14.41 Problem Set #1 Answers
10/1/04

1a) The social optimum is where each firm's marginal cost of abatement is equal to the marginal benefit of abatement:

$$MC_A=d/dx[x^3]=3x^2$$
$$\rightarrow \text{at the optimum, where } MC_A=SMB,$$
$$3x^2=300$$
$$x=10$$

$$MC_B=d/dx[x^2]=2x$$
$$\rightarrow \text{at the optimum, where } MC_B=SMB,$$
$$2x=300$$
$$x=150$$

The social optimum is 160 units of abatement, 10 by Firm A and and 150 by Firm B.

b) At 80, Firm A's marginal cost of abatement is:

$$MC_A=3(80^2)=19200$$

while Firm B's marginal cost is:

$$MC_B=2(80)=160$$

Since Firm B's marginal cost is below Firm A's, Firm A could abate one unit less and Firm B one unit more and society would save 19200-160=$19,040 while achieving the same level of abatement. Thus this outcome is not socially optimal.

c) Here, Firm A will set marginal cost to $300, as will Firm B, and (as shown in part a) Firm A will abate 10 while Firm B will abate 150, for a total of 160. This is socially optimal, as each firm has internalized the $300 social benefit of abatement because of the Pigouvian tax (subsidy).

d) Firm A will choose to abate x_A units such that x_A minimizes its costs, which are:

$$P_{credit}*\#(100- x_A) + x_A^3$$
$$-P_{credit} + 3 x_A^2 = 0$$
$$\text{so at equilibrium, } P_{credit}= 3x_A^2$$

Firm B will choose to abate x_B units such that x_B minimizes its costs, which are:

$$P_{credit}*\#(60- x_B) + x_B^2$$
$$-P_{credit} + 2 x_B = 0$$
$$\text{so at equilibrium, } P_{credit}= 2x_B$$

$$\text{so } 2x_B=3x_A^2$$
$$\text{and } x_A+ x_B=160$$
$$\text{substituting, } 2(160- x_A)= 3x_A^2$$

$$3x_A^2+2\,x_A-320=0$$
$$x_A=10$$
$$x_B=150$$
$$\text{and } P_{credit}=2x_B=300$$

which is the social optimum from A.

In practice the market may not be perfectively competitive when there are only two participants. Since Firm B will have monopoly power, the market may not function properly. Another potential problem is political credibility: the market will only function if the participants believe the government will honor the property rights conveyed by the permits. If the firms believe the president will not be re-elected and a new president will not honor the property rights conveyed by the permits, the market will not function properly.

2a) $PMB_{Joe}=dU_{Joe}/dx=5x^{-1/2}$
$PMC_{Joe}=0$

$SMCB=dU_{Joe}/dx=5x^{-1/2}$
$SMC=dU_{Maria}/dx=1$

Joe will set his volume to the maximum possible, 100.

b) Once Joe and Maria begin trading, the requirements for the Coase theorem to hold are in place: property rights are clearly assigned (to Joe), and there are zero bargaining costs (they're already talking to each other, and have an agreed-upon system of trade—t-shirts). Thus Joe and Maria will trade until $PMB_{Joe}=PMC_{Maria}$, the social optimum. The new volume will be such that:

$$5x^{-1/2}=1$$
$$x=25$$

Joe's loss of utility from reducing the volume is:

$$10*100^{1/2}-10*25^{1/2}=50$$

Maria's gain of utility from reducing the volume is:

$$100-25-(100-100)=75$$

Thus Joe would require at least two t-shirts to turn down the volume; Maria would be willing to give him up to three t-shirts. Maria is willing to give him more than he requires because trade here is a Pareto improvement and creates a surplus of 25. Who actually ends up with the third t-shirt will depend on who is a better bargainer.

c) $SMC=dU_{Maria}/dx+dU_{Jerome}/dx=1+2=3$

Joe and Jerome will trade until $PMB_{Joe}=PMC_{Jerome}$:

$$5x^{-1/2}=2$$

$$x=6.25$$

Joe's loss of utility from the additional reduction in volume is:

$$10*25^{1/2}-10*6.25^{1/2}=25$$

Jerome's gain of utility from reducing the volume is:

$$100-2*6.25-(100-2*25)=37.5$$

Thus Joe would require at least 100 stickers to turn down the volume; Jerome would be willing to give him up to 150.

There is a gain from trade, but bargaining between Joe and Jerome will *not* result in the social optimum, because Jerome is not considering the benefit Maria gets when he trades with Joe (nor did Maria consider her affect on Jerome). The requirements for the Coase theorem no longer hold: bargaining between all the relevant parties is no longer costless—Maria isn't there, and even if she were she and Jerome would probably have a free-rider problem.

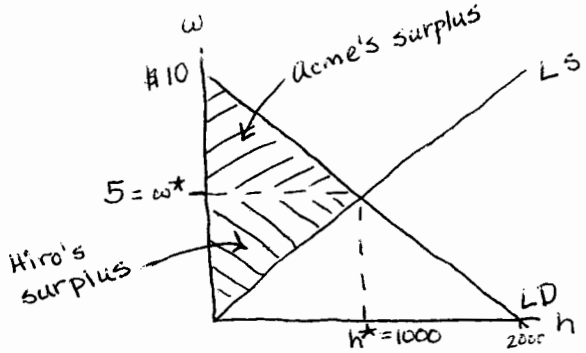
d) The landlady should set SMB=SMC:

$$5x^{-1/2}=3$$

$$x=2.78$$

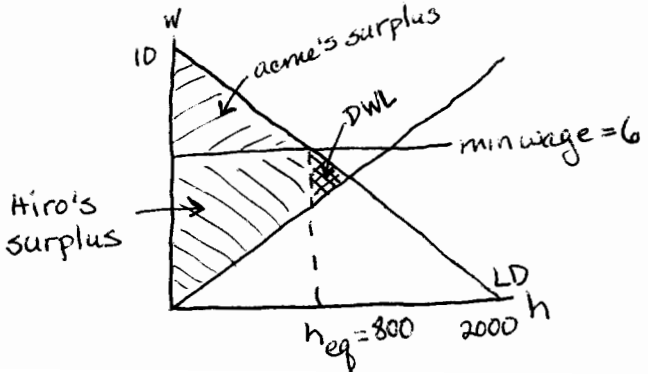
There is no uncertainty about the costs of noise abatement, so it doesn't matter if she engages in price or quantity regulation. For example, she could set a rule that no stereo can be played above 2.78. Alternatively, she could increase Joe's rent by 3 for each unit of volume.

3a)



b) Acme gets surplus ½*1000*5=2500, the area between its demand curve and the equilibrium wage. Hiro gets surplus ½*1000*5=2500, the area between his supply curve and the equilibrium wage.

c)

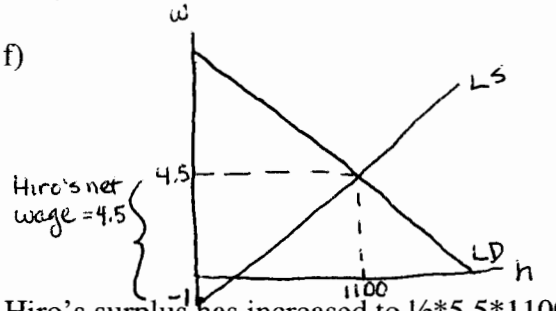


Acme will now only be willing to employ Hiro for 800 hours. Acme now gets surplus ½*4*800=1600, the area between its demand curve and the minimum wage at the equilibrium. Hiro gets surplus ½*4*800 + 2*800=3200, the area between his supply curve and the minimum wage at the equilibrium.

d) While Hiro's surplus has increased, Acme's surplus has fallen by more. Total surplus has decreased, because the trade of hours 801 through 1000, which made both Hiro and Acme better off, no longer occurs.

e) Hiro now supplies labor LS= 200(w+1), so the equilibrium wage Acme pays him will be 4.5, the wage he receives will be 5.5 (including the subsidy from the government), and he will supply 1100 hours.

f)



Hiro's surplus has increased to ½*5.5*1100=3025. Acme's surplus has increased to ½*5.5*1100=3025. The government is subsidizing the market, giving gains to both Hiro and Acme. What's not shown on this graph, however, is the opportunity cost of this subsidy—the money must come from some other use. The government cannot subsidize every market, because some market must be taxed to pay for the subsidies. Whether it is worthwhile to subsidize this market depends on the gain in surplus here relative to the loss in surplus in the market that's taxed.

4)a)i) There is a negative externality to driving a hybrid to the extent that it adds greenhouse gases to the atmosphere, although the externality is not as great as with a conventional car.
a)ii) It is difficult for a market to arise to internalize this externality as many drivers contribute in very small ways to a problem that affects the whole world.
a)iii) As discussed in class, the marginal social benefit to reducing greenhouse gases is flat, so price regulation (i.e. taxes) is most effective. The optimal tax would be lower than the optimal tax on conventional cars, as conventional cars have a greater negative externality (they produce more greenhouse gases).
b)i) Going to a crowded party while sick with the flu exerts a negative externality on the other people at the party, assuming that you do not take into account the costs of other people becoming sick.
b)ii) It is unlikely that a private market will arise to internalize the externality—there are too many people at the party to negotiate costlessly (just negotiating might get them sick!) and it's hard to assign blame (when they get sick several days later, it's difficult to prove you were the culprit).

b)iii) The people at the party are probably much better off if you don't attend than if you do—if they guess a price regulation, and it turns out to be too low to dissuade you from coming, they will be much worse off. Therefore quantity regulation is probably best—that is, requiring that you not attend while sick. Party-attendance credits probably can't work either, since people attending other parties affect an entirely different group of people. Thus a command-and-control rule is best.
c)i) Conducting economics research may have a positive externality (if it's good research!), in that you cannot capture all the returns to the research—it also increases knowledge for other people.
c)ii) It is possible that a private market will arise—if you have clearly assigned property rights to your research, you may be able to charge others to read your research, thereby internalizing the positive externality. But this may decrease gains, as some people would benefit from your research but don't find out because it's kept quiet.
c)iii) There is probably a constant social marginal benefit of research, so price regulation (say, subsidizing student research through a UROP program) is probably best.
d)i) Home smoke alarms make it easy to catch fires early, preventing them from spreading. Whether this benefits other people depends on your home's location. If your house is on a big rock in the middle of nowhere, so that if it burns it will harm neither nature nor other people, there may not be an externality. If you live in an apartment building in the middle of a city, there probably is a positive externality.
d)ii) If you have only one neighbor, you could probably internalize the neighbor's benefit through trade. But if you have many neighbors, it will be hard for the market to solve the problem.
d)iii) Answers will vary based on assumptions made about the nature of the externality.