14.03 Fall 2004

Problem Set 6 - Solutions

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1 Moral hazard and insurance

Donald is a risk-averse person who has \$100 in monetary wealth and owns a house worth \$300. The probability that his house is destroyed by fire (equivalent to a loss of \$300) is $p_{ne} = 0.5$. If he exerts an effort level $e = 2$ to keep his house safe, the probability falls to $p_e = 0.2$. His utility function is:

$$U = w^{\frac{1}{2}} - e$$

where e is effort level exerted (zero in the case of no effort and 2 in the case of effort).

1. Expected utility under no effort and under effort:

$$\begin{aligned} U_{ne} &= 0.5\,(400)^{\frac{1}{2}} + 0.5\,(100)^{\frac{1}{2}} = 15 \\ U_e &= 0.8\,(400)^{\frac{1}{2}} + 0.2\,(100)^{\frac{1}{2}} - 0.3 = 17.7 \end{aligned}$$

   Therefore Donald will choose to exert effort.

2. Given the premium α:

$$\begin{aligned} U_{ne} &= 0.5\,(400 - \alpha)^{\frac{1}{2}} + 0.5\,(400 - \alpha)^{\frac{1}{2}} = (400 - \alpha)^{\frac{1}{2}} \\ U_e &= 0.8\,(400 - \alpha)^{\frac{1}{2}} + 0.2\,(400 - \alpha)^{\frac{1}{2}} - 1 = (400 - \alpha)^{\frac{1}{2}} - 0.3 \end{aligned}$$

   Insurees will always choose to exert no effort and therefore will cause an increase in the probability of fire.

3. The incentive compatibility constraint to induce the choice of effort $e = 0.3$ is the following:

$$\begin{aligned} 0.8\,(400-\alpha)^{\frac{1}{2}} + 0.2\,(400-D-\alpha)^{\frac{1}{2}} - 0.3 &\geq 0.5\,(400-\alpha)^{\frac{1}{2}} + 0.5\,(400-D-\alpha)^{\frac{1}{2}} \\ D &\geq 2\,(400-\alpha)^{\frac{1}{2}} - 1 \end{aligned}$$

4.

$$\begin{aligned} \max E(\pi) &= \alpha - 0.2\,(300-D) \\ s.t.\; 0.8\,(400-\alpha)^{\frac{1}{2}} + 0.2\,(400-D-\alpha)^{\frac{1}{2}} - 0.3 &\geq 17.7 && \text{(IR)} \\ 0.8\,(400-\alpha)^{\frac{1}{2}} + 0.2\,(400-\alpha-D)^{\frac{1}{2}} - 0.3 &\geq 0.5\,(400-\alpha)^{\frac{1}{2}} + 0.5\,(400-\alpha-D)^{\frac{1}{2}} && \text{(IC)} \end{aligned}$$

This is the problem the insurance company solves: it maximizes expected profits subject to two constraints. The first constraint is the Individual Rationality constraint (IR) and establishes that in order to make the insuree buy the insurance policy, it must guarantee at least the same utility level that he would achieve with no insurance. The second constraint is the Incentive Compatibility costraint (IC) and establishes that in order to induce the insuree to exert effort it must reduce the payoff in the case of fire. The IC allows to find the optimal amount of D: this is the lowest amount of deductible compatible with the choice of effort $e = 0.3$. Since the insuree is risk averse this is the smallest variation in income (compatible with incentives) and therefore optimal. D is the same found in part 3. Knowing D the premium α is found substituting D in the IR constraint and finding the maximum α that induces the insuree to buy the insurance plan.

5. Without deductible the insurance knows the insuree will do nothing to avoid fire and therefore the actuarially fair insurance premium takes into account that the probability of fire is 0.5:

$$\alpha = 0.5 \cdot 300 = 150$$

Under this premium Donald's utility would be:

$$0.5\,(400-150)^{\frac{1}{2}} + 0.5\,(100+300-150)^{\frac{1}{2}} = 15.81$$

Donald would not buy such insurance because he is better off exerting some effort and lowering the probability of fire.

6. Consumers are better off buying insurance plans with deductibles that induce them to "behave". They know that if full insurance were offered, they would have to pay a higher premium.

2 Rothschildia health plan

Expected utility for consumer with health p is $(1-p)\,U(1) + pU(1-1) = 1-p$. Expected utility for this consumer if she purchases full insurance at premium r is $(1-p)\,U(1-r) + pU(1-r-1+1) = U(1-r) = \sqrt{1-r}$. So a consumer purchases insurance iff $\sqrt{1-r} \geq 1-p \iff p \geq 1-\sqrt{1-r}$. Define p_c to be the probability of sickness for the most healthy person to buy insurance, i.e. $p_c = 1- \sqrt{1-r}$. Then the average health of those who enroll when the premium is r is $\frac{1+p_c}{2} = \frac{2-\sqrt{1-r}}{2}$The average profit for the insurance plan is given by average revenue (equal to the premium) minus average cost (equal to average health), or $r - \frac{1+p_c}{2} = \frac{2r-2+\sqrt{1-r}}{2}$.

1. Premium $= \frac{1}{2} \Longrightarrow p_c = \frac{2-\sqrt{2}}{2}$

   (a) Most healthy enrollee: $p = \frac{2-\sqrt{2}}{2}$

   Least healthy enrollee: $p = 1$

   (b) Average health of enrollees $= \frac{4-\sqrt{2}}{4}$

   (c) Average profit $=\frac{\sqrt{2}-2}{4} < 0$, plan loses money.

   (a) Premium $= \frac{4-\sqrt{2}}{4} \Longrightarrow p_c = \frac{2-\sqrt[4]{2}}{2}$

   (b) Most healthy enrollee: $p = \frac{2-\sqrt[4]{2}}{2}$

   Least healthy enrollee: $p = 1$

   (c) Average health of enrollees $= \frac{4-\sqrt[4]{2}}{4}$

   (d) Average profit $=\frac{\sqrt[4]{2}-\sqrt{2}}{4} < 0$, plan loses money.

   (a) Premium $= \frac{4-\sqrt[4]{2}}{4} \Longrightarrow p_c = \frac{2-\sqrt[8]{2}}{2}$

   (b) Most healthy enrollee: $p = \frac{2-\sqrt[8]{2}}{2}$

   Least healthy enrollee: $p = 1$

   (c) Average health of enrollees $= \frac{4-\sqrt[8]{2}}{4}$

   (d) Average profit $=\frac{\sqrt[8]{2}-\sqrt[4]{2}}{4} < 0$, plan loses money.

2. The premium such that the pool of citizens who enroll at that premium cost on average exactly that premium is r such that $r = \frac{2-\sqrt{1-r}}{2}$. The relevant root of this equation gives us a premium of $\frac{3}{4}$.

   Most healthy enrollee: $p = \frac{1}{2}$

   Least healthy enrollee: $p = 1$

   Average health of enrollees $= \frac{3}{4}$

   (a) 1) If there is no health plan, expected utility is $1-p$, so average expected utility is $\frac{1}{2}$. 2) Under the current (voluntary) break-even plan, $r = \frac{3}{4}$ and $p_c = \frac{1}{2}$. The expected utility for enrollees is $U(1 - \frac{3}{4}) = \frac{1}{2}$. The expected utility for non-enrollees is $1-p$, and since $p < \frac{1}{2}$ for non-enrollees, the average expected utility for non-enrollees is $\frac{3}{4}$. The average expected utility for all consumers (which is equal to the average expected utility for enrollees times the probability of being an enrollee, plus the average expected utility for non-enrollees times the probability of being an non-enrollee) is $(1/2)(1/2) + (3/4)(1/2) = 5/8$. 3) The mandatory break-even plan would set the premium equal to the average cost when all citizens enroll, which is $\frac{1}{2}$. This yields an average expected utility of $U\left(1 - \frac{1}{2}\right) = \frac{\sqrt{2}}{2}$

   (b) If you want to maximize average expected utility, you should recommend the mandatory break-even plan.

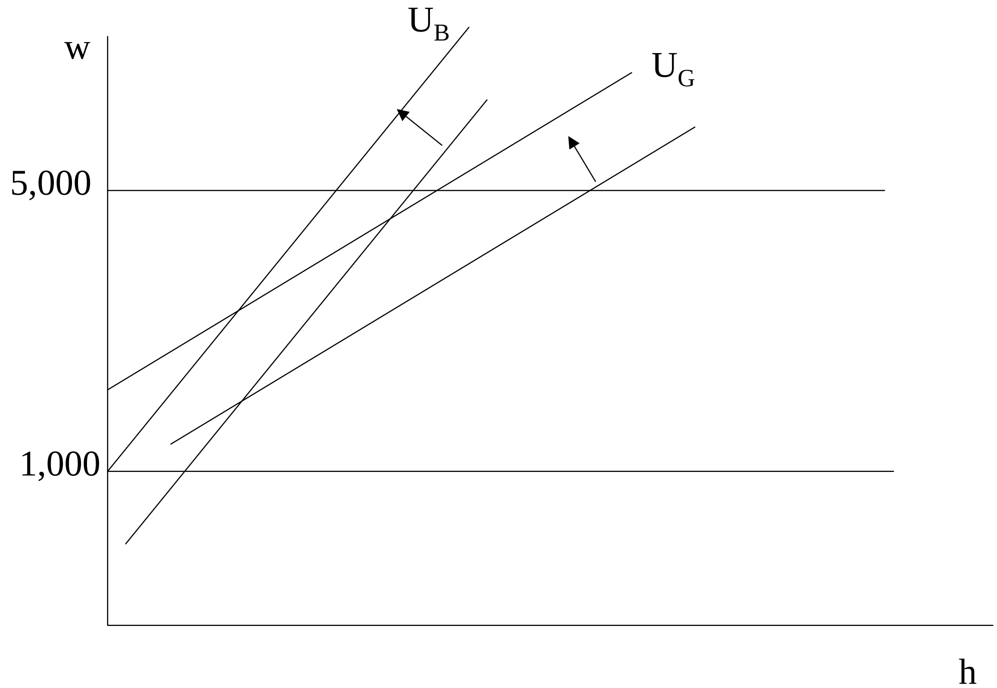
3. Mandatory insurance increases average expected utility because it eliminates the adverse selection problem. Since low-risk citizens can no longer opt out, the cost of providing insurance to everyone goes down. In fact, one of the strongest arguments for public insurance programs like national health insurance is that they can prevent adverse selection from spoiling (or reducing the social efficiency) of the insurance market by requiring people to enroll. Since private insurance providers cannot require people to enroll, it may often be the case that governments can improve net (or average) social welfare by requiring everyone to buy insurance. In fact, governments do this quite frequently by using taxes (compulsory) to pay for social insurance plans like flood and earthquake insurance. As we have discussed in class, private markets for flood and earthquake insurance do not exist.

   However, you should observe that the mandatory plan is not a strict Pareto improvement as compared to the voluntary break-even plan since some healthy citizens are made worse off under the mandatory plan. At the same time, not everyone who is compelled to buy the plan is made worse off. In fact, under the mandatory plan, a person with illness probability .292 or higher is strictly better off under the social insurance plan, whereas someone with an illness probability of $< .292$ is better off without having to pay the $\frac{1}{2}$ Stiglitz insurance premium. Notice however that under the private break-even insurance plan, no one with illness probability below $\frac{1}{2}$ would buy the plan. Hence, people in the region $\frac{1}{2} > p > .292$ are made better off by the mandated plan whereas people in with $p < .292$ are made worse off. (It is actually easy to come up with examples where everyone is made better off by mandated insurance.)

Finally, observe that since average (and hence total) social welfare is greater under the mandatory plan than under the break-even plan, it must be the case that those helped by the mandatory plan could in theory compensate those hurt by the mandatory plan and still be better off. Hence, the plan represents a potential Pareto improvement, although not a strict Pareto improvement. This test of "potential Pareto improvements" is called the Kaldor Compensation test and it is used frequently for evaluating policy interventions that help some citizens while making others worse off. The mandatory insurance plan above passes the Kaldor criterion.

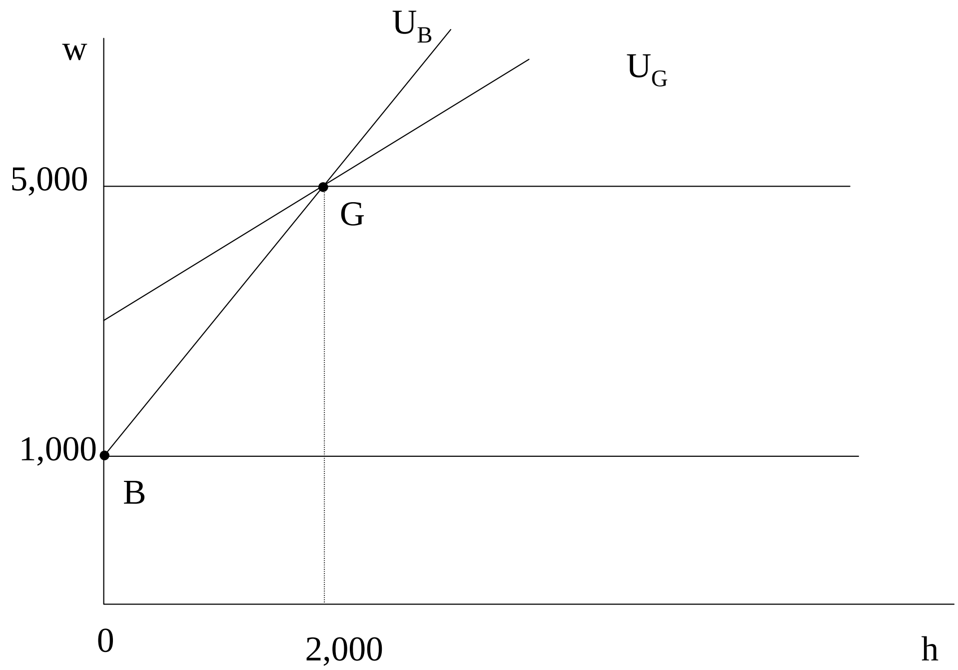
3 The job market for Santa Claus

1.



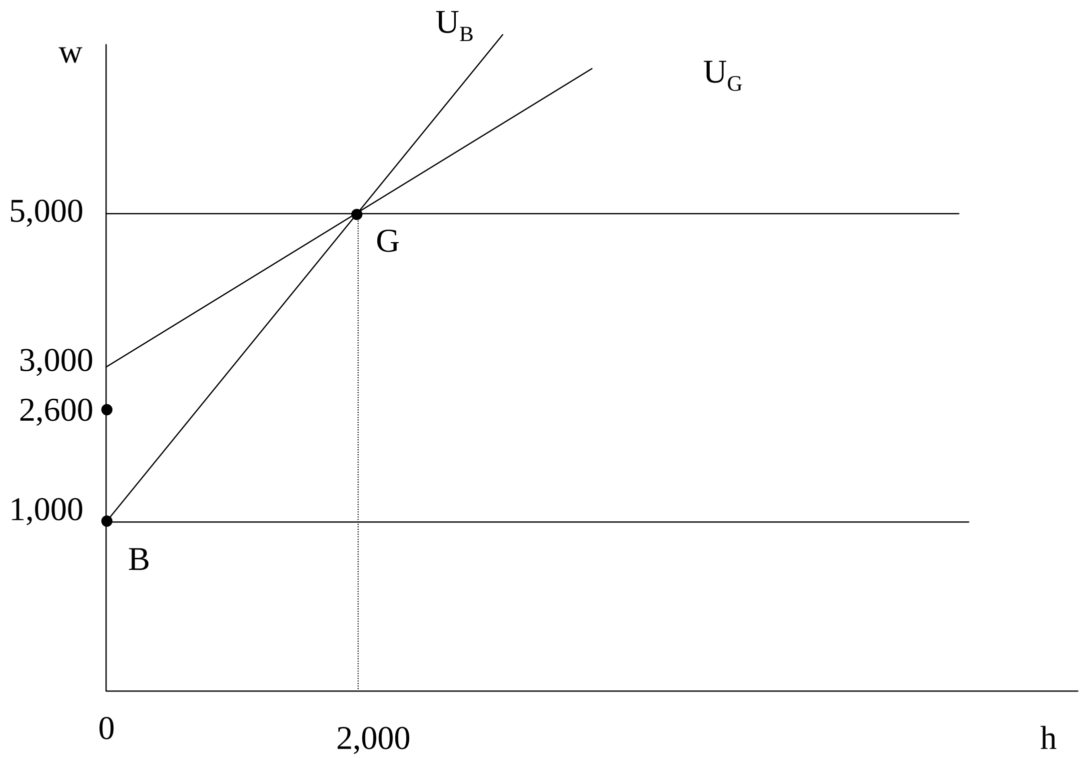
2. If the committee could tell the candidates apart, there would be no need to signal productivity by spending time in the cold.

3. The good Santas are going to spend time in the cold to signal their higher productivity (and therefore receive a wage $5,000$). The minimum amount of hours spent in the cold necessary to signal higher productivity, without inducing also the bad Santas to signal and stay out in the cold is $2,000$. This is the number of hours h^* that induce the committee to believe that they are faced with a high productivity Santa and give him the high wage $5,000$. The bad Santas will spend no time in the cold and receive a wage of $1,000$.



4. Good Santas would be happier if productivity were observable. That way they would receive a wage of 5,000 and would not have to spend any time in the cold. The Bad Santas are indifferent between the full information and the asymmetric information cases. They receive in both cases a wage of 1,000 and spend no time in the cold.

5. If all Santas were paid the same expected productivity 2,600 then there would be no need for any Santa to stay out in the cold because that would no induce any differential in the wage. This would reduce the welfare loss associated to standing out in the cold (which has no social benefit, other than as a signalling device). In particular, if the 2/3 of the Bad Santas gave 401 of their wage to the Good Santas, then every candidate would be better off.



6. The non-protesters are paid $1,000$ because they are the Bad Santas. The Good Santas have an incentive to coordinate and return to the previous equilibrium where they stood out in the cold, but receive the higher wage. The graph shows that the pooling equilibrium (where all Santas are paid the same average productivity wage) is on a lower indifference curve for the Good Santas with respect to the separating equilibrium.

7. The flat wage equilibrium is worse from the point of Good Santas. They have an incentive to signal their higher productivity even if this is socially inefficient.