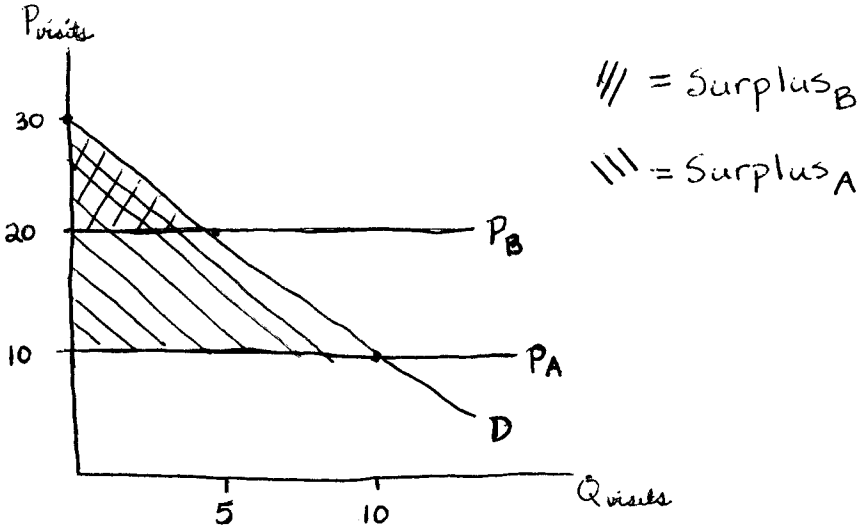
14.41 Problem Set #2 Solutions
Due October 3, 2003

1a) The roundtrip cost for inhabitants of city A is $.50*(10+10)=$10
The roundtrip cost for inhabitants of city A is $.50*(20+20)=$20

b) The individual demand curve for visits is Q=30-P*2
The surplus of each inhabitant of city A is ½*20*10=$100
The surplus of each inhabitant of city B is ½*10*5=$25
The total surplus is 200,000*100 + 200,000*25=$25 million



c) The NPV of the park is (25,000,000 – 1,500,000)*(1 + .1)/.1=$258.5 million
→The park's NPV is greater than the offer, so the park should not be sold

d) Cons: People may not be able to give sensible answers to this question, as it is hard to think about the value of something that is not traded in a market. Also, people who value the park and know it may be sold have an incentive to overstate their valuation. If we try to eliminate this incentive by charging people according to their valuations, they have an incentive to understate their valuation (this is free-riding, hoping that others will state high enough valuations that the park will not be sold).

Pros: This approach can capture valuations by people who do not actually visit the park—say, people who do not care to see it but are glad that nature is preserved, or people who plan one day to move to city A in order to take advantage of the park.

2a)
The social benefit each day is 100*1/6 hour*$6 per hour=$100
At a 10% discount rate, the NPV is $100*(1 + .1)/.1=$1100
At a 20% discount rate, the NPV is $100*(1 + .2)/.2=$600

b) At a 10% discount rate, the NPV of $1100 tomorrow is $1100/(1 + .1)=$1000
→The project is worth it if it costs less than $1000
At a 20% discount rate, the NPV of $600 tomorrow is $600/(1 + .1)=$500
→The project is worth it if it costs less than $500

c)
If residents are risk-neutral, evaluate the cost at its expected value:

$$EV=2/3*\$500 + 1/3*\$1500=\$833$$

Thus the road is worth it if the discount rate is 10% ($833<$1000), but not if the discount rate is 20% ($833>500).

d)
The social benefit each day is $100 + 101*1/6 hour*$2 per hour=$134.
The NPV if the benefits began accruing today is $134*(1 + .1)/.1=$1470.
Since the benefits do not begin to accrue until tomorrow, the NPV is
$1470/(1 + .1)=$1337
Clearly the road is worth it at a 10% discount rate, since it was already worth it before the new people began to benefit.

The residents will each have to pay $833/201=$4.14 for the road. The median voter is one of the new residents (since there are 101 new residents, and only 100 old residents) who earns $2 per hour and therefore gets a daily benefit of $0.34.
The NPV of a stream of $0.34 that starts tomorrow is ($0.34*(1 +.1)/.1)/(1 + .1)=$3.34

Thus the road is not worth it to the median voter ($4.14>$3.34), even though it is worth it to society ($833<$1337)

e)
It will cost each person $5 if the project costs $500; it will cost each person $15 if the project costs $1500. The social utility loss is equal to the sum of the individual utility losses:

$$\Delta U_{C=500} = 100 * [\ln(48-5) - \ln(48)] = -11$$
$$\Delta U_{C=1500} = 100 * [\ln(48-15) - \ln(48)] = -37$$

The expected change in utility is:

$$E[\Delta U] = 2/3*\Delta U_{C=500} + 1/3*\Delta U_{C=1500} = 2/3*-11 + 1/3*-37 = -20$$

The certainty equivalent g for each individual is such that:

$$100 * [\ln(48-g) - \ln(48)] = -20$$
$$\ln(48-g)=3.67$$
$$48-g=39.37$$
$$\rightarrow g=\$8.63$$

Thus the certainty equivalent for society is 100*$8.63=$863, and the road is worth it ($863<$1000)

3a)
Red wants to max $U= c + 20 \log(r) + 10 \log(b)$ s.t. $Y=c + r_r + b_r$:

$$\max U = Y - r_r - b_r + 20 \log(r_r + r_b) + 10 \log(b_r + b_b)$$

FOCs: $dU/d\, r_r = -1 + 20/(r_r + r_b)=0 \rightarrow (r_r + r_b)=20$
$dU/d\, b_r = -1 + 10/(b_r + b_b)=0 \rightarrow (b_r + b_b)=10$

By symmetry, the FOCs for Blue are:
$dU/d\, r_b = -1 + 10/(r_r + r_b)=0 \rightarrow (r_r + r_b)=10$
$dU/d\, b_b = -1 + 20/(b_r + b_b)=0 \rightarrow (b_r + b_b)=20$

Clearly, these FOCs cannot all hold, since they give you 20=10. Thus there is no interior solution.

But there is a corner solution: Red purchases 20 red fireworks and 0 blue fireworks, and spends all other income on c; Blue purchases 20 blue fireworks and 0 red fireworks, and spends all other income on c.

Check that this is a Nash equilibrium: Given what Blue is doing, Red continues to want to purchase 20 red fireworks, since that is the point at which the marginal utility of red fireworks equals the marginal utility of c. Red does not want to purchase more blue fireworks than the 20 Blue is already purchasing, since Red's marginal utility of blue fireworks is already below the marginal utility of both red fireworks and c.

→Given what Blue is doing, Red does not want to change strategies.

By symmetry:

→Given what Red is doing, Blue does not want to change strategies.

Thus this satisfies the definition of a Nash equilibrium.

b)
The social welfare function is the sum of the individual utility functions:

$$SW = c_r + c_b + 20\ln(r) + 10\ln(r) + 20\ln(b) + 10\ln(b) \text{ s.t. } Y = c_r + r/2 + b/2,\ Y = c_b + r/2 + b/2$$

$$\max SW = 2*(Y - r/2 - b/2) + 30\ln(r) + 30\ln(b)$$

FOCs: $dSW/dr = -1 + 30/r = 0 \rightarrow r = 30$
$dSW/db = -1 + 30/b = 0 \rightarrow b = 30$

In each town, the socially optimal provision is 30 red fireworks and 30 blue fireworks.

c)
In Redtown:

$$SW_{Reds} = c_r + c_r + 20\ln(r) + 20\ln(r) + 10\ln(b) + 10\ln(b) \text{ s.t. } Y = c_r + r/2 + b/2$$

$$\max SW_{Reds} = 2*(Y - r/2 - b/2) + 40\ln(r) + 20\ln(b)$$

FOCs: $d\,SW_{Reds}/dr = -1 + 40/r = 0 \rightarrow r = 40$
$d\,SW_{Reds}/db = -1 + 20/b = 0 \rightarrow b = 20$

Redtown's optimal provision is 40 red fireworks and 20 blue fireworks. By symmetry, Bluetown's optimal provision is 20 red fireworks and 40 blue fireworks.

d)
The joint social welfare function is:

$$SW_{joint} = 2c_r + 2c_b + 40\ln(r) + 20\ln(r) + 20\ln(b) + 40\ln(b) \text{ s.t. } Y = c_r + r/4 + b/4,\ Y = c_b + r/4 + b/4$$

$$\max SW = 4*(Y - r/4 - b/4) + 60\ln(r) + 60\ln(b)$$

FOCs: $dSW_{joint}/dr = -1 + 60/r = 0 \rightarrow r = 60$
$dSW_{join}/db = -1 + 60/b = 0 \rightarrow b = 60$

The optimal joint provision is 60 red fireworks and 60 blue fireworks.

e)
A has the lowest total social welfare, since too much is being spent on the private good and not enough on the public good.

In B, C, and D, the same total amount is spent on fireworks. C is an improvement over B, because Reds get a higher ratio of red to blue for the same price, and Blues get a lower ratio for the same price.

D is an improvement over C, because they get more of everything for the same price.

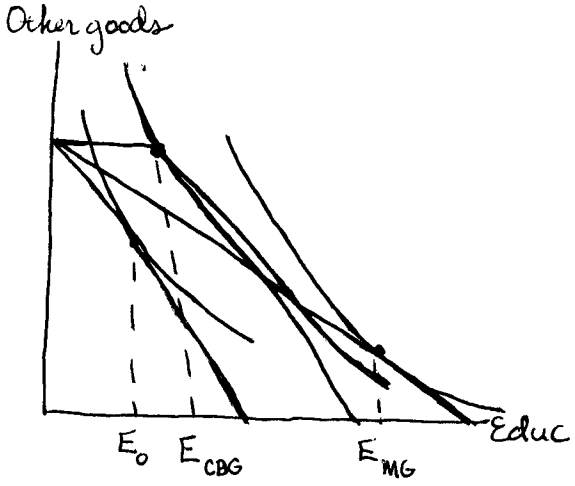
In more general terms:
A → B: social provision of a public good results in higher utility than private provision
B → C: sorting increases the efficiency of public good provision
C → D: sharing the costs of provision over more people increases the efficiency of public good provision

4)
a) Congressman A is incorrect. While there may be good reasons to subsidize college education, this is not one of them. The benefit he is referring to is a private benefit—increased earnings—not a social benefit, and therefore can be handled within the market mechanism (unless he makes another argument that there is market failure, such as credit constraints).

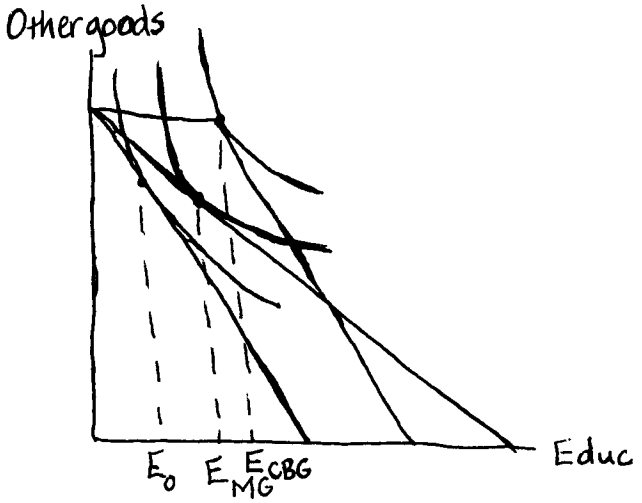
b) Congressman B is incorrect. This is an argument about redistribution, not education. While there may be good reasons for redistribution, he does not give a good reason for using education as a redistributional mechanism. It is probably more efficient, for example, to redistribute through the tax system.

c) Congressman C is making a Tiebout argument—that families sort into school districts that provide the level of elementary education they desire, and therefore that provision is efficient without federal intervention. While there is evidence that families sort, there are several reasons that sorting may not be perfect (and therefore that there may be room for federal intervention). First, there may not be as many towns as there are preferences over education. Second, towns may offer bundles of goods of which education is only one, so that families pick the best available bundle but cannot maximize over education separately. Third, poor people who value education may be credit-constrained, and unable to move into a town with the level of education they prefer.

d) In most cases, matching will lead to a larger increase:



But if spending is very low and a town has strong preferences for other goods rather than education, a conditional block grant may lead to the larger increase:



e)
Both proposals decrease the cost of college relative to other goods, and therefore will increase college attendance as long as one assumes that there are some people currently on the margin between attending and not attending college.

Lowering tuition at state universities makes the higher education system look more like the primary education system: students can get a fixed quantity of education for low cost (or, in the case of the primary system, for free), but to increase the level of education at the margin they must spend much more. Lowering the cost of public education relative to private education will cause people to substitute toward public education. If we assume that state schools cost less than private schools, this will lower spending by people who otherwise would have attended private schools. Plus, the tuition at state schools has gone down, which lowers spending by people who were attending state schools anyway. But note that we also expect to increase college attendance, so the net effect on spending is ambiguous.

Grants for any university in the state are a type of voucher system. The grants will shift some students from attending school out-of-state to attending school (public or private) within state. They will also decrease the spending of people who were already attending schools in-state, since they no longer pay the last $1000 of tuition (although note that in general equilibrium tuition is likely to increase). Again, however, there will be an increase in college attendance—in fact, probably a larger effect, since some people who previously did not attend college may not be interested in attending state schools, but may be on the margin of attending private school. Thus the net effect on spending is ambiguous.

These proposals have the standard financial pros and cons of increased public school spending versus vouchers:

1) Increased public school spending may crowd out private spending, as some students move from higher spending on a private school to the cheaper—but lower-quality—public provision.
2) Vouchers go even to those who are very well-off and unlikely to significantly increase their consumption of education—for them, when the government spends more on education, they simply spend less on education (and more on other goods).

But there is one difference between higher education and primary education, which is that many people do not go to college. So any program that decreases the cost of college, although it decreases spending by those already in the system, will also increase the spending of those who now enter the system.