, housing prices may fall in the rich communities and rise in the poor communities. (Note that this process may offset itself because the community transfers are based on the property tax base. As the housing values of rich communities fall and the housing values of poor communities rise, the size of the transfers will decrease, which will feed back into housing prices.)

There are a couple of other things you might have mentioned. If the rich towns receive a “warm glow” from their contributions, housing prices may not fall in the rich towns. The transfers are essentially an unconditional block grant. There will only be an income effect on the poor communities. The grant may be spent on many things – road repair, lowering local taxes, etc, - not just on education. You also might note the similarity between the New Hampshire court decision and the Serrano vs. Priest decision in California which may have led to Prop. 13. Prop. 13 capped taxes and thereby reduced local spending. A similar process may occur in New Hampshire. Finally, you might note that the large externalities to education may provide a rationale for reducing the reliance on local financing for education.

## Pollution Externalities

Answers :

a.

	X	Y
Private	$p/2$	P
Social	$(p-5)/2$	$p/7$

b.

$$\text{pigouvian tax} = 6(w_y^*) = 6(p/7) = t$$

A pigouvian tax is the marginal damage evaluated at the socially optimal level of production.

c.

$$t=6; MC = 2 w_x + t = 2 w_x + 6; MB = p = 7$$

$$MC = MB \rightarrow w_x = 1/2$$

$$DWL = \frac{1}{2} * |\Delta t \text{ (from social optimum)}| * |\Delta Q \text{ (from social optimum)}| =$$

$$\frac{1}{2} * |(5-6)| * (1 - 1/2) = 1/4$$

If the DWL formula is not clear to you, draw the standard DWL graph and recall that the area of a triangle is  $\frac{1}{2} * \text{base} * \text{height}$ .

d.

	X	Y
Units of abatement with new technology	$5/2$	4
Socially optimal output level	$15/2$	4

It is socially optimal to abate until the marginal damage of the pollution equals the marginal cost of abatement *as long as production is socially optimal at this level of abatement*.

Socially optimal production is where the social MC = social MB (p in this case). First determine the level of pollution abatement that is optimal, ignoring the decision on the socially optimal level of production.

$MD_x = 5$  ; MC of abatement =  $2 w_x \rightarrow$  abate until  $5/2$  units have been produced  
 $MD_y = 6 w_y$  ; MC of abatement =  $w_y^2$ ; abate until 6 units have been produced

Now, determine the socially optimal level of production. Note that, for firm x, past  $5/2$  units of production, the social MC must be calculated with the marginal damage of pollution, not the marginal cost of abatement, because abatement ceases at  $5/2$  units. At  $5/2$  units, the  $SMC = 2 w_x + 5 = 2 w_x + 2 w_x = 10$  ;  $SMB = p = 20 \rightarrow MC < MB \rightarrow$  the firm will continue to produce after the point where it is optimal to use the new technology to abate pollution.

$$SMC = SMB \rightarrow 2 w_x + 5 = 20$$

$$w_x = 15/2$$

Firm x will produce  $15/2$  units of output and abate  $5/2$  units of pollution.

For firm y, the SMC of production at 6 units is :  $SMC = w_y + 6 w_y = w_y + w_y^2 = 42$ ;  $SMB = p = 20 \rightarrow SMB < SMC$ . Clearly it is not optimal for firm y to produce 6 units of output. Calculate the optimal production, assuming that ALL units of production will use the pollution abatement technology.

$$SMC = w_y + w_y^2 ; SMB = 20$$

$$w_y + w_y^2 = 20 \rightarrow w_y = 4$$

Firm y will produce four units of output and abate four units of pollution.

The government could enforce the optimum via price or quantity regulation. It could also use a permit system. The permit system is preferred because it equalizes the marginal cost of abatement across firms.

Public Goods

a.

Solve for the Nash equilibrium which results from private provision of the public good:

Bob :

$$\max o_b + 5(v_b + v_g)^{\frac{1}{2}} \quad \text{s.t.} \quad o_b + 10v_b = 90 \quad (1)$$

$$L = o_b + 5(v_b + v_g)^{\frac{1}{2}} - \lambda(o_b + 10v_b - 90) \quad (2)$$

FOC:

$$\frac{1}{2o_b^{\frac{1}{2}}} - \lambda = 0 \quad (3)$$

$$\frac{5}{2(v_b + v_g)^{\frac{1}{2}}} - 10\lambda = 0 \quad (4)$$

$$o_b + 10v_b = 90 \quad (5)$$

manipulating the equations:

$$\lambda = \frac{1}{2o_b^{\frac{1}{2}}} \quad (6)$$

$$\lambda = \frac{1}{4(v_b + v_g)^{\frac{1}{2}}}$$

$$\frac{1}{2o_b^{\frac{1}{2}}} = \frac{1}{4(v_b + v_g)^{\frac{1}{2}}} \quad (7)$$

$$2o_b^{\frac{1}{2}} = 4(v_b + v_g)^{\frac{1}{2}} \quad (8)$$

using the budget constraint :

$$2(90 - 10v_b)^{\frac{1}{2}} = 4(v_b + v_g)^{\frac{1}{2}} \quad (9)$$

$$90 - 10v_b = 4(v_b + v_g) \quad (10)$$

$$14v_b + 4v_g = 90 \quad (11)$$

yields Bob's reaction function:

$$v_b = \frac{90 - 4v_g}{14} \quad (12)$$

Gene :

In the same fashion, it can be shown that Gene's reaction function is:

$$v_g = \frac{90 - v_b}{11} \quad (13)$$

Solve these equations simultaneously:

$$v_b = 4.2 \quad (14)$$

$$v_g = 7.8 \quad (15)$$

$$V = 12 \quad (16)$$

b.

The socially optimal number can be solved for using the Samuleson Condition ( $\sum \text{MRS} = \text{MRT}$ ) or by solving the social planner's problem. Using the social planner's problem framework:

$$\max o_b^{\frac{1}{2}} + o_g^{\frac{1}{2}} + 15V^{\frac{1}{2}} \text{ s.t. } o_b + o_g + 10V = 180 \quad (17)$$

$$L = o_b^{\frac{1}{2}} + o_g^{\frac{1}{2}} + 15V^{\frac{1}{2}} - \lambda(o_b + o_g + 10V - 180) \quad (18)$$

Solving yields :

$$V = 16.53 \quad (19)$$

c. and d.

Both c and d require further calculations of Nash equilibrium. Using the approach shown in a., it can be shown that the general reaction functions are as follows (note that, as in part a., the math falls out much cleaner for the specific cases than the general one shown here):

Bob's reaction function:

$$\frac{25 * y - p^2 * v_g}{p^2 + 25p} \quad (20)$$

Gene's reaction function:

$$\frac{100 * y - p^2 * v_b}{p^2 + 100p} \quad (21)$$

Solving simultaneously:

$$v_b = \frac{25 * y - p^2 * (\frac{100 * y - p^2 * v_b}{p^2 + 100p})}{p^2 + 25p} \quad (22)$$

$$v_g = \frac{100 * y - p^2 * (\frac{25 * y - p^2 * v_g}{p^2 + 25p})}{p^2 + 100p} \quad (23)$$

$$v_b = -\frac{1}{5} \frac{y}{p} \frac{3p - 100}{p + 20} \quad (24)$$

$$v_g = \frac{1}{5} \frac{y}{p} \frac{3p + 100}{p + 20} \quad (25)$$

To show that these equations are the correct, general solution, part a. can be solved using them :

$$p=10, y=90$$

$$v_b = -\frac{1}{5} \frac{90}{10} \frac{3 * 10 - 100}{10 + 20} = 4.2 \quad (26)$$

$$v_g = \frac{1}{5} \frac{90}{10} \frac{3 * 10 + 100}{10 + 20} = 7.8 \quad (27)$$

$$V = 12 \quad (28)$$

Solve c. using the general equations. The per-person 30 subsidy means that both Bob and Gene have incomes of 120:

$$p=10, y=120$$

$$v_b = -\frac{1}{5} \frac{120}{10} \frac{3 * 10 - 100}{10 + 20} = 5.6 \quad (29)$$

$$v_g = \frac{1}{5} \frac{120}{10} \frac{3 * 10 + 100}{10 + 20} = 10.4 \quad (30)$$

$$V = 16 \quad (31)$$

Finally, note that the government's cost =  $30 * 2 = 60$

Solve part d. using the general equations. With the price reduction:

$$p=8, y=90$$

$$v_b = -\frac{1}{5} \frac{90}{8} \frac{3 * 8 - 100}{8 + 20} = 6.1071 \quad (32)$$

$$v_g = \frac{1}{5} \frac{90}{8} \frac{3 * 8 + 100}{8 + 20} = 9.9643 \quad (33)$$

$$V = 6.1071 + 9.9643 = 16.071 \quad (34)$$

Finally, note that the government cost = (per-unit subsidy \* number of units purchased) =  $(10 - 8) * 16.071 = 32.142$

e.

As shown above, the two plans result in approximately the same number of voting machines. However, the subsidy has a government cost that is approximately  $\frac{1}{2}$  the government cost of the income transfer plan. The subsidy is more efficient because it induces a substitution effect (as well as an income effect), while the income transfer only induces an income effect. Another way to view this is that the subsidy has lower infra-marginal effects.