# 14.03 Fall 2004 

Problem Set 4
Due Friday, November 5th at 5pm

## 1. Gains from International Trade

Claudia (C) and Jesse (J) live in the land of Carvel, where only ice cream scoops (I) and brownies (B) are consumed. Claudia only enjoys brownies and ice cream together: one brownie with one scoop of ice cream on top. Jesse also enjoys both separately.

Their utility functions and initial endowments are the following:
Claudia: $U_{C}=\min \left(I_{C}, B_{C}\right) ; E_{C}^{I}=4 \quad E_{C}^{B}=6$

Jesse: $U_{J}=0.5 * \ln \left(I_{J}\right)+0.5 * \ln \left(B_{J}\right) ; E_{J}^{I}=6 \quad E_{J}^{B}=4$
To simplify the analysis assume $P_{B}=1$.
(a) Draw the Edgeworth Box, locate the endowment point and draw indifference curves for both people.
(b) Find Claudia's and Jesse's demand functions. (i.e. $I_{C}, B_{C}, I_{J}, B_{J}$ as functions of $p_{I}$ ).[Hint: what is the optimal relation between $I_{C}$ and $B_{C}$ ?]
(c) Using your estimations in (b) and the market clearing conditions, find the market equilibrium price $\left(p_{I}^{*}\right)$. Replace this value in the demand functions and find the equilibrium consumption bundles $I_{C}, B_{C}, I_{J}, B_{J}$. Represent the equilibrium in the Edgeworth box. What are the utility levels for Claudia and Jesse at this equilibrium?
(d) Now suppose that Claudia and Jesse are considering opening their economy to world trade. The world price of ice cream $\left(p_{I}^{W}\right)$ is 2 and the world price of brownies $\left(p_{B}^{W}\right)$ is 1 . Using the demand functions calculated in (b) find Claudia's and Jesse's demands for brownies and ice cream at the world prices. Represent them in the Edgeworth box.
(e) Show that if redistribution is not possible, Jesse would agree to open to trade, but Claudia would not. If you were a central planner and only cared for the sum of their utilities would you open the economy?
(f) Now assume that redistribution is possible. Find all possible transfers from Jesse to Claudia that would make Claudia exactly indifferent between free trade and remaining under autarky. Assume Jesse makes one of these transfers. Does he benefit from free trade?

## 2. International Trade and Growth (Based on Frankel and Romer)

As in Frankel and Romer we want to estimate the causal effect of Trade on Economic growth. We have information at the country level for gdp per capita growth $(G)$ and a measure of openness to trade $(F)$. The measure, developed by Sachs and Warner (1995) takes a value of one if NONE of the following conditions are true:

- average tariff rates are $40 \%$ or higher on imports of intermediate and capital goods
- non-tariff barriers cover $40 \%$ or more of imports of intermediate and capital goods
- a black market exchange rate is depreciated by $20 \%$ or more relative to the official exchange rate
- a socialist economic system
- the state has a monopoly on major exports.
and a value of zero if at least one of them is true. We have data for 62 countries, 36 for which $F=0$ [The variable was constructed for 1990. Many countries opened their borders in the 90 's]. Examples of countries with $F=1$ are the US, Thailand,Australia and Taiwan. Examples with $F=0$ are Argentina, Vietnam, China, Slovenia.
(a) Suppose we only have this two variables, and we calculate the following:
$\sum_{F=1} G_{i} / 26=3.268$
$\sum_{F=0} G_{i} / 36=0.786$
Propose a possible estimator for the causal effect of free trade on growth using these sums. What is the value of your estimator? What are the potential problems with this estimator? Do you think your estimator will understate or overstate the causal effect of trade on growth?
(b) Following Frankel and Romer, we look for an instrumental variable that can help us improve the plausibility of the causal estimation.
i. What are the conditions we want the instrumental variable to satisfy (please use formal notation to state the necessary conditions)?

We find data on whether a country is landlocked or not. If a country has direct access to the sea $L=0$, if it doesn't $L=1$. There are 11 landlocked countries in our data.
ii. Do you think it is plausible that this variable satisfied the conditions you stated in $(i)$ ? Why or why not [or you can give reasons for and against]?
(c) You average the trade index for the landlocked and non-landlocked countries and find:
$\sum_{L=1} F_{i} / 11=0.181$
$\sum_{L=0} F_{i} / 51=0.47$
i. What is the causal effect of being landlocked on free trade under the assumptions you outlined in (b)? Why do you think landlocked countries are less likely to be open to trade? Can you think of any reasons why this relationship might go in the opposite direction?
ii. You calculate the following averages:
$\sum_{L=1} G_{i} / 11=0.968$
$\sum_{L=0} G_{i} / 51=2.012$
Propose and calculate an IV estimator for the causal effect of trade on growth. Interpret its sign and magnitude.
iii. Compare the magnitude of your IV estimate with your estimate in part (a). Did you expect to find this result? Explain.

## 3. Short Questions

(a) Consider the four choices:
a. $\$ 1,000,000$ for sure
b. $10 \%$ chance of $\$ 5,000,000$
$89 \%$ chance of $\$ 1,000,000$
$1 \%$ chance of $\$ 0$
c. $10 \%$ chance of $\$ 5,000,000$
$90 \%$ chance of $\$ 0$
d. $11 \%$ chance of $\$ 1,000,000$
$89 \%$ chance of $\$ 0$
Before reading further, choose which you would prefer between $\mathbf{a}$ and $\mathbf{b}$, and then choose which you would prefer between $\mathbf{c}$ and $\mathbf{d}$. [The choice you make does not affect your grade.]

It is commonly observed that people prefer $\mathbf{a}$ to $\mathbf{b}$, and prefer $\mathbf{c}$ to $\mathbf{d}$. Show that this pair of choices is inconsistent with expected utility maximization.
(b) Richie, an expected utility maximizer, places an even bet of $\$ 50,000$ on the Red Sox winning the World Series. If he has a utility function that is logarithmic in wealth [e.g., $U=\ln (W)$ ]
and his current wealth is $\$ 200,000$, what is the minimum probability he must place on the Red Sox wining the Series? [That is, Richie must believe that the probability that the Sox win is greater than equal to $\rho$, a number you calculate.]
(c) Angelo has a utility function of the form $U=\ln (W)$ He is offered a lottery that gives $\$ 50$ with a probability of $\frac{1}{2}$ and $\$ 10$ with a probability of $\frac{1}{2}$. What is the maximum amount of money he is willing to pay to participate? (Assume his initial wealth is $\$ 30$ ).

## 4. Uncertainty and Insurance

Forecasters predict there is a 50 percent probability that the upcoming growing season will be a drought. Assume that Farmer Jane is an expected utility maximizer with utility function $U(W)=\ln (W)$. Her initial wealth is $\$ 0$.
(a) Is Jane risk averse (yes/no)?

Jane initially has the choice between two crops with payoffs:

|  | Normal Rain | Drought |
| :---: | :---: | :---: |
| Potatoes | $\$ 5,000$ | $\$ 40,000$ |
| Strawberries | $\$ 20,000$ | $\$ 12,000$ |

b. If she can only plant one crop, which crop should she plant?
c. Assume she can instead plant half her land with each crop. Which crop mix gives the highest expected income (all potatoes, all strawberries half of each). Which crop mix should Jane choose? Explain.
d. Assume Jane can choose any combination of Potatoe and Strawberry crops, provided that their total sums to 100 percent. What mix of crops maximizes Jane's expected utility?
e. Assume Jane decides to plant half her land with each crop. She is offered Strawberry insurance. This insurance costs $\$ 5,000$ and pays $\$ 10,000$ in the case of a drought. [Hence, the policy is actuarily fair.] Should Jane buy it? Explain your answer in light of your response to queston (a).
5. Bill is a Von-Neumann Morgenstern expected utility maximizer with a well-behaved, continuously differentiable utility function (i.e., no kinks or inflection points). Bill is presented with the following choices:
a. $\$ 1,000$ for sure
b. $50 \%$ chance of $\$ 800,50 \%$ chance of $\$ 1,500$
c. $\$ 500$ for sure
d. $50 \%$ chance of $\$ 400,50 \%$ chance of $\$ 900$

Bill is indifferent between $\mathbf{a}$ and $\mathbf{b}$ and is also indifferent between $\mathbf{c}$ and $\mathbf{d}$. (Note: this does not imply that he is indifferent between $\mathbf{a}$ and $\mathbf{c}$ or $\mathbf{b}$ and $\mathbf{d}$.)
(a) Is Bill risk neutral, risk averse, risk loving, or can't you tell? Explain.
(b) He is now faced with the following choice:
e. $\$ 750$ for sure
f. $25 \%$ chance of $\$ 400,25 \%$ chance of $\$ 900,25 \%$ chance of $\$ 800,25 \%$ chance of $\$ 1,500$

Will Bill choose $\mathbf{e}$ or $\mathbf{f}$, or is he indifferent between them, or is not possible to tell? (Prove your answer)

