FLIGHT TRANSPORTATION LABORATORY
REPORT R 92-4

PRESENTATIONS FROM

THE 1992 MIT / INDUSTRY

COOPERATIVE RESEARCH PROGRAM

ANNUAL MEETING

FTL

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FTL REPORT 92-4

PRESENTATIONS FROM THE ANNUAL MEETING OF THE MIT / INDUSTRY COOPERATIVE RESEARCH PROGRAM

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TABLE OF CONTENTS

Airline Network Seat Inventory Control: Methodologies and Revenue Impacts

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Outline

Introduction

The Seat Inventory Control Problem

Network Seat Inventory Control

Previous Work

Network Seat Inventory Control Approaches

Multiple Leg Flight Analysis

Network Optimization Approaches

Leg-Based Optimization Approaches

Hub-and-Spoke Analysis

Conclusion

Summary

Contributions

Future Work

Yield/Revenue Management

Pricing:

Determining the number and type of fares available in each market.

Reservations Control:

Determining how much of each product to sell.

Seat Inventory Control

Traditionally:

Practice of allocating seats to different fare classes.

Benefits:

Increase LF

Increase Yields

Competitive

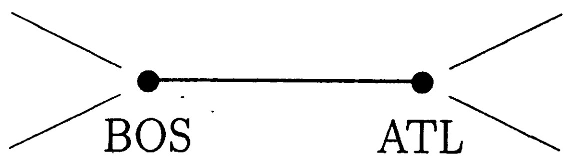
Need:

Supply $\neq$ Demand

- Probabilistic Demand
- Scheduling Constraints

Current Approach

Airlines currently control bookings by individual flight legs.

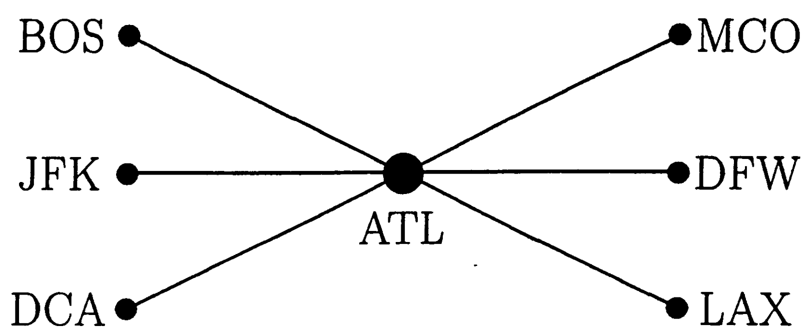


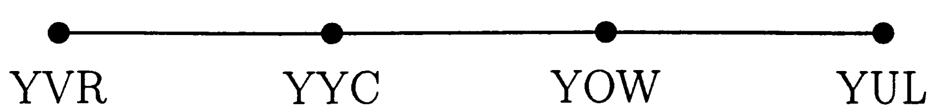
- Manage and maintain seat inventories by fare class.

- Maximize revenue by flight leg.

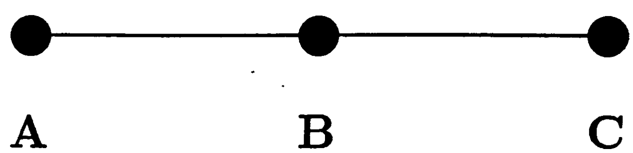
However, the seat inventory control problem is really a network problem.

- Passenger demand is based on itineraries.
- Flights are scheduled to connect with other flights through a hub and spoke structure or a multi-leg structure.





Maximizing flight leg revenue is not necessarily the same as maximizing total network system revenues.



AB	\$100
AC	\$150
BC	\$100

The Seat Inventory Control Problem

Not simply allocating seats between fare classes.

Decisions involve allocating seats between single leg itineraries and multi-leg or connecting itineraries.

Problem has grown with development of large hub-and-spoke operations:

- 50 market destinations per flight.
- 10 different fare classes in the coach cabin.
- 2500 flights per day.
- Controls applied beginning 330 days before departure.

Network Seat Inventory Control

Origin-Destination Control

Segment Control

Point-of-Sale Control

Characteristics and Complexities:

Passenger demand is probabilistic.

Demand is dynamic.

Fast solution times are necessary.

Multi-stage problem.

Seat allocations need to be integral.

Nested environment.

Size and complexity of **optimal** probabilistic, dynamic, nested, network seat inventory control problem is impractical.

Previous Research

Littlewood (1972)
Bhatia and Parakh (1973)
Richter (1982)

Belobaba (1987)

Brumelle and McGill (1988)
Wollmer (1988)
Curry (1988)

Buhr (1982)
Wang (1983)

Glover, et al. (1982)
Wollmer (1985)
D'Sylva (1982)

Curry (1990)

Network Optimization

Using traditional operations research, the problem can be formulated as a mathematical program.

Deterministic Formulation

$$\text{Maximize } \sum_{ODF} f_{ODF} \cdot x_{ODF}$$

subject to:

$$\sum_{ODF} x_{ODF} \leq C_j \qquad \text{for all ODF's on flight leg } j, \text{ for all flight legs } j.$$

$$x_{ODF} \leq D_{ODF} \qquad \text{for all ODF's.}$$

The solution is a set of distinct seat allocations for each ODF.

In order to take into account the uncertainty of demand, the problem can also be formulated probabilistically.

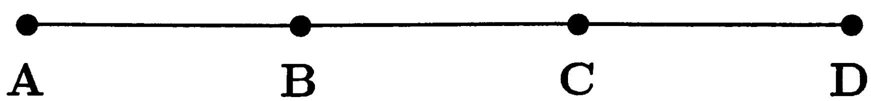
$$EMR(i_{ODF}) = f_{ODF} \cdot \overline{P}(i_{ODF})$$

$$\text{Maximize} \sum_{ODF} \sum_{i=1}^{C_j} EMR(i_{ODF}) \cdot x_{i,ODF}$$

subject to:

$\sum_{ODF} \sum_{i=1}^{C_j} x_{i,ODF} \leq CAP_j$	for all ODF's on flight leg j, for all flight legs j.
$x_{i,ODF} \leq 1$	for all ODF's, $i = 1, 2, ..., C_j$.

Multiple Leg Example



6 OD itineraries: AB, BC, CD
AC, BD
AD

4 Fare Classes: Y, M, B, Q

	Y	M	B	Q
AB	25.21	2.66	6.78	25.67
	7.26	4.94	14.02	11.37
	216.00	203.00	194.00	152.00
AC	2.15	1.45	4.16	14.44
	2.67	5.84	3.42	10.45
	519.00	344.00	262.00	231.00
AD	2.61	1.27	3.68	2.32
	3.25	1.56	6.62	2.69
	582.00	379.00	302.00	269.00
BC	9.64	22.48	11.55	32.50
	5.08	18.99	9.55	16.37
	440.00	315.00	223.00	197.00
BD	5.78	4.49	4.50	5.81
	4.77	5.78	5.53	5.52
	485.00	340.00	247.00	209.00
CD	19.42	55.70	7.43	5.63
	10.78	31.63	13.34	3.93
	251.00	179.00	164.00	134.00

	A-B	B-C	C-D
Local:	60.32	76.17	88.18
Through:	32.08	52.66	30.46
Total:	92.40	128.83	118.64

Deterministic Network Solution

	Y	M	B	Q
AB	25	3	7	26
AC	2	1	4	14
AD	3	1	0	0
BC	10	22	12	15
BD	6	0	0	0
CD	19	56	5	0

(Capacity = 90)

Probabilistic Network Solution

	Y	M	B	Q
AB	30	6	14	29
AC	3	1	2	2
AD	3	0	0	0
BC	13	25	9	24
BD	6	2	0	0
CD	22	50	4	3

(Capacity = 90)

Evaluating
Seat Inventory Control Approaches

Cost of developing new reservations systems, as well as updating support systems, is quite high.

Want to determine realistic revenue expectations in advance.

Modeled booking process of an airline and developed an integrated optimization/booking process simulation:

Network Based
Dynamic

Integrated Optimization/Booking Process Simulation

- Inputs: Network of ODF combinations.
  Fares.
  Incremental means and standard deviations of forecasted demand.
  Aircraft/Cabin capacities.
  Number of revision points.
- ODF seat allocations and booking limits are calculated based on the remaining capacity of each flight leg and the total forecasted demand to come.
- Demand for each ODF is randomly generated for the booking period at hand.
- Demand is booked, given seats are available.
- The booking process is repeated for each booking period.
- The complete booking and revision process for a single network of departures is repeated a number of iterations.

Multi-Leg Flight Simulation Results

Based on real airline data providing both a realistic mix or traffic and a realistic representation of ODF booking profiles.

Demand assumed to follow a Poisson distribution.

Bookings on hand and bookings to come assumed to be independent.

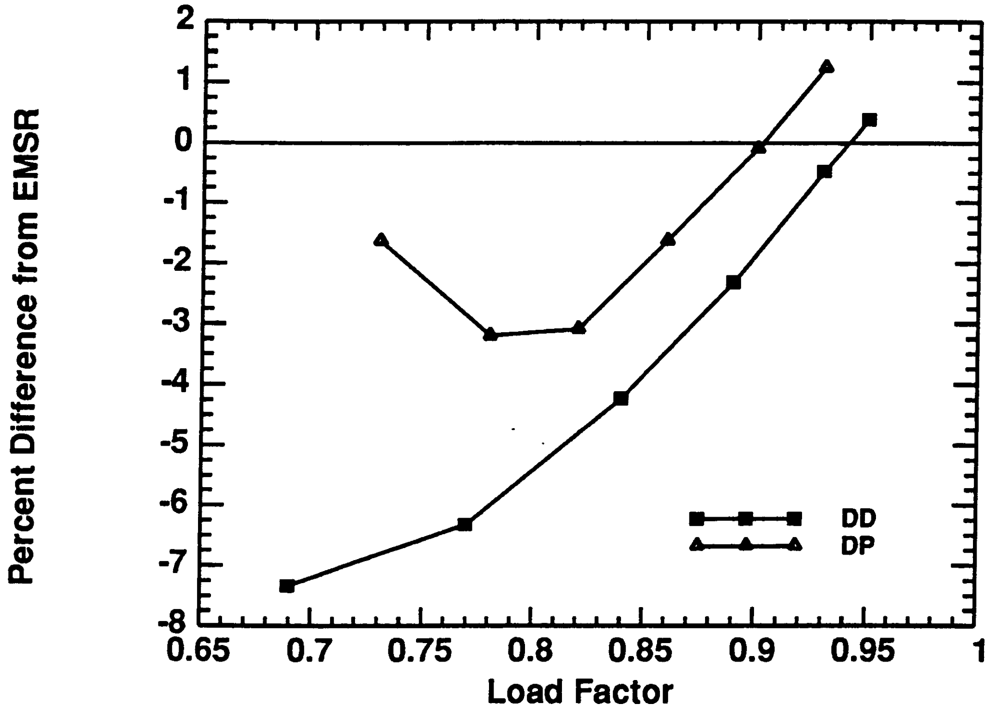
Within each booking period the lowest fare class books first.

15 booking periods.

500 iterations.

Revenue impacts compared to leg-based EMSR fare class control approach.

Distinct Network Methods

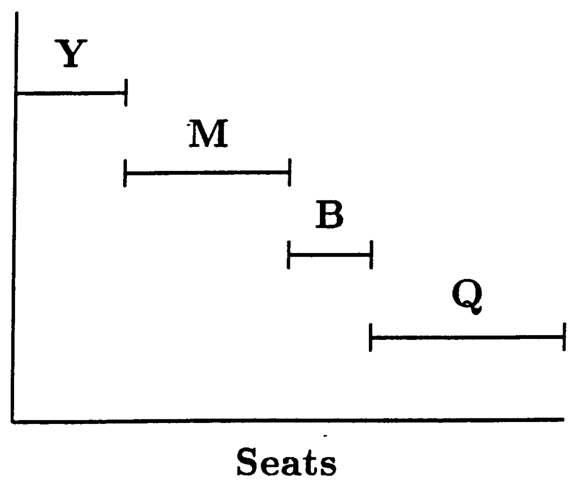


Directly applying distinct booking limits can result in negative revenue impacts.

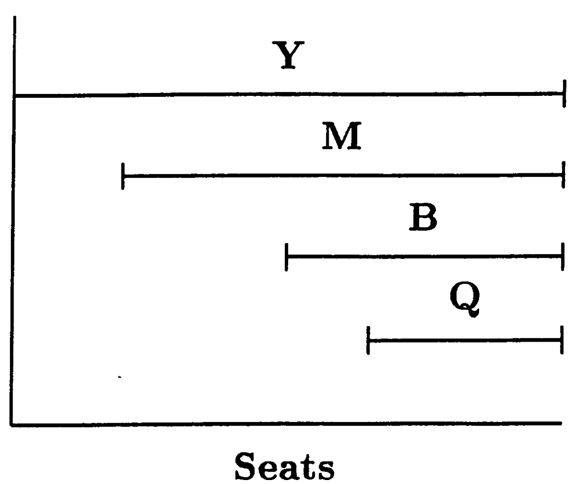
To overcome this problem, and still get the benefits of incorporating network flows, we use the idea from leg-based control methodologies of **nesting**.

Inventories are nested so that as long as there are seats available, a higher revenue, more desirable request will not be denied.

Non-nested, distinct structure:



Nested structure:



Nesting Possibilities

Fare Classes

- Aggregate ODF allocations back to the fare class level on each flight leg.
- No longer have the control of different itineraries.

Fares

- Nest based on total itinerary fare value.
- Introduces aspect of "greediness" where long-haul itineraries will receive priority over local itineraries.

Shadow Prices

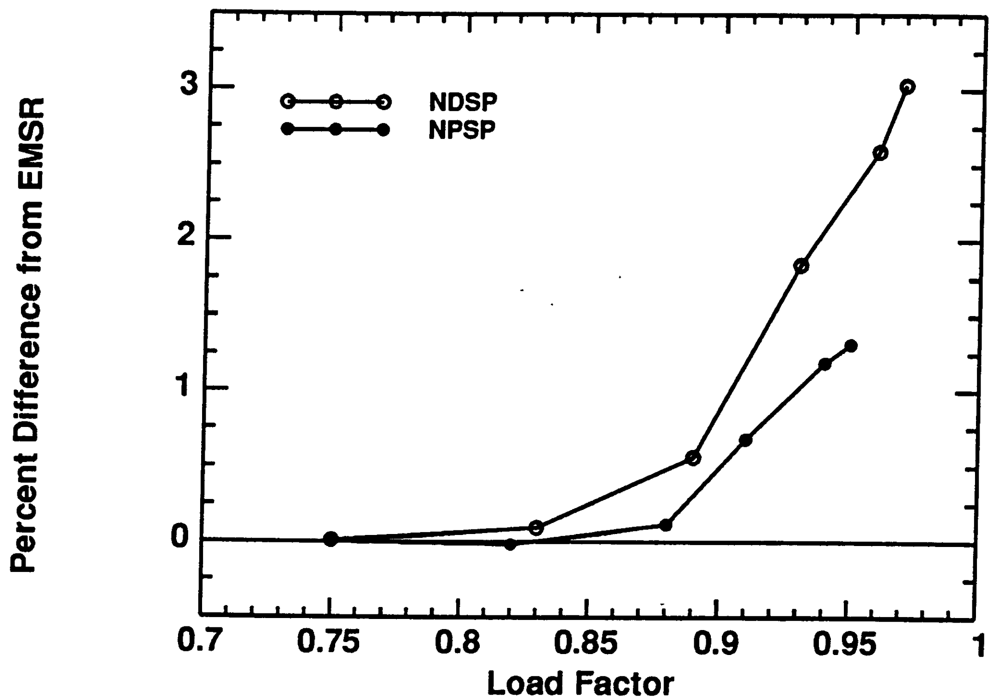
- Use information from the dual, nest based on the shadow price of the demand constraints.
- ODF's with higher shadow prices have a higher potential value to the network.

Nesting Deterministic by Shadow Prices

Leg B-C

ODF	Fare	Shadow Price	Reduced Cost	Seats Allocated	Booking Limit
ACY	$519	322	0	2	90
BCY	$440	243	0	10	88
ADY	$582	221	0	3	78
ACM	$344	147	0	1	75
BDY	$485	124	0	6	74
BCM	$315	118	0	22	68
ACB	$262	65	0	4	46
ACQ	$231	34	0	14	42
BCB	$223	26	0	12	28
ADM	$379	18	0	1	16
BCQ	$197	0	0	15	15
BDM	$340	0	-21	0	0
ADB	$302	0	-59	0	0
ADQ	$269	0	-92	0	0
BDB	$247	0	-114	0	0
BDQ	$209	0	-152	0	0

Network Methods
Nested by Shadow Prices



The partitioned probabilistic optimization approach tends to overprotect seats for the more desirable and higher fare class ODF's.

Leg A-B

Partitioned Deterministic

	Y	**M**	**B**	**Q**
AB	$\boxed{25}$	3	7	26
AC	$\boxed{2}$	1	4	14
AD	$\boxed{3}$	1	0	0

Partitioned Probabilistic

	Y	**M**	**B**	**Q**
AB	$\boxed{30}$	6	14	29
AC	$\boxed{3}$	1	2	2
AD	$\boxed{3}$	0	0	0

This overprotection of seats is compounded as the booking process proceeds.

Mean Demand	Deterministic Allocation	Probabilistic Allocation
25.2	25	28
25.1	25	28
24.8	25	28
24.0	24	28
22.8	23	28
22.0	22	26
20.4	20	26
19.3	19	26
16.9	17	25
15.6	16	23
12.3	12	21
9.2	9	19
8.6	9	18
5.9	6	15
2.6	3	11

Network Bid Price

Bid Price is a Shadow Price for the capacity constraint.

The marginal value of the last seat of a given flight leg.

Bid Prices establish a "cutoff" value for each flight leg, on which decisions can be made whether to accept or reject a given ODF request.

For a single leg itinerary, a fare class is open for bookings if the corresponding fare is greater than the bid price, or shadow price, for the leg.

For a multi-leg itinerary, fares must be greater than the sum of the bid prices of the respective flight legs.

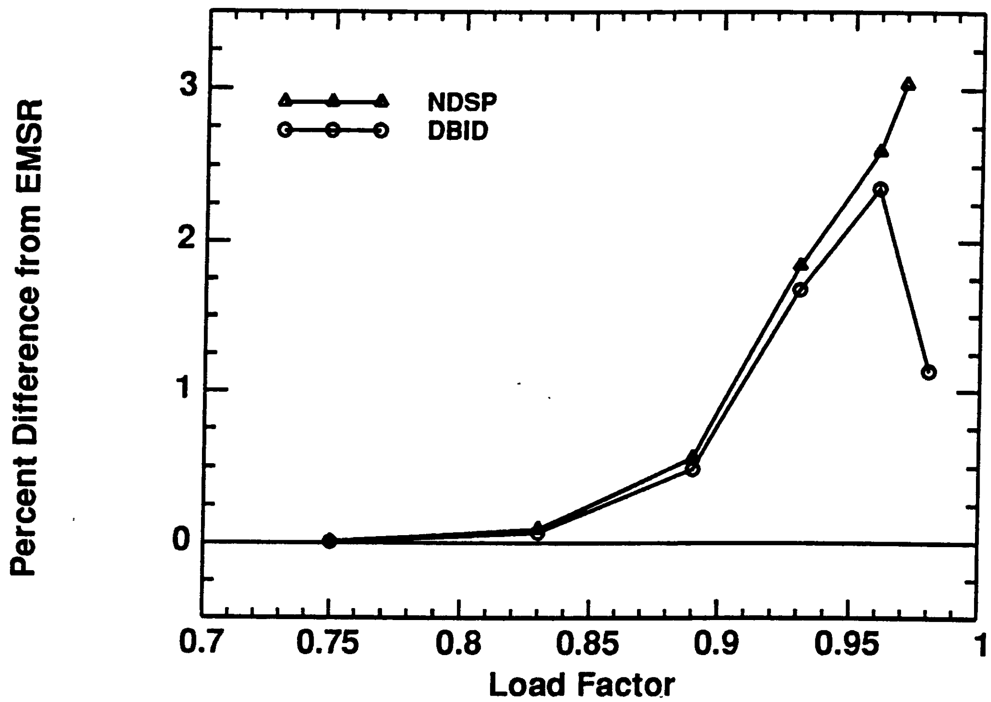
Bid Price Example

Bid Prices

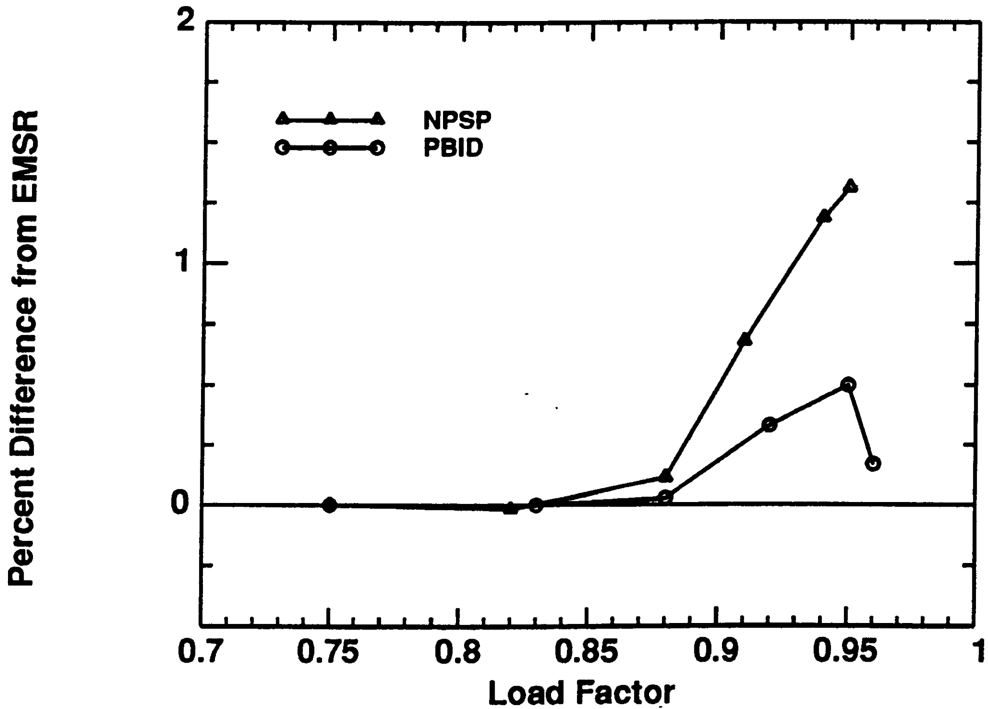
A-B:	65
B-C:	197
C-D:	138

BCY	$440	**ACY**	$519	**ADY**	$582
BCM	$315	**ACM**	$344	**ADM**	$379
BCB	$223	**ACB**	$262	**ADB**	$302
BCQ	$197	**ACQ**	$231	**ADQ**	$269

Deterministic Network Methods

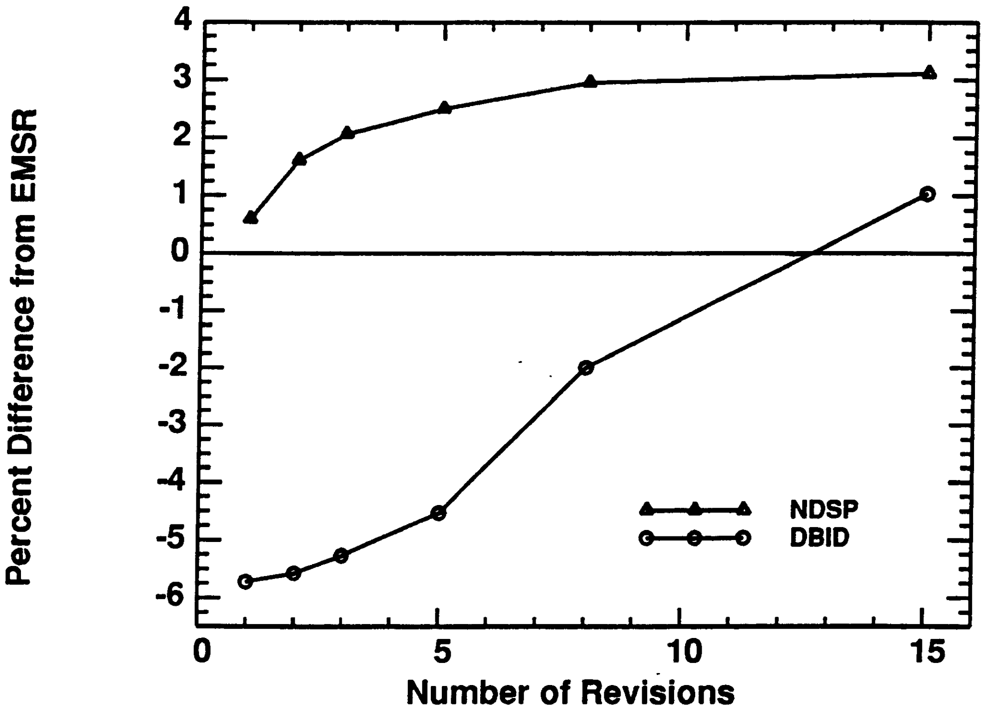


Probabilistic Network Methods



Revenue Impacts vs. Revisions

96/97% Load Factor



The Nested Deterministic by Shadow Prices approach allows for better control of bookings

Immediate Problems with Network Optimization Control Methods

Data on the itinerary/fare class level is not currently collected.

The small numbers and large variations of ODF demand forecasts.

Current inventory structures control bookings at the flight leg level.

Communications with computer reservations systems of other airlines.

Leg-Based OD Control Heuristics

Nesting of network allocations is a heuristic in itself.

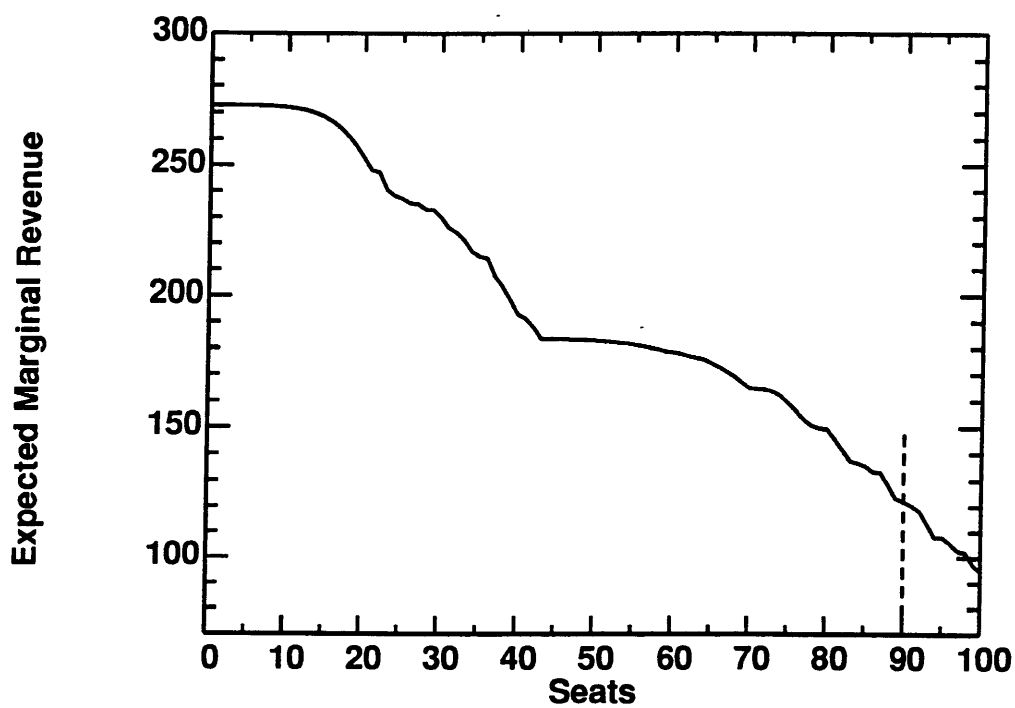
Use general ideas and concepts from network optimization, but at the leg level, such that:

- Information about passenger demand and traffic flows are taken into account.
- Optimization and control remains at the leg level.

Leg-Based Bid Price

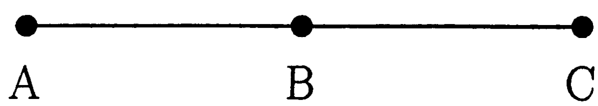
Similar to network bid prices, information at the leg level can be used to determine the marginal value of the last seat on a given flight leg.

$$EMR(S_i) = f_i \cdot \overline{P}(S_i)$$



$EMR(C)$ gives us a "cut-off" value for the flight leg.

The leg-based bid price values can be used in the same manner as network bid prices.

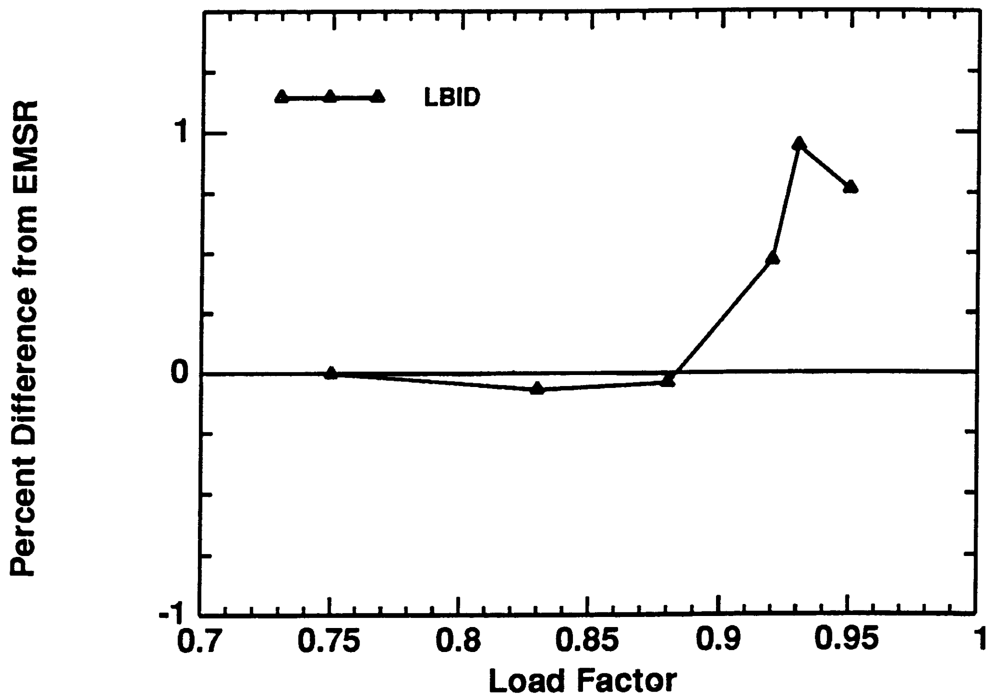


$$\begin{aligned} EMR(C_{A-B}) &= \quad 214 \\ EMR(C_{B-C}) &= \quad 86 \end{aligned}$$

Cut-off Values

AB	\$214
AC	\$300
BC	\$86

Leg-Based Bid Price



Combined Leg-Based Bid Price/Booking Limit Approach

The leg-based bid price acceptance rule is used to assess the approximate value of different ODF's to the network.

$$f_{ODF} \geq \sum_{j} EMR(C_j)$$ for all flight legs j over which the ODF traverses.

ODF seat availability is limited by the maximum of the respective fare class booking limits from the appropriate flight legs.

$$BL_{ODF} = Max(BL_{i,j})$$ for the respective fare class i and all flight legs j of ODF.

Virtual Nesting on the "Value Net of Opportunity Cost"

The $EMR(C)$ value can also be used as an estimate of a displacement cost, or opportunity cost, in a virtual nesting system.

Under the "greedy" virtual inventory system, total itinerary ticket revenues are used to assign, or map, each ODF to a virtual inventory bucket.

ABY	$200
ACY	$350
BCY	$250

	A-B	**B-C**
V1: 300-	ACY $350	ACY $350
V2: 250-299		BCY $250
V3: 200-249	ABY $200	
V4: 160-199		
V5: 130-159		
	⋮	⋮

Under Virtual Nesting on the "Value Net of Opportunity Cost" approach, each ODF is mapped to the virtual buckets based on total itinerary revenue minus upline and downline displacement costs.

ABY $200
ACY $350
BCY $250

	A-B $EMR(C)$=**150**	**B-C** $EMR(C)$=**100**
V1: 300-		
V2: 250-299	ACY $250	BCY $250
V3: 200-249	ABY $200	ACY $200
V4: 160-199		
V5: 130-159		
⋮	⋮	⋮

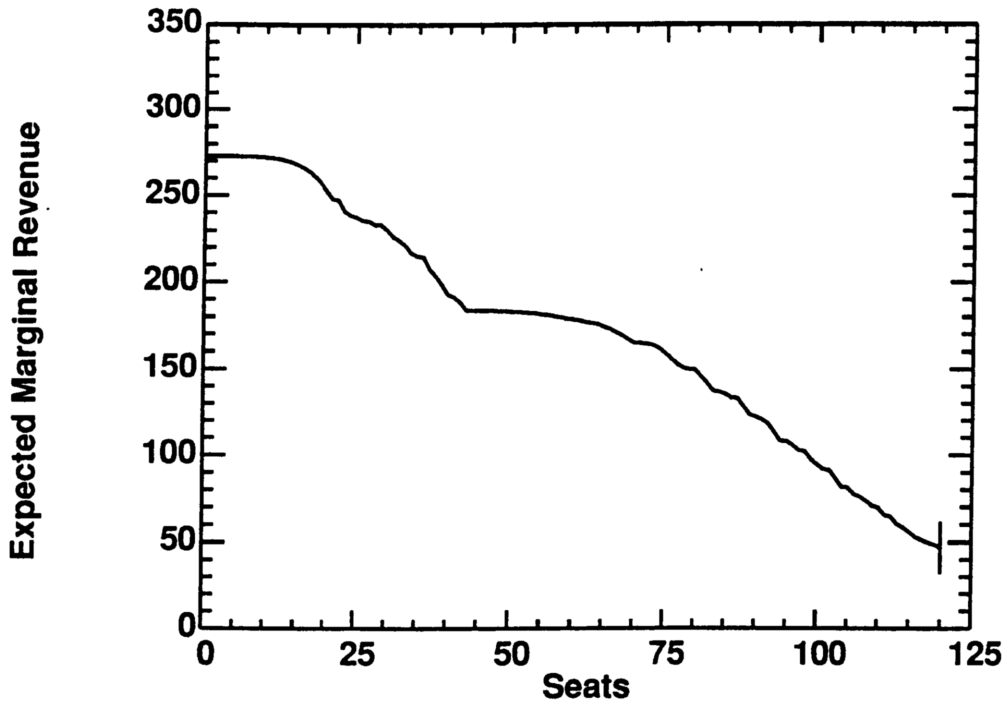
Using leg based optimization methods, such as EMSR, booking limits for each virtual inventory bucket are then determined.

Nested Leg Based Itinerary Limits

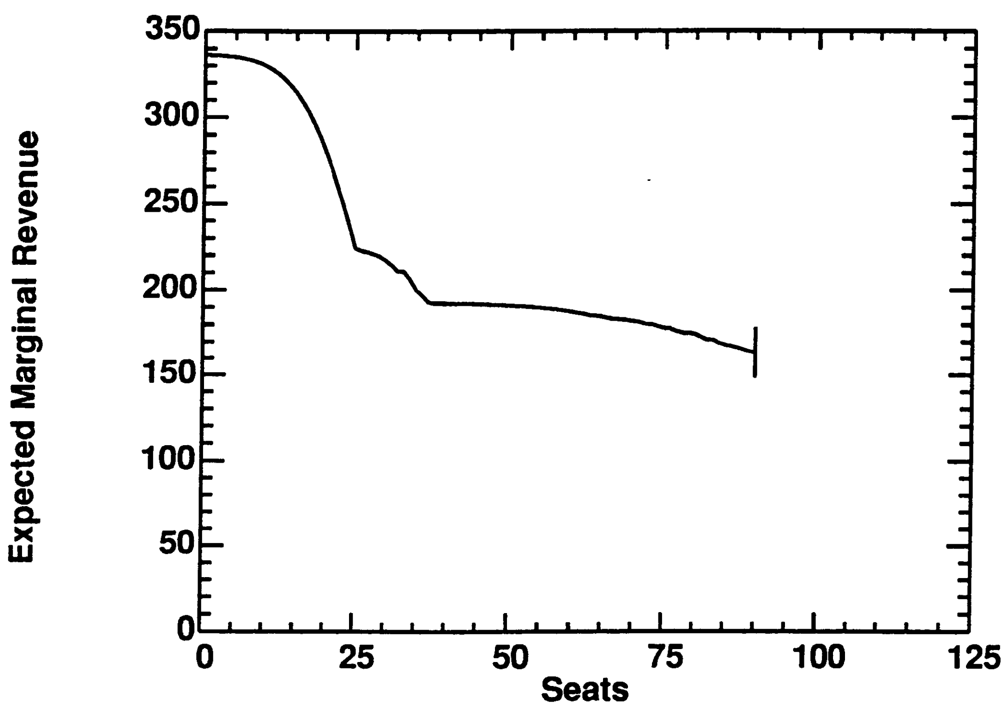
Based on the EMR Bid Price logic, but using information from the entire EMR curve, leg based itinerary limits can be determined.

- For single leg itineraries, booking limits remain the same.
- For multi-leg itineraries, the EMR curves from the respective flight legs are summed and booking limits determined based on where the itinerary revenue value intersects the total EMR curve for the itinerary.

EMR Curve - Leg A-B

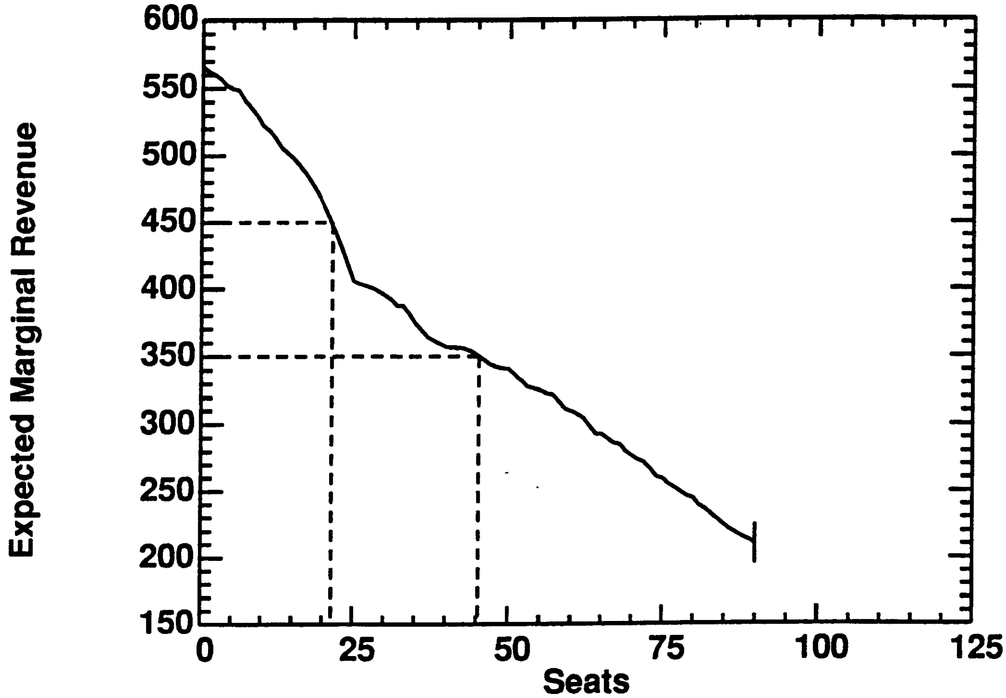


EMR Curve - Leg B-C



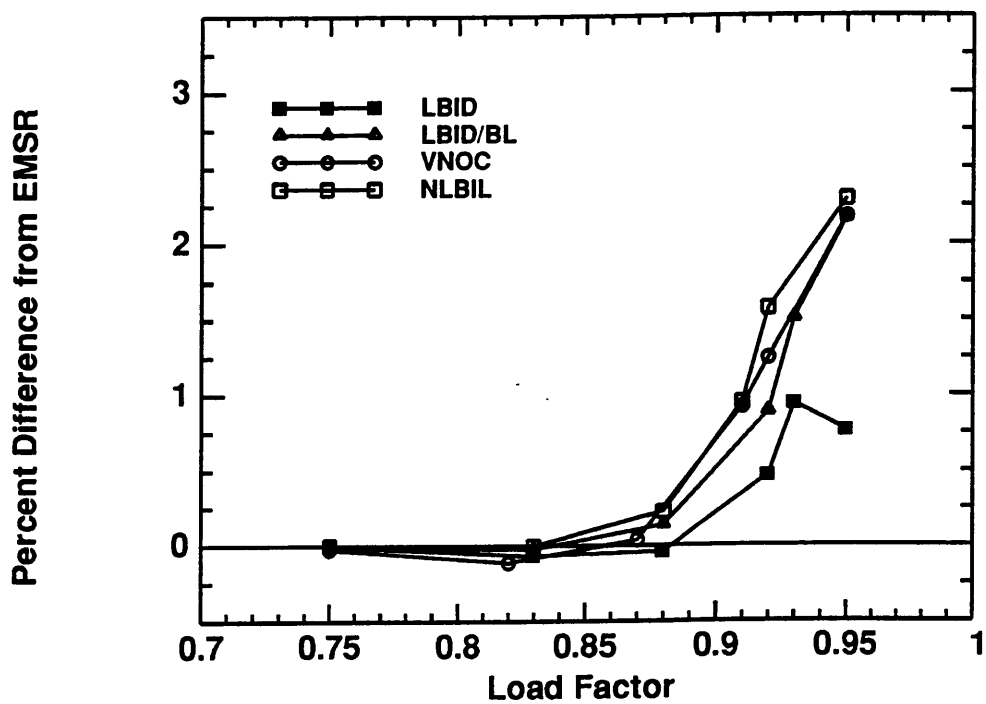
Sum beginning with the last seat on each flight leg.

Total EMR Curve - AC Itinerary



ACY	\$450	69 Seats
ACM	\$350	45 Seats

Leg-Based OD Control Methods



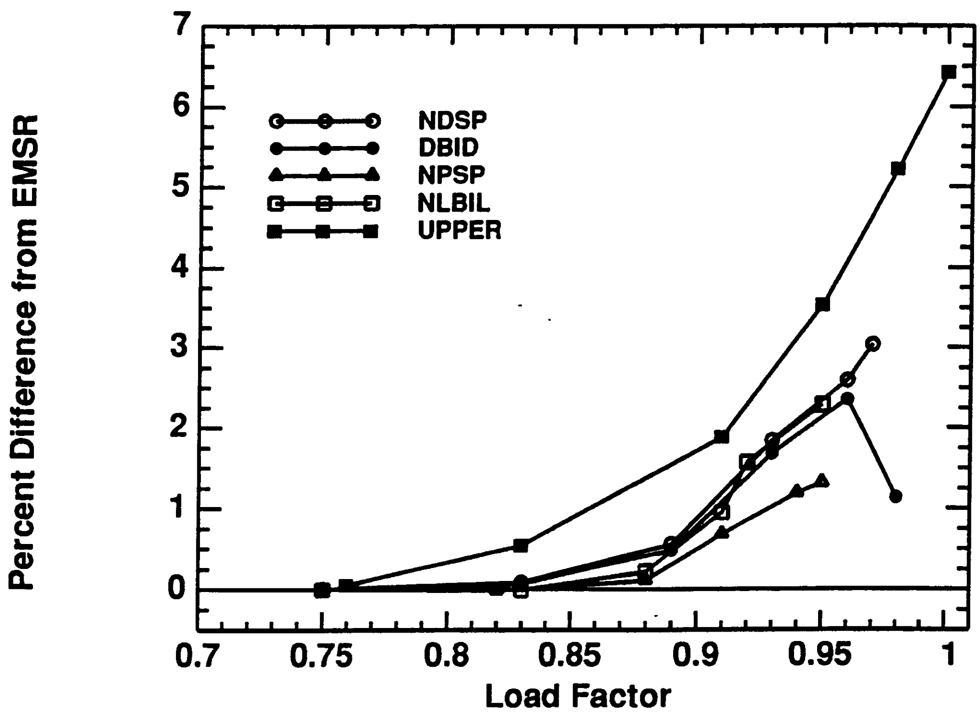
Upper Bound

The "upper bound" is the revenue obtained from decisions based on perfect information, i.e. what would we have done in hind sight.

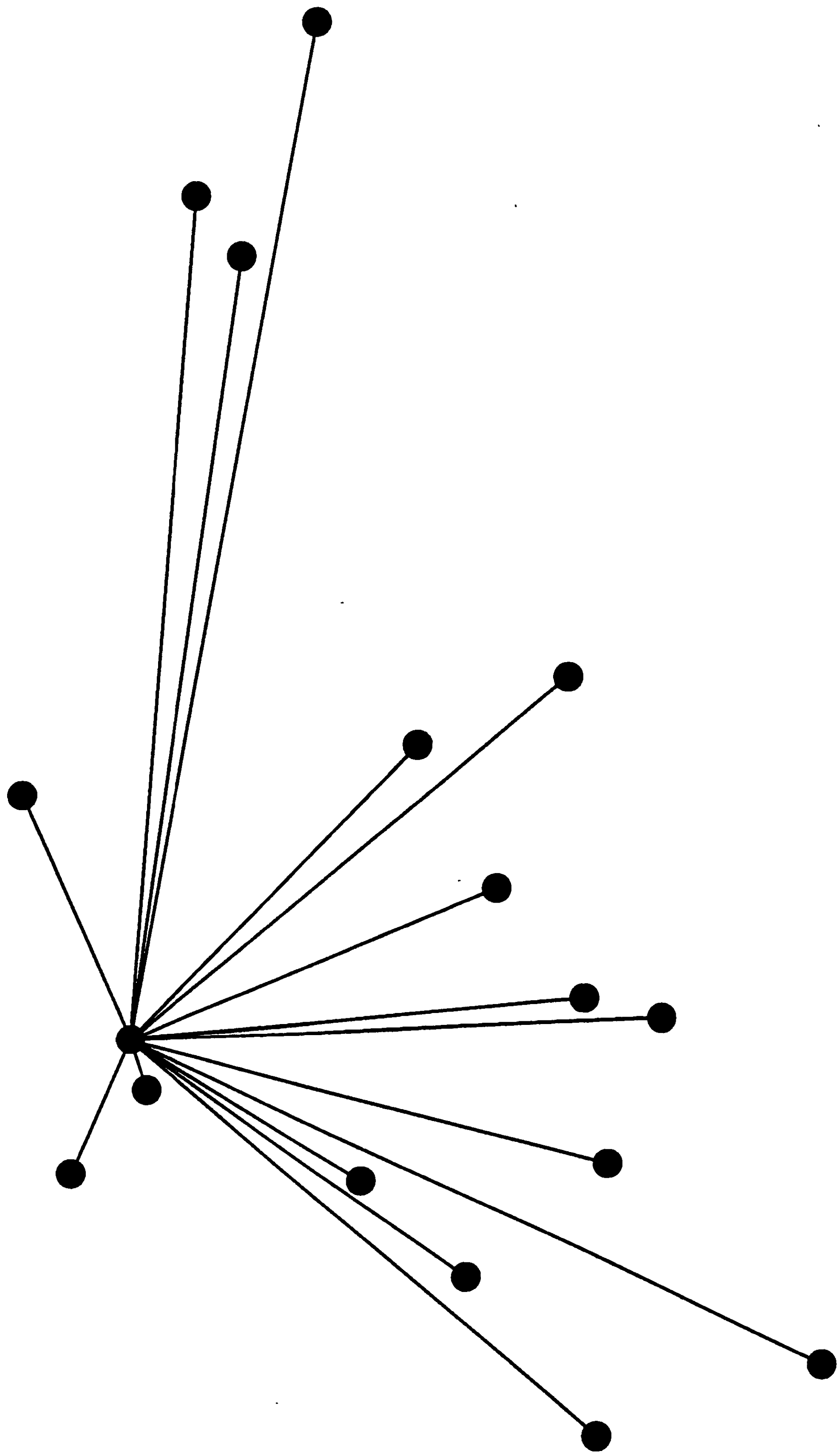
- All requests for the full booking process are randomly generated.
- The optimial combination of ODF requests which maximizes the revenue of the network is booked.

The "upper bound" represents the maximum possible revenue for a particular set of requests across a network.

Summary Comparison for the Multiple Leg Flight



Hub Network



Hub Network

16 flights in/16 flights out.

Demand exists for 196 of the 272 OD pairs.

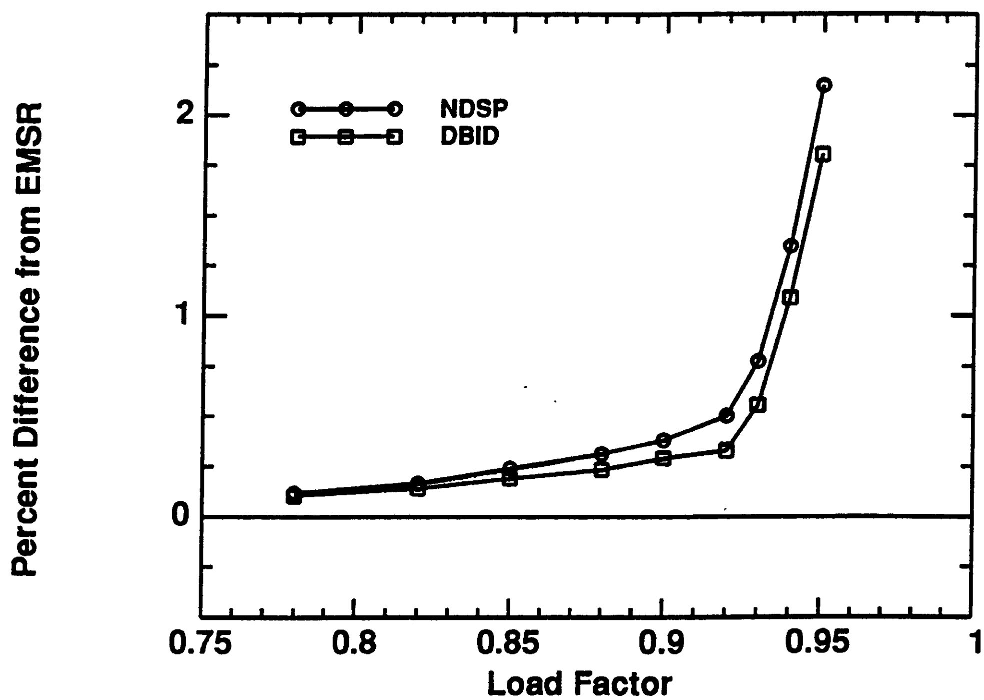
10 fare classes.

Base case:

> Demand factor on the different flight legs ranges from 0.56 to 1.46, with an overall average demand factor of 0.95.

Incremental ODF demand data for 20 booking periods.

Deterministic Network Methods



NDSP

Base Case: 88% Load Factor
0.3% Improvement
$7500 per day
$2.7 million per year

Aggregated Network Optimization Methods

For a hub-and-spoke network, problems arise with forecasting individual ODF demand due to the small numbers problem.

Aggregating ODF's together on a global level while preserving differences in the level of attractiveness to the network of each ODF is difficult.

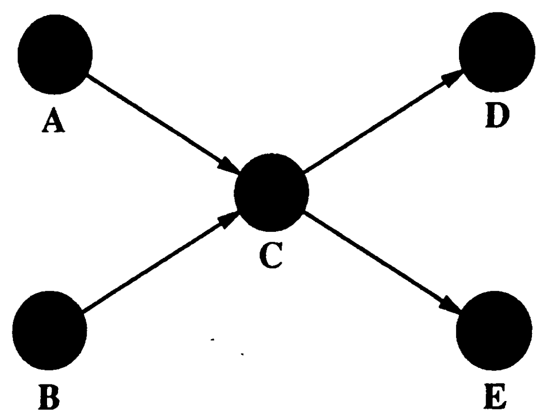
However, from the perspective of an individual flight leg, combinations of ODF's that have the same level of attractiveness can be aggregated.

Using the property that the mean value of the sum of two variables is equal to the sum of their mean values:

$$\mu_{(A+B)} = \mu_A + \mu_B,$$

formulating an aggregated deterministic network optimization is a straightforward extension of the full deterministic network model.

Aggregated Deterministic Network Optimization



	Mean Demand	Fare
ACY	40	$100
ADY	30	$150
AEY	40	$150
BCY	30	$100
BDY	10	$150
BEY	20	$150
CDY	20	$100
CEY	50	$100

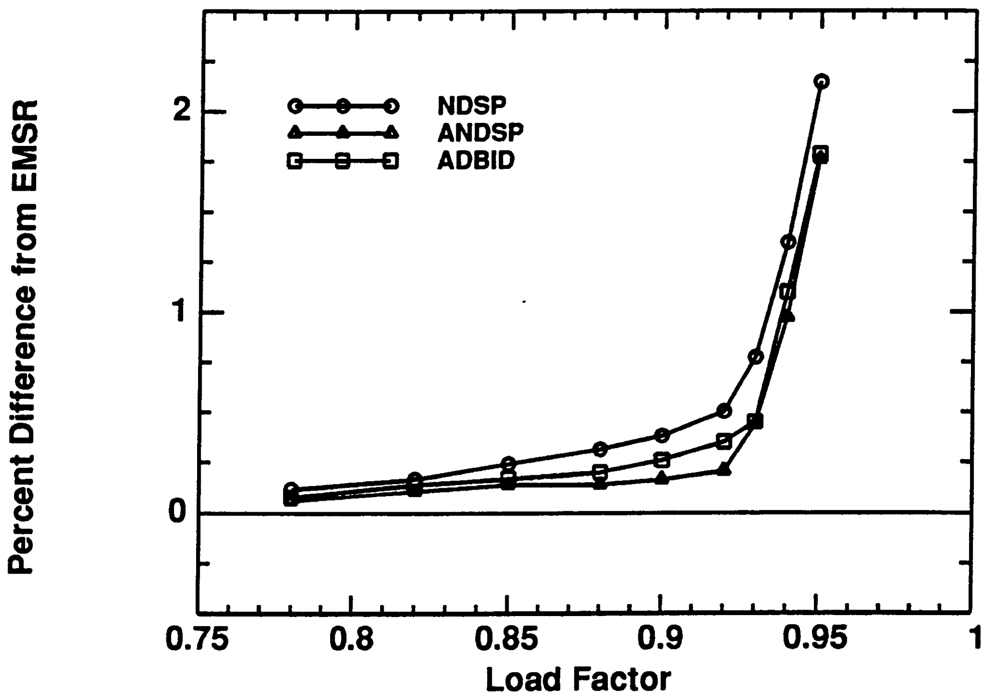
Maximize $100x_{ACY} + 150x_{ADY} + 150x_{AEY} +$
$100x_{BCY} + 150x_{BDY} + 150x_{BEY} + 100x_{CDY} + 100x_{CEY}$

subject to:

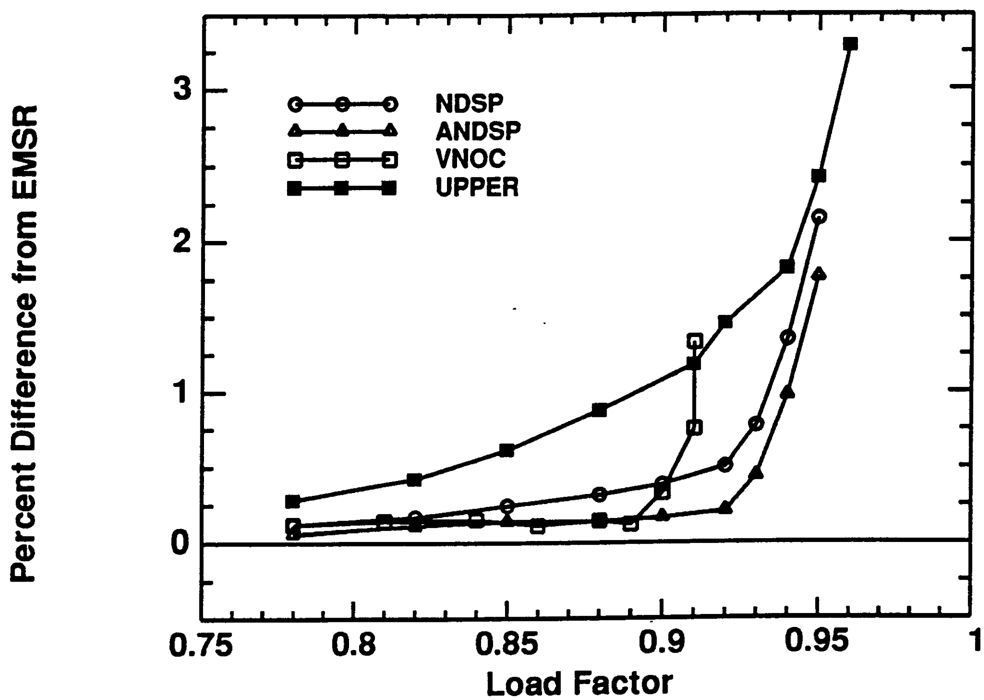
$$\begin{aligned} x_{ACY} + x_{ADY} + x_{AEY} &\leq 100, \\ x_{BCY} + x_{BDY} + x_{BEY} &\leq 100, \\ x_{ADY} + x_{BDY} + x_{CDY} &\leq 100, \\ x_{AEY} + x_{BEY} + x_{CEY} &\leq 100 \end{aligned} \tag{1}$$

$$\begin{aligned} x_{ACY} &\leq 40, \\ x_{ADY} + x_{AEY} &\leq 70, \\ x_{BCY} &\leq 30, \\ x_{BDY} + x_{BEY} &\leq 30, \\ x_{CDY} &\leq 20, \\ x_{ADY} + x_{BDY} &\leq 40, \\ x_{CEY} &\leq 50, \\ x_{AEY} + x_{BEY} &\leq 60. \end{aligned} \tag{2}$$

Aggregated Deterministic Network Methods



Summary Comparison for the Hub Network



Summary

Through the upper bound analysis, the true potential of better seat inventory control is obtained. This maximum potential ranges from 4-8%.

Direct application of traditional network seat inventory control solutions yields significant negative revenue impacts.

By using information from the dual, network solutions can be applied to the seat inventory control problem, providing revenue benefits of approximately 1/2 of the maximum potential.

Due to practical constraints, it is currently difficult to implement such approaches.

By using concepts from the network optimization approaches to develop leg-based heuristics which incorporate information about traffic flows, approximately 1/3 of the maximum potential revenue can be obtained.

Contributions

Developed the theory for a realistic representation of the interaction between airline reservations control and the booking process which is modeled through a computer simulation.

Demonstrated that direct application of traditional network seat allocation solutions provide significant negative results.

Introduced several new practical approaches to network seat inventory control which can provide significant positive revenue impacts over current leg-based approaches:

- Network optimization approaches
- Leg-based heuristics

Showed that using a **partitioned** probabilistic network solution as the basis for nested control applications is not as effective as a deterministic network solution.

Generated realistic estimates of the revenue impacts of controlling seat inventories at the network level.

Future Work

- Extensions of leg-based heuristics to virtual nesting.
- Value definitions for virtual inventory buckets.
- Most efficient mathematical algorithms for network approaches.
- Compound Poisson distribution.

- Effects on revenue impacts of ODF forecasting accuracy.
- Effects on revenue impacts other airline computer reservations systems.

- Including first class cabin in network seat inventory control problem.
- Incorporating overbooking.
- Practical dynamic programming approaches.
- Full integration of reservations control with pricing and scheduling.

Modeling Airline Group Passenger Demand For Seat Inventory Control

Prepared For The

MIT Cooperative Research Meeting

Tom Svrcek
MIT Flight Transportation Laboratory
May 22, 1992

Outline Of Presentation

Individual Passenger Demand

Group Passenger Demand

Group Demand Decomposition

The Group Demand Model

Group Demand Forecasting

Application To Seat Inventory Control

Conclusions

Further Research

Individual Passenger Demand

Individual Passenger Demand Is Subject To Many Sources Of Variability.

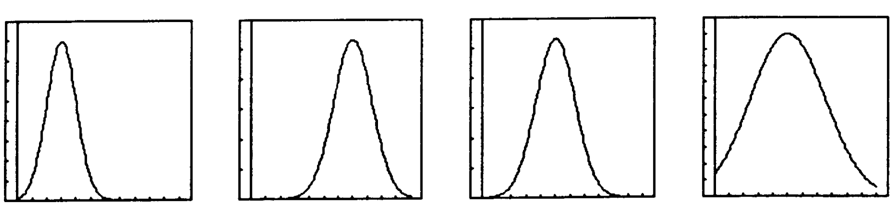
(Time Of Day, Day Of Week, Season)

Market	Date	Day	Dept	Load
ORD–BOS	12 FEB 92	MON	07:00	135
ORD–BOS	12 FEB 92	MON	14:00	115
ORD–BOS	13 FEB 92	TUE	07:00	120
ORD–BOS	11 AUG 92	MON	07:00	155

(Random)

ORD–BOS	12 FEB 92	MON	07:00	135
ORD–BOS	19 FEB 92	MON	07:00	131

To Account For Such Variability, Most Airlines Employ A Normal Distribution Assumption When Forecasting Demand.



Y Class M Class B Class Q Class

Empirical Results Support Such An Assumption

Group Passenger Demand

How Is Group Passenger Demand Different From Individual Passenger Demand ?

- Groups Negotiate For A Lower Than Published Fare (Bulk Pricing)

- Group Demand Is Realized Many Months In Advance Examples : Carnaval, Olympics, Oktoberfest ...

- Unused Bookings Are Absent From Seat Inventory For Months, Potentially Displacing Individual Passengers

- Cancellation Penalties Often Difficult To Enforce Due To Competitive Environment

Group Demand Decomposition

Forecasting Group Passenger Demand Using The Normal Assumption May Be Inappropriate :

- Number Of Groups On A Given Flight Is Relatively Small
- Spikes Occur At "Popular" Group Sizes, Skewing The Distribution
- Significant Cancellations Occur Throughout The Booking Period

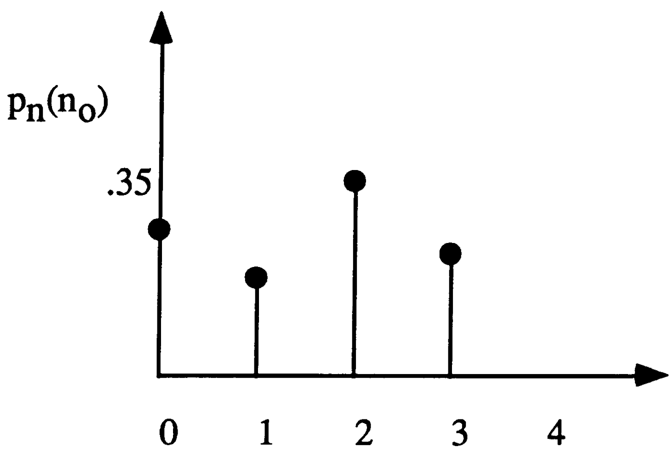
Question : What Is The Appropriate Model For Group Passenger Demand ?

To Forecast Group Demand, We Look At Three Constituent Components Separately :

1) Number Of Group Requests, n
2) Size Of Any Single Group Request, s
3) Utilization Rate Of Any Single Request, u

Number Of Group Requests (n)

The Number Of Group Requests, *n*, For A Particular Flight Can Be Modeled As A Probability Mass Function :



From Above ...

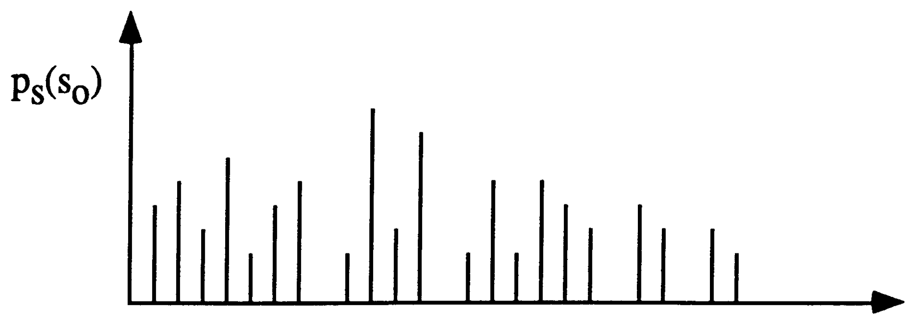
Historically, 35% Of Departures Had Two Group Requests

- or -

The Probability Of Receiving Two Group Requests For This Flight Is . 35

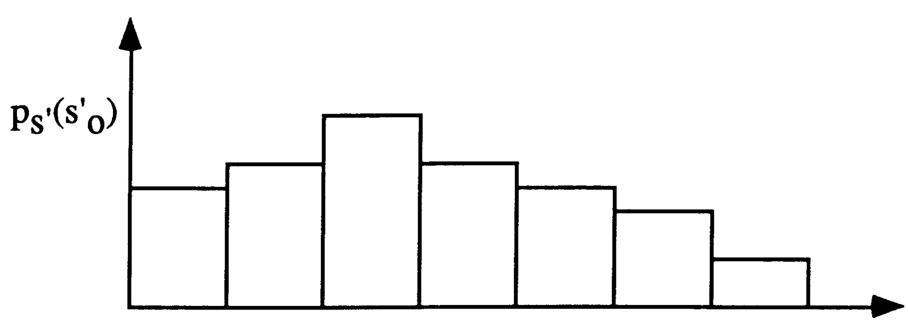
Size Of Any Single Request (s)

Similarly, Size Of Any Individual Request, *s*, Also Modeled As A PMF :



But ... - Bounds On *s* : [0 , CAP]
- Missing Values Easily Misinterpreted

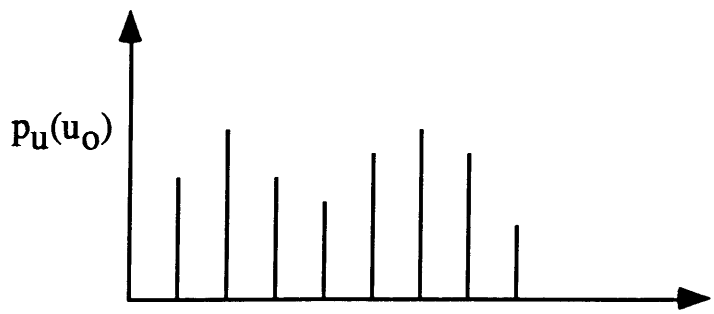
Aggregation Yields



Advantages - Reduces Number Of Observations
- Eliminates "Absence" Problems

Utilization Of A Group Request (u)

Utilization Rate, *u*, Also Modeled As A PMF :



There Are At Least Three Separate Utilization Rates, With Associated Costs Corresponding To Three Distinct Time Periods During The Group Booking Process :

1) Time Between Negotiation And Placement Of Non-Refundable Deposit (c_1)

2) Time Between Placement Of Deposit And Actual Purchase Of Tickets (c_2)

3) Time Between Actual Ticket Purchase And Date Of Departure (c_3)

We Believe :

$$c_2 >> c_3 > c_1$$

Group Demand Model

Discrete Transform Analysis :

Discrete Random Variables With Only Non-Negative, Integer Values Can Be Completely Defined By A Discrete Or z-Transform,

$$p_x^T(z) = E(z^x) = \sum_{x_0=0}^{\infty} z^{x_0} \cdot p_x(x_0)$$

Furthermore, It Is Possible To Determine The Individual Terms Of A PMF From Its Transform By :

$$p_x(x_0) = \frac{1}{x_0!}\left[\frac{\partial^{x_0}}{\partial z^{x_0}} p_x^T(z)\right]_{z=0} \qquad x_0 = 0,1,2\ \ldots$$

Group Demand Model

The Distribution For The Sum Of A Random Number Of Independent, Identically Distributed Random Variables Can Be Determined Using Transform Analysis.

Let r Be The Sum Of n (A Random Variable) Independent Values Of Random Variable x.

The z-Transform For The PMF Of r Is :

$$p_r^T(z) = p_n^T\left[p_x^T(z)\right]$$

Using The Chain Rule For Differentiation, We Can Obtain Expressions For The Expectation And Variance For r :

$$E(r) = E(n) \cdot E(x)$$

$$\sigma_r^2 = E(n) \cdot \sigma_x^2 \quad + \quad [E(x)]^2 \cdot \sigma_n^2$$

Group Demand Model

Distribution Of Group Booking Requests (r)

Assume The Number Of Group Requests And The Size Of Any Individual Request Are Statistically Independent.

The Distribution Of Total Group Seats Requested (r) Is Described By :

$$p_r^T(z) = p_n^T\left[p_s^T(z)\right]$$

The Expressions For The Expectation And Variance Are :

$$E(r) = E(n) \cdot E(s)$$

$$\sigma_r^2 = E(n) \cdot \sigma_s^2 \quad + \quad [E(s)]^2 \cdot \sigma_n^2$$

Group Demand Model

Distribution Of Group Passengers (g)

Incorporating The Utilization Rate Into Our Model Involves A Random Sum Of The Random Variable *r*.

The Distribution For *g*, The Level Of Group Passenger Demand Can Be Expressed By The Transform :

$$p_g^T(z) = p_u^T\left[p_r^T(z)\right]$$

Expressions For The Expectation And Variance Of *g* Are :

$$E(g) = E(u) \cdot E(r)$$

$$\sigma_g^2 = E(u) \cdot \sigma_r^2 \quad + \quad [E(r)]^2 \cdot \sigma_u^2$$

Group Demand Forecast Application

Ability To Forecast Group Passenger Demand Allows Application Of Results To Seat Inventory Control Models :

Types Of Models:

Planning :

Used In Advance Of Demand Realization. Allocates Seats To The "Optimal" Mix Of Passengers To Come.

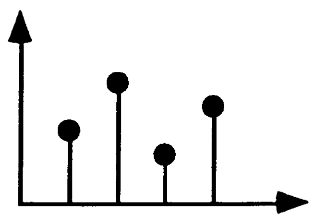
Decision Making

"Yes" Or "No" Result For Actual Request. Can Use Planing Model To Determine Revenue Potential With Or Without Request.

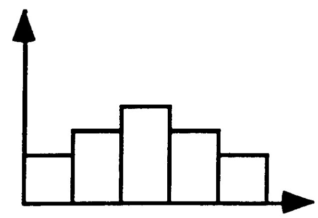
Accept/Reject Accordingly

Group Demand Forecasting

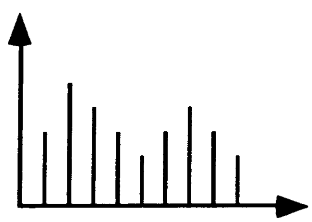
In General, Much Easier To Forecast Each Of The Three Constituent Components Individually, Using Historical Data :



$p_n(n_o)$



$p_s(s_o)$



$p_u(u_o)$

Combine These Three Distributions Using Discrete Transforms To Obtain A Forecast For Group Passenger Demand

Group Demand Forecast Application

Extend Traditional Math Programming Techniques For Seat Inventory Control To Include A G-Class :

Maximize

$$\sum_{i=1}^{k} F_i \cdot x_i \quad + \quad F_g \cdot \mu_g \cdot x_g$$

Subject to

$$\mathrm{x_i} \le \mu_i \qquad \text{for } i = 1,\ldots,k$$

$$\sum_{i=1}^{k} x_i + (\mu_g \cdot x_g) \le C$$

$$x_i \ge 0 \ \text{ and integer} \quad \text{for } i = 1,\ldots,k$$

$$x_g = 0 \text{ or } 1$$

Group Demand Forecast Application

Can Also Be Applied To Probabilistic Integer Programming Model (Group Demand Deterministic)

Maximize

$$\sum_{i=1}^{k}\sum_{j=1}^{C}\left[EMR_{i,j}\cdot x_{i,j}\right]+F_g\cdot\mu_g\cdot x_g$$

Subject to

$$\sum_{i=1}^{k}\sum_{j=1}^{C}x_{i,j}+x_g\cdot\mu_g\leq C$$

$$x_{i,j},x_g=0,1$$

Group Demand Forecast Application

With A Completely Defined Distribution Of Group Demand, We Can Include Expected Revenues From "Group" Seats As Well :

Maximize

$$\sum_{i=1}^{k,g} \sum_{j=1}^{C} \left[EMR_{i,j} \cdot x_{i,j} \right]$$

Subject to

$$\sum_{i=1}^{k,g} \sum_{j=1}^{C} x_{i,j} \leq C$$

$$x_{i,j}, x_{g,j} = 0,1$$

Conclusions

- Group Demand Differs Significantly From Individual Passenger Demand
- Differences Motivate A Different Distribution Assumption
- Demand Decomposition Into
    - Number Of Requests
    - Size Of Individual Request
    - Utilization Rate Of Request
- Discrete Transform Analysis To Obtain Distribution
- Use In Group Demand Forecasting
- Application To Seat Inventory Control Techniques

Further Research

- Empirical Testing
- Incorporation Into Nested Inventory Environments
- Fare Structure Issues

IMPROVING AIRSPACE CAPACITY:

FUTURE DIRECTIONS FOR RESEARCH AT NASA

MIT/ Industry Cooperative Research Program
Annual Program Review
May 1992

Robert W. Simpson
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IMPACTS OF TECHNOLOGY ON THE CAPACITY NEEDS
of the
U.S. NATIONAL AIRSPACE SYSTEM

NASA Grant NAG-1-1143
Langley Research Center

OBJECTIVES:

- Overview of US Air Transportation System focusing on the congestion problem
- Identify technology research areas and topics, near term and long term, concerned with increasing the capacity of the US National Airspace System (NAS).

Ray Ausrotas
Robert Simpson
MIT Flight Transportation Laboratory
November,1991

The Capacity Problem of the US National Airspace System (NAS)

OBSERVATIONS:

1. LACK OF CAPACITY FOR RUNWAY OPERATIONS
2. EFFECTS OF BAD WEATHER ON HOURLY CAPACITY
3. SLOTS ESTABLISHED ONLY AT 4 AIRPORTS
4. DELAYS OCCUR PRIMARILY AT MAJOR HUB AIRPORTS
5. AIRLINES ARE DELIBERATELY SCHEDULING HOURLY PEAKS
6. INADEQUATE CONSTRUCTION OF NEW AIRPORTS AT MAJOR CITIES

STUDY FINDINGS

IMPROVEMENTS IN TECHNOLOGY ARE NEEDED IN THE FOLLOWING AREAS:

1. NOISE

Reduction of jet transport and Tilt Rotor noise on takeoff and departure to assist in gaining community acceptance of new airports, runways, and vertiports.

2. PRECISION FLIGHT PATH CAPABILITY

To increase bad weather capacities, it is necessary to reduce the ATC separations between aircraft during arrival and departure at busy terminal areas. This requires increased precision in defining and flying 3-D and 4-D profiles.

3. CIVIL TILT ROTOR SHORT HAUL AIR TRANSPORT SYSTEM

An efficient, environmentally acceptable, short haul air system for business travellers can divert perhaps 50 % of current demand from current conventional airports.

OBSERVATION 1

1. The problem is a lack of traffic flow capacity in terms of runway approach and departure <u>operations per hour</u> at major airports, not a lack of capacity in Enroute airspace.

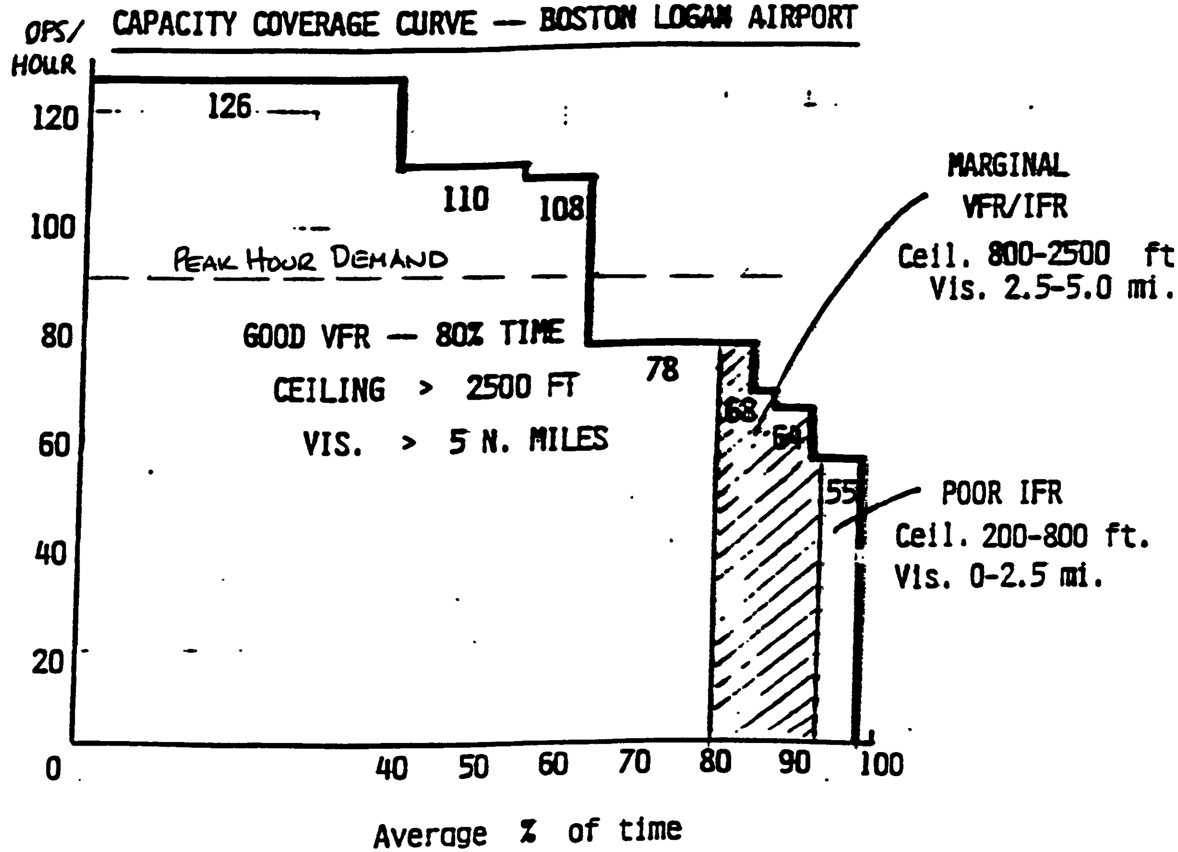
OBSERVATION 2

2. The inadequacy is a lack of capacity at times of bad weather. There usually is sufficient runway capacity in good weather to meet scheduled traffic at major US airports.

AIRPORT HOURLY CAPACITY VARIES STRONGLY WITH WEATHER

THERE IS A 3/1 OR 2/1 RATIO BETWEEN GOOD WEATHER/ BAD WEATHER CAPACITIES.

CAPACITY COVERAGE CURVE — BOSTON LOGAN AIRPORT



OBSERVATION 3

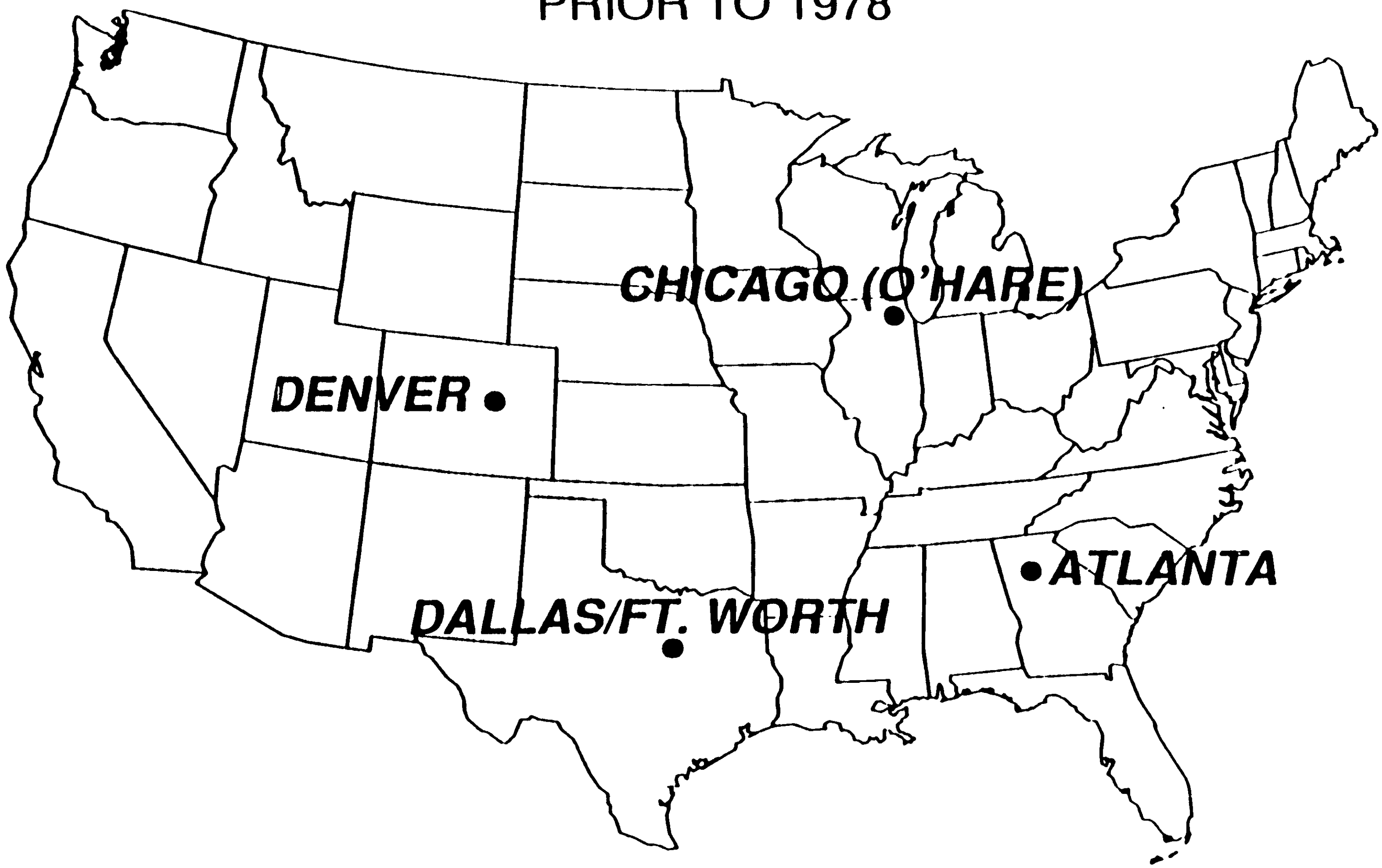
3. **In the 1980's, to minimize the costs of the lack of airport capacity, US airlines correctly reversed their prior policy of limiting schedules to bad weather capacities to ensure all weather reliability. Bad weather slots now exist only at four major US airports - LGA, JFK, ORD, DCA**

OBSERVATION 4

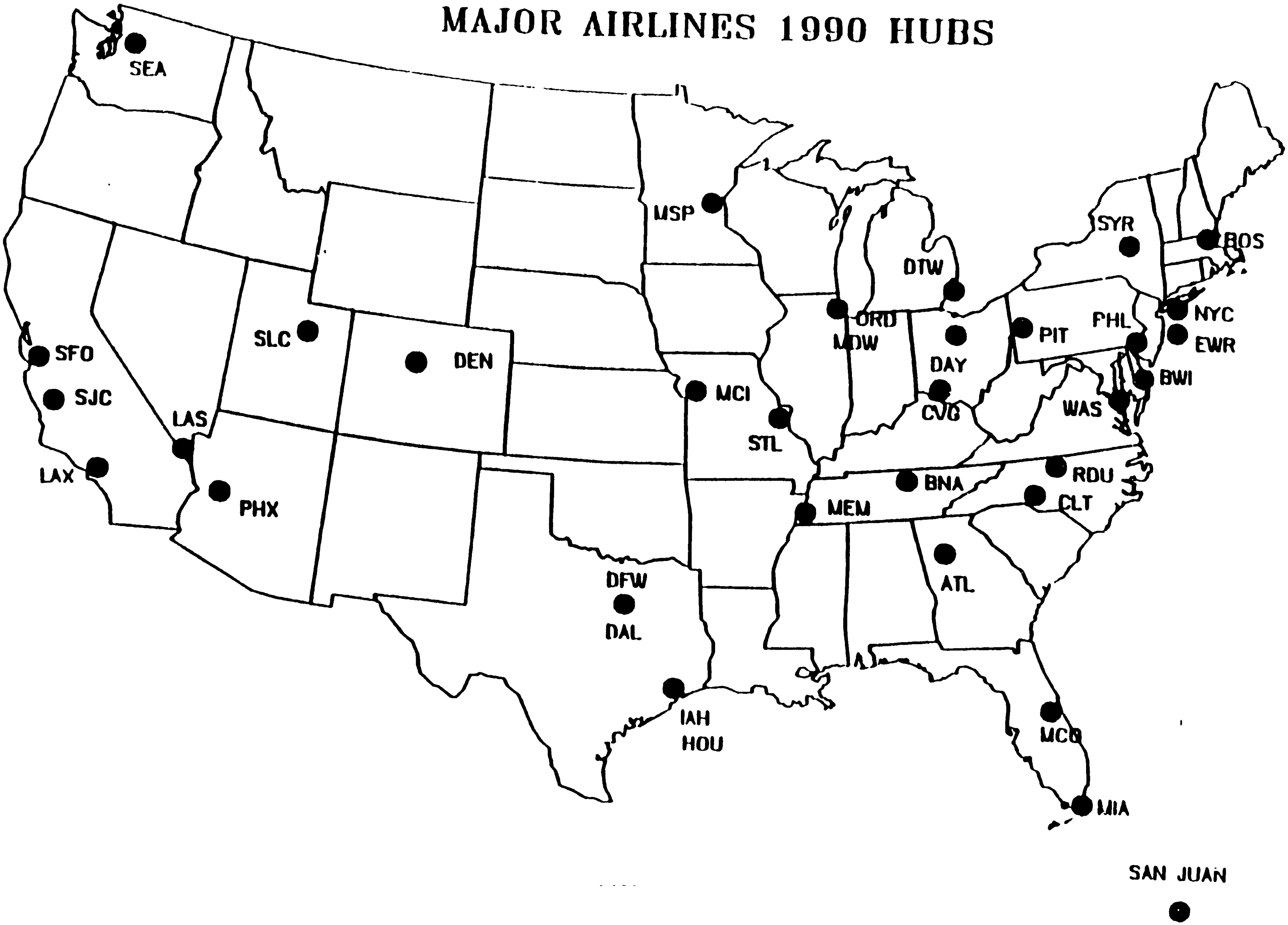
4. Today, the problem exists at the top US "hub" airports where, since deregulation, major airlines have expanded into new markets by offering competitive connecting services using smaller jet transport aircraft.

MAJOR AIRLINE HUBS

PRIOR TO 1978

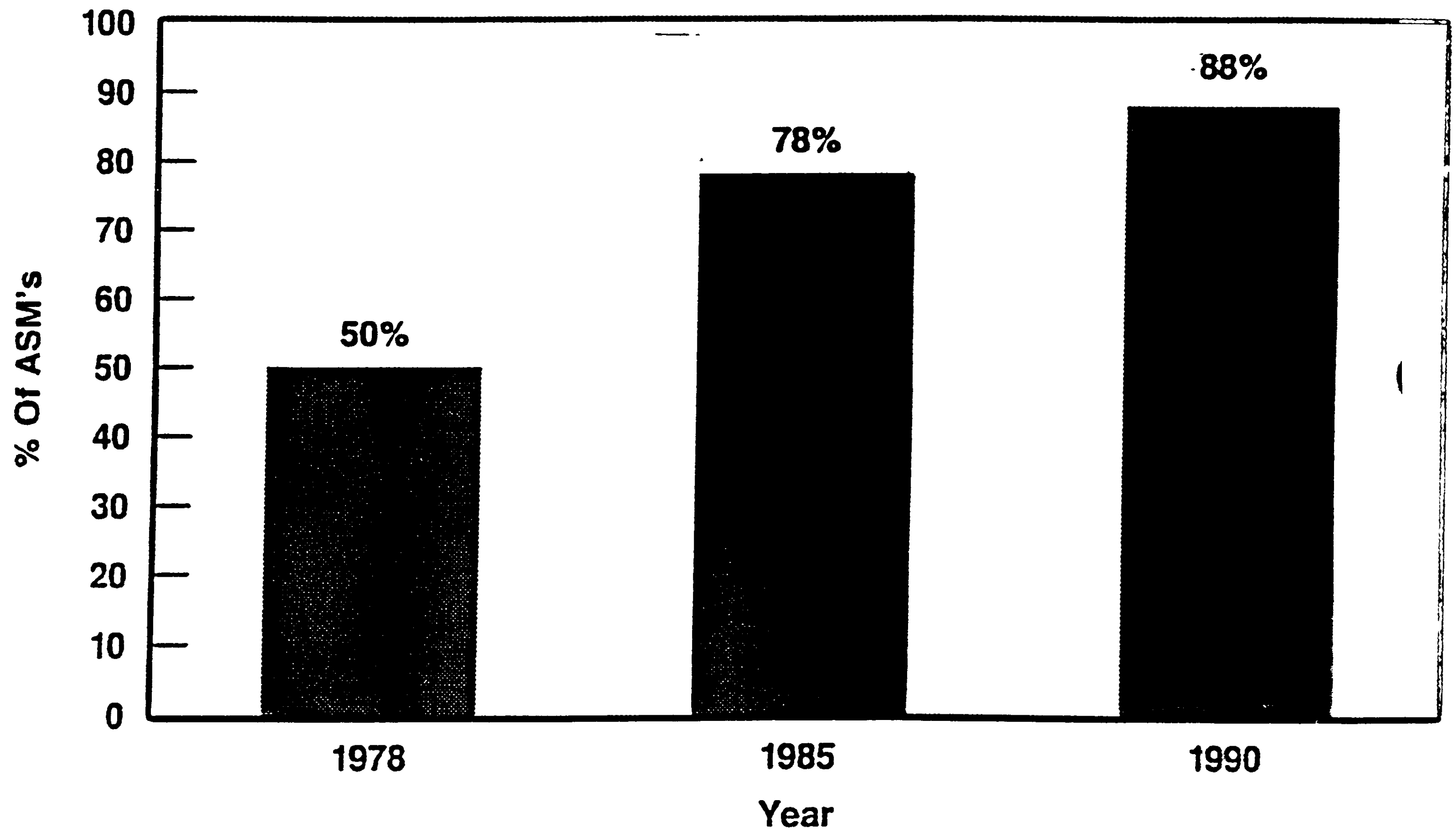


MAJOR AIRLINES 1990 HUBS

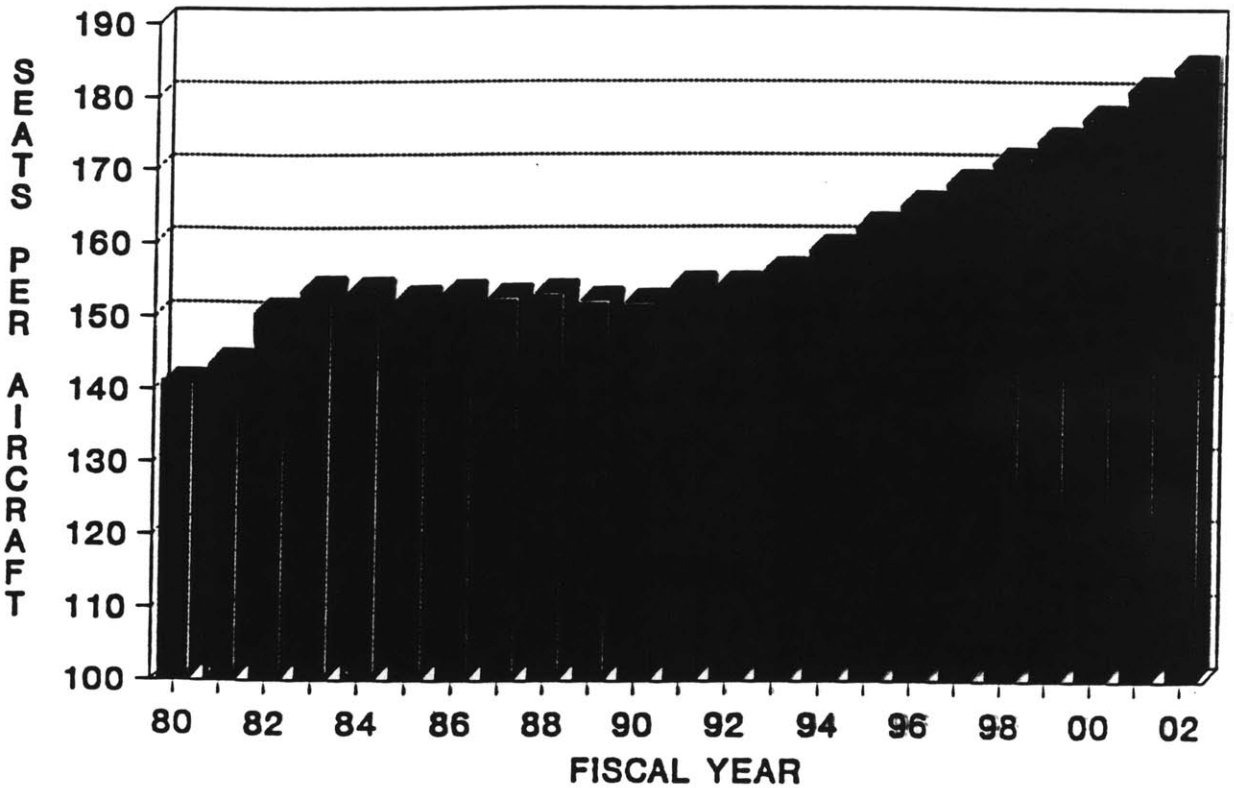


% OF DOMESTIC CAPACITY RELATED TO HUBS

U.S. Major Carriers



U.S. COMMERCIAL AIR CARRIERS
SEATS PER AIRCRAFT - DOMESTIC SERVICE

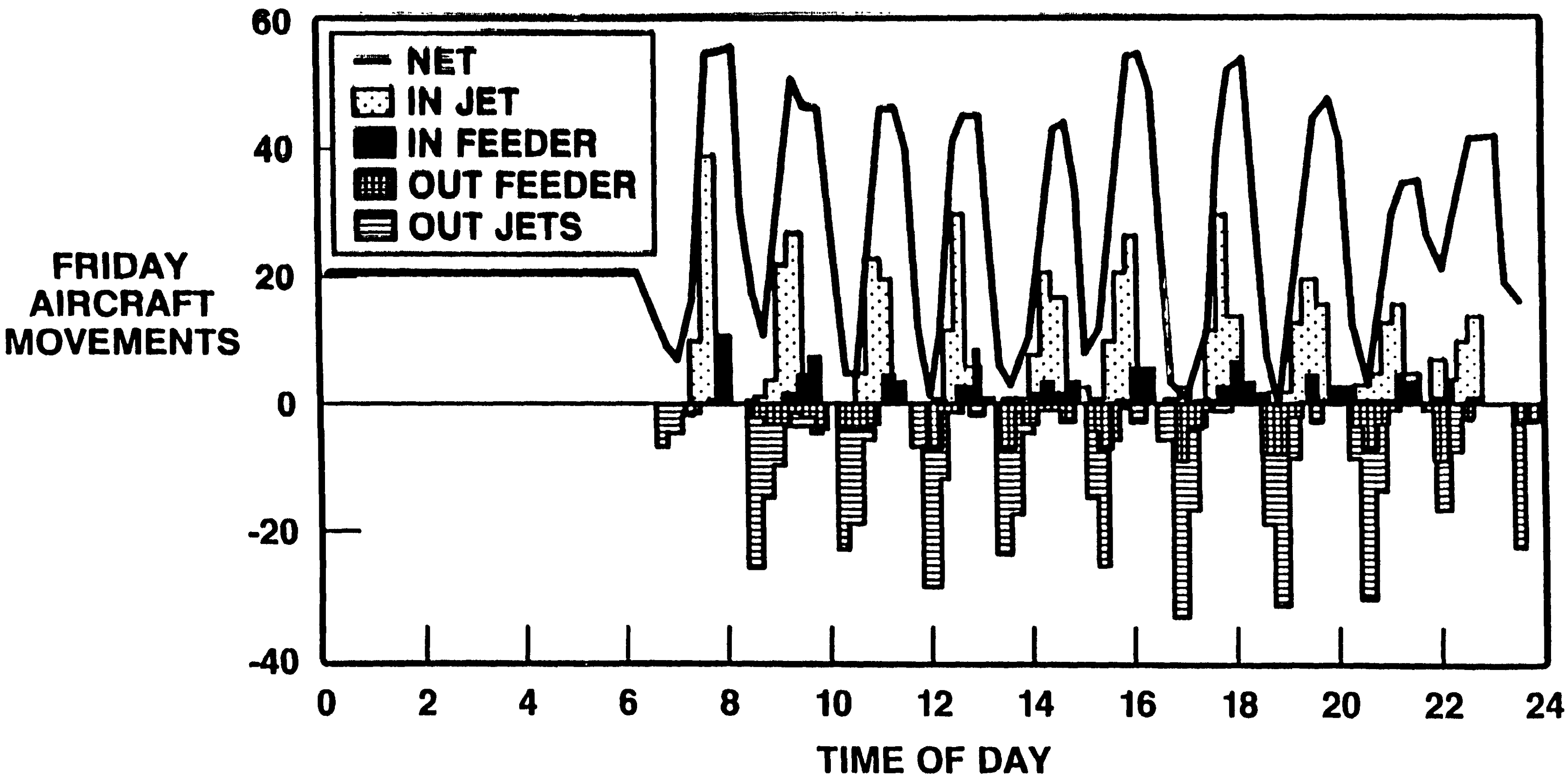


FAA Forecast Conference 1991.

OBSERVATION 5

5. There is deliberate peaking of daily schedules by airlines at these hub airports to create "connecting complexes" which causes hourly demand to exceed good weather hourly capacities for short periods.

PASSENGER AIRCRAFT MOVEMENTS
DELTA AIR LINES - ATLANTA
AUGUST 1988



CB1287.15

90

OBSERVATION 6

6. There has been inadequate construction of new airport and runway capacity to serve major US cities in the last 30 years due to local airport community objections to the noise of jet transport aircraft on approach and departure operations.

THERE ARE 6 ALTERNATIVE COURSES OF ACTION

(for the Air Transport Industry, federal and local governments)

A-1. Increase AIRCRAFT SIZE as number of carriers is reduced

A-2. Create NEW HUBS at secondary airports

G-1. Impose SLOTS at congested airports

G-2. Increase BAD WEATHER CAPACITIES at hub airports

G-3. Construct NEW AIRPORTS near hub cities

G-4. Construct new CIVIL TILT ROTOR short haul air system

Topics for Noise Research

TOPIC N-1	Transient Annoyance to Short Term Noise Exposure
TOPIC N-2	Incorporate "Intrusion" into Airport Annoyance Measures
TOPIC N-3	Establish Takeoff and Sideline Goals for Stage 4, Stage 5
TOPIC N-4	Decelerating, Low power Approach Paths for CTR
TOPIC N-5	Noise-Oriented Maneuver Departure Paths
TOPIC N-6	Active Suppression for Fan Engine Noise
TOPIC N-7	Novel Suppression Techniques for Propellor, Rotor Noise

Topics for Precision Flight Path Research

TOPIC H-1 Monitoring & Intervention of Abnormal Divergences

TOPIC H-2 Precision Guidance in the Departure Area

TOPIC H-3 Deviation Detection, Oceanic Parallel Track Systems

TOPIC H-4 Airborne Surveillance, Random Oceanic Tracks

TOPIC H-5 Hybrid Navigation Management Systems

TOPIC H-7 Trajectory Prediction for Climb/Descent

TOPIC L-1 Airborne Wake Vortex Prediction on Final Approach

TOPIC L-2 Reduction of In-Trail Separations on Final Approach

TOPIC L-3 Integration of Voice/Digital Clearances

Topics for Civil Tilt Rotor Research

TOPIC T-1	Precision Guidance for Decelerating Approach Transitions
TOPIC T-2	Wake Effects from Simultaneous Operations at a Vertiport
TOPIC T-3	CTR Approach and Departure Noise for Vertiport Operations
TOPIC T-4	Improved Rotor Performance and Noise
TOPIC T-5	Evaluation of Acquisition and Operational Costs for CTR System

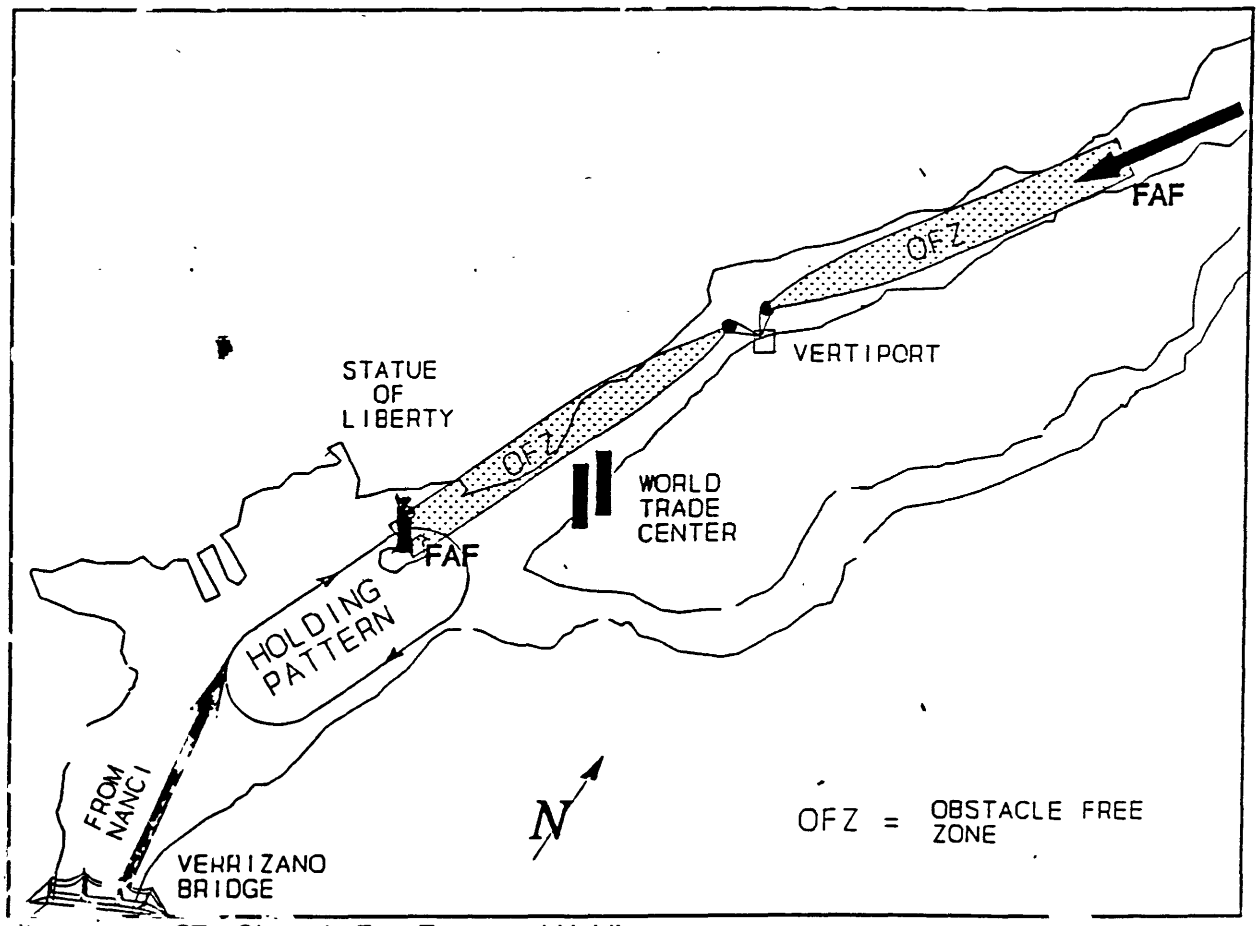


Figure 2.11 CTR Obstacle Free Zones and Holding

On the final stabilized segment, the CTR would turn to the westbound departure direction, from the vertiport, and initiate a climb. The single-engine capabilities of the CTR allow a climb-out angle of 7° at 40 knots, while turning within a 525 foot radius. The single engine climb rate would be better than 500 feet per minute, and the normal climb rate could be much higher. Upon reaching 500 feet, the CTR could be cleared to return to the final stabilized portion (perhaps on the opposite approach), or could continue its climb to clear other arriving traffic by at least 500 feet, and then proceed down the Hudson towards a CTR holding pattern established at the Statue of Liberty above 2,200 feet. The CTR would transition to conventional mode at roughly 180 knots. Holding could be done within the New York harbor area while awaiting sequencing back into any other CTR traffic flow. Alternatively, a diversion to one of the regional airports could be planned. The harbor holding pattern is also shown in Figure 2.11.

2.3.5 Spacing on Final Approach

Under current Instrument Flight Rules (IFR), successive aircraft conducting approaches to the same runway/landing area must be separated by 3 to 6 miles (depending on the aircraft mix) if radar is used. Without radar, the standard separation is 2 to 3 minutes between successive approaches, again depending on the aircraft mix[9]. These rules are designed for fixed-wing operations, however, and do not consider the

[9] When smaller aircraft follow larger, heavier aircraft, the standard radar separation of 3 miles or non-radar separation of 2 minutes is increased to allow for wake turbulence.

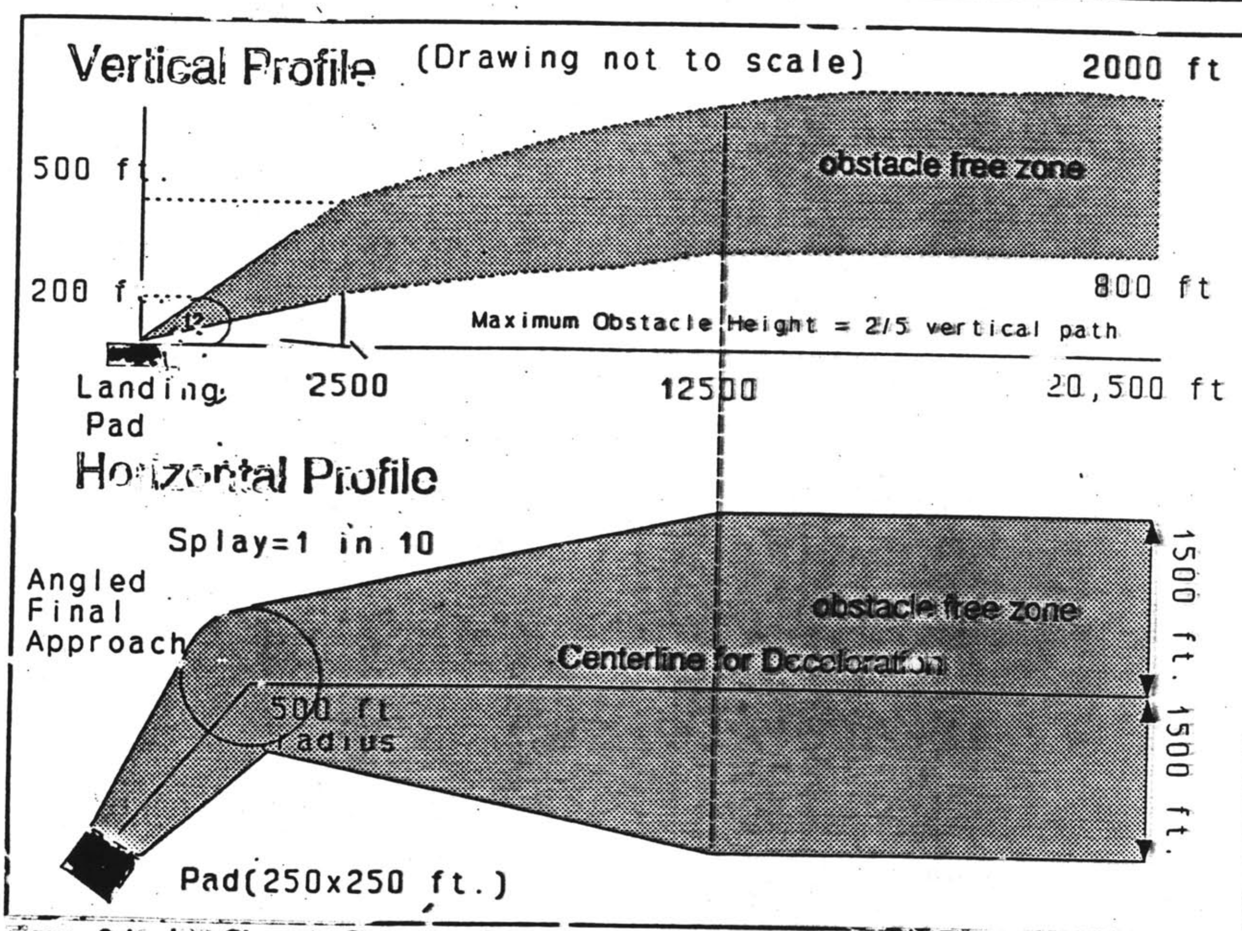


Figure 2.16, IFR Obstacle Clearance Criteria for the CTR

2.3.3 *Final Approach Segment*

The proposed final approach segment for a CTR consists of two portions as described in Section 2.1.2. In Figure 2.11, the FAF is shown 3.4 nm from the vertiport. As can be seen in the figure, the southern FAF is at the Statue of Liberty while the deceleration segment and its Obstacle Free Zone (OFZ) are over an industrial area along the western shore of the Hudson River. The northern FAF is centered in the Hudson River near West 135th Street, and its OFZ is contained within the river edges. Both deceleration segments specify the beginning of the stabilized portion of the final approach at 500 feet above the center of the Hudson River, followed by a 45 degree turn at 30 knots towards the vertiport. There is approximately 30 seconds of flight on the stabilized portion before the 200 foot decision height and visual acquisition of the vertiport surface.

2.3.4 *Missed Approach Segment*

Missed approaches will occur rarely, but must be provided for by ATC procedures. They can be caused by several factors; inability to acquire visual contact at the decision height; mechanical/electrical problems with the CTR while on approach; landing surface obstructions (e.g. stalled vehicle, CTR, or personnel in the touchdown zone). An escape from the approach at any point is possible by climbing. With radar coverage, the CTR would then be vectored to maintain safe separation from other CTR or conventional traffic.

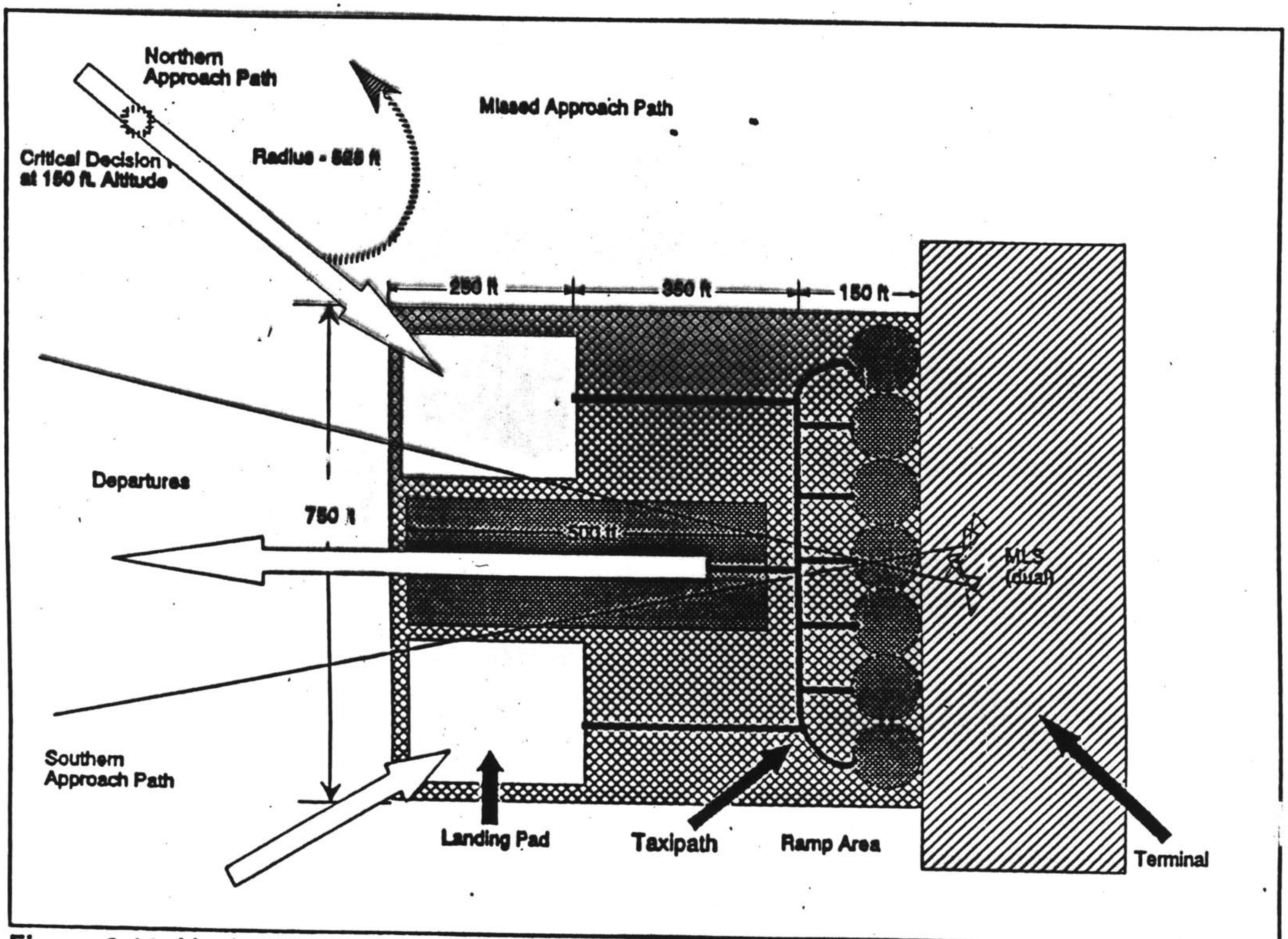


Figure 2.12 Vertiport Operations

STUDY CONCLUSIONS

1. Community reactions to noise around airports and vertiports is the long term barrier to increasing the capacity of the nation's air transport system. More airports or vertiports must be built around major cities to accommodate the long term growth expected in air transport. **Noise Research** is needed to understand community long term and transient annoyance to quieter operations

2. There are valuable returns from exploiting existing technology to reduce current ATC separation criteria used in Oceanic and Terminal areas. To demonstrate safe reductions, it is necessary to introduce the capability for **Precision Flight** along 3-D and 4-D paths to a majority of aircraft in the traffic flow.

3. There is a need to provide evidence of the economic, environmental, and operational viability of a **CTR Short Haul Air Transport System** to support decisions by federal and local government, and aviation industry to embark on a long term development program.

Presented To

MIT/Industry Cooperative Research Program

May 21, 1992

Gary Waldman

MIT Flight Transportation Lab | *R/T Phraseology Study*

Background Information

- Problem Statement - There is a mounting safety concern related to the increasing use of non-standard phraseology in ATC communications

- Study Goal - This study will attempt to determine the associated error rates due to grouped versus serial presentation of numerical information of varying complexity

Grouped Form - Similar to normally spoken language
Example - 125 as One-Hundred Twenty Five
Serial Form - Strict Numerical Presentation by Digit
Example - 125 as One Two Five

MIT Flight Transportation Lab	*R/T Phraseology Study*

Background Information

The concern for safety emerges when, for example, "30" is communicated as thirty and interpreted as thirteen. Some reasons for this are:

1. Obvious similarity in the pronunciation of words
1. Inability to distinguish message because of noisy radio channel/speed of information delivery
2. Pilot workload too high for careful monitoring

Basic Ground Rules for This Study:

- Only current commercial pilots will be tested
- Audio tapes containing instructions to pilots will be recorded by certified controllers
- Instructions will be random in nature (i.e. a real ATC environment will not be simulated)
- No side task involved
- No artificial background noise will be present nor will there be any visual stimuli

Data Collection

- Elements of messages may require change of instrument settings for items such as:
  - Radio Frequency
  - Heading
  - Speed
  - Altitude
  - Crossing Points, Transponder, etc.
- A readback of information from the subject will be required. An observer will note accuracy of readback
- Timed reading/storage of instrument settings will show whether required action carried out correctly

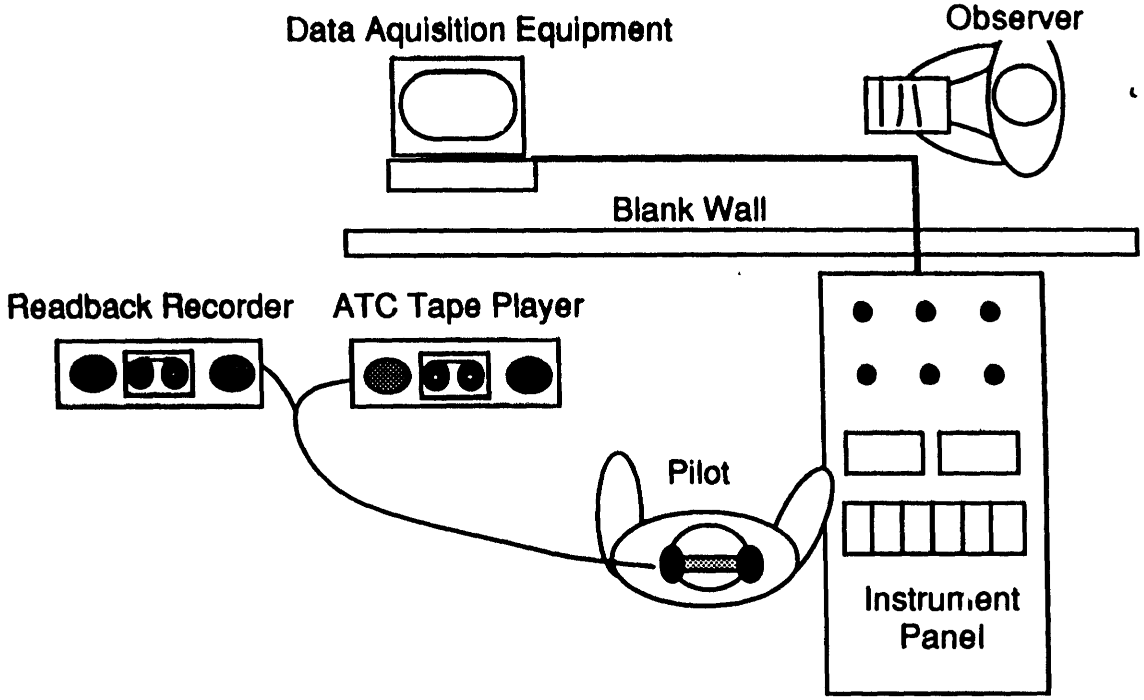
MIT Flight Transportation Lab	*R/T Phraseology Study*

Protocol

- Each pilot will participate in four separate tests - length of each test has not been determined
- Messages will be transmitted at the rate of 162 per hour
- The subject will be required to act upon 54 (33%) of the messages
- Of the 54 messages, there will be 18 each in
  - Sequential form
  - Grouped form
  - Partially Restated form
- In each group of 18 messages, 6 each will contain
  - 3 Pieces of numerical information
  - 4 Pieces " "
  - 5 Pieces " "

MIT Flight Transportation Lab	*R/T Phraseology Study*

Experimental Set-Up



THE INTEGRATION OF FLIGHT MANAGEMENT SYSTEMS INTO ADVANCED ATC OPERATIONS

MIT/Industry Cooperative Research Program
Annual Program Review
May, 1992

Robert W. Simpson
Flight Transportation Laboratory
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(617) 253-3756

Integration of FMS into Advanced ATC

Emerging Technology - #1 -Flight Management Systems & Digital Avionics

Functions of a FMS

- **a) Flight Planning**
- **b) Navigation**
- **c) Guidance**
- **d) Performance Management**
- **e) Display**
- **f) Aircraft Database Management**

Transport Aircraft with Flight Management Systems

Boeing	Douglas	Airbus	Fokker	Other
737-300	MD82	A-300	F-100	ATR-42
737-400	MD-88	A- 310	F-130	G-IV
737-500	MD-90	A-320		BAE-125
757	MD-11	A-330		Citation3
767	MD-12	A-340		Canadair RJ

20 % of the worlds fleet is now FMS equipped
50 % will be equipped by 1995

Emerging Technology - #2 - Digital Data Link for Advanced ATC Systems

There is a committment to introducing digital data link systems into newer forms of ATC

Digital Data Links for ATC

- **VHF data links (ACARS) already here**
- **Mode S - SSR surveillance system is a world standard and provides a digital data link**
- **ICAO has agreed that digital SATCOM will be part of the Future Air Navigation System**

Integration of FMS into Advanced ATC

Emerging Technology - #3 - Improved Weather Data Gathering Systems

- it appears that there will be a sigificant improvement in the collection of weather data by satellites, aircraft, and remote sensing earth stations in coverage and in frequency

- and hopefully an improvement in weather forecasting accuracy

- NEXRAD weather radar network
- Weather Profiler network
- aircraft datalinked winds, temperatures, turbulence
- FAA's aviation weather program

Integration of FMS into Advanced ATC

Emerging Technology - #4 - Automated Airline Flight Operation Centers

- airlines are automating the flight planning, dispatch, and in-flight monitoring of progress of all aircraft

- introduction of centralized dispatch center
- introduction of electronic flight plans loaded directly into the FMS
- introduction of paperless cockpits
- real time communications with the cockpit at all times and at all points in the world

Integration of FMS into Advanced ATC

The Generic Objectives of the FAA's FTMI Project

(Flight Operations and Air Traffic Management Integration)

1) define extended functionalities for an AFMS which is compatible with future automated ATM environments worldwide.

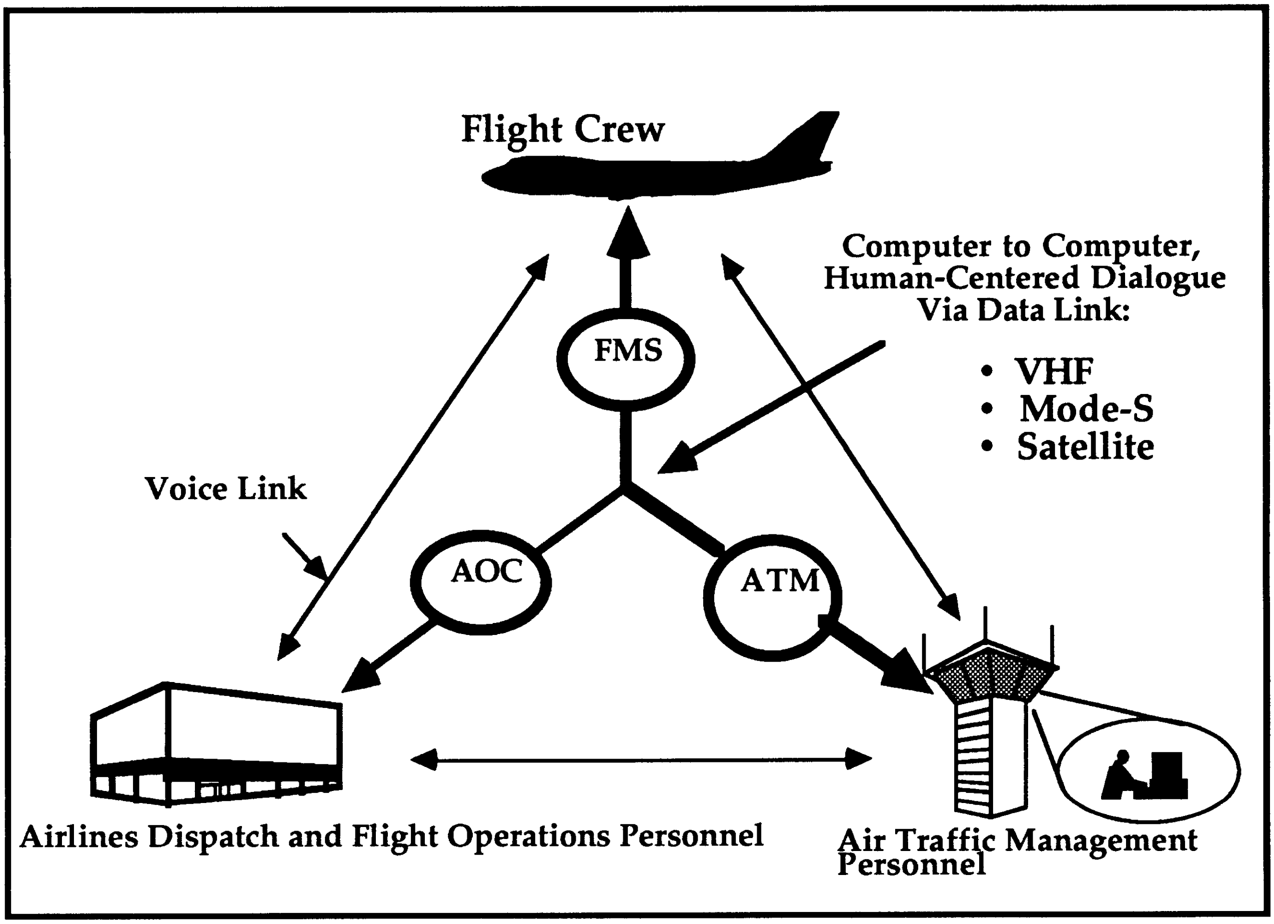
2) define the specific nature of the datalink messages between the AFMS, AOCC and various ATM facilities.

3) define new operational applications for Oceanic, Enroute and Terminal Area airspace based on the extended functionalities.

4) provide evidence from various demonstrations to support international adoption of standards for an AFMS, and adoption of operational procedures for its applications.

Integration of FMS into Advanced ATC

There is a need for a Systems Integration Effort by Civil Aviation Agencies to determine the best way to use these emerging Technologies



Integration of FMS into Advanced ATC

Examples of Extended Functionalities of an AFMS (EFFs)

- Mid-Flight Refiling of flight plans
- accept a updated weather forecast for rest of flight
- accept and fly tactical modifications of SIDS/STARS
- accept and fly a " Digital Vector"
- accept and fly a "Required Time of Arrival"
- fly a Digital Holding Pattern
- accept and fly a "Stationkeeping Clearance"
- send next waypoints/altitudes on intended path
- send current track, groundspeed, vertical speed
- send windspeed, direction, temperature, turbulence at requested points or frequency
- send best estimate for intended climb/descent profiles
- send earliest/latest possible times at future waypoints
- send flight plan request for rest of trip

Examples of New Operational Applications (NOAs)

Oceanic Fixed Track System

- **Longitudinal Stationkeeping**
- **Same Track Passing and Step Climbs**
- **Oceanic Track - Required Entry Time**
- **Mid-Flight Refiling of flight plans**
- **Emergency Diversion**

Oceanic Free Tracks

- **Conflict Identification and Local Resolution**
- **Cruise-Climb Paths**

Integration of FMS into Advanced ATC

Examples of New Operational Applications (NOAs)

Domestic Enroute Airspace

- **Conflict Free Climbs and Descents**
- **Digital Resolution Advisories from AERA**

Extended Terminal Area Airspace

- **Complex Precision Paths for Arrival and Departure**
- **Paired Stationkeeping Departures**
- **RTAs at Metering Fixes**

Planning a Systems Integration Project - FTMI

There are many activities to be defined and funded over the next several years

1. Set Detailed Objectives

2. Develop a Project Plan

3. Establish Liason with On-going FAA R&D Projects

4. Study Specific Applications (NOAs)

- **establish Technical Requirement**
- **establish Operational requirements**
- **conduct simulations, demonstrations, validation tests**
- **conduct Cost/Benefit studies**

5. Generate Operational Specifications

6. Facililtate International Agreement

AIRCRAFT GROUND MOVEMENT SIMULATOR A TESTBED FOR RESEARCH IN AIRPORT PLANNING AUTOMATION

Dr. D. F. X. Mathaisel
Dr. J. D. Pararas

May, 1992

OVERVIEW

1. The Mission of GMS
2. Planning Airport Surface Traffic
3. Overall System Design
4. Operational Characteristics
5. Software Design Characteristics
6. Future Work

THE MISSION OF GMS

- Real-time man-in-the-loop simulation

- Realistic simulation of surface traffic at major airports
    - Current control environments

    - Future planning

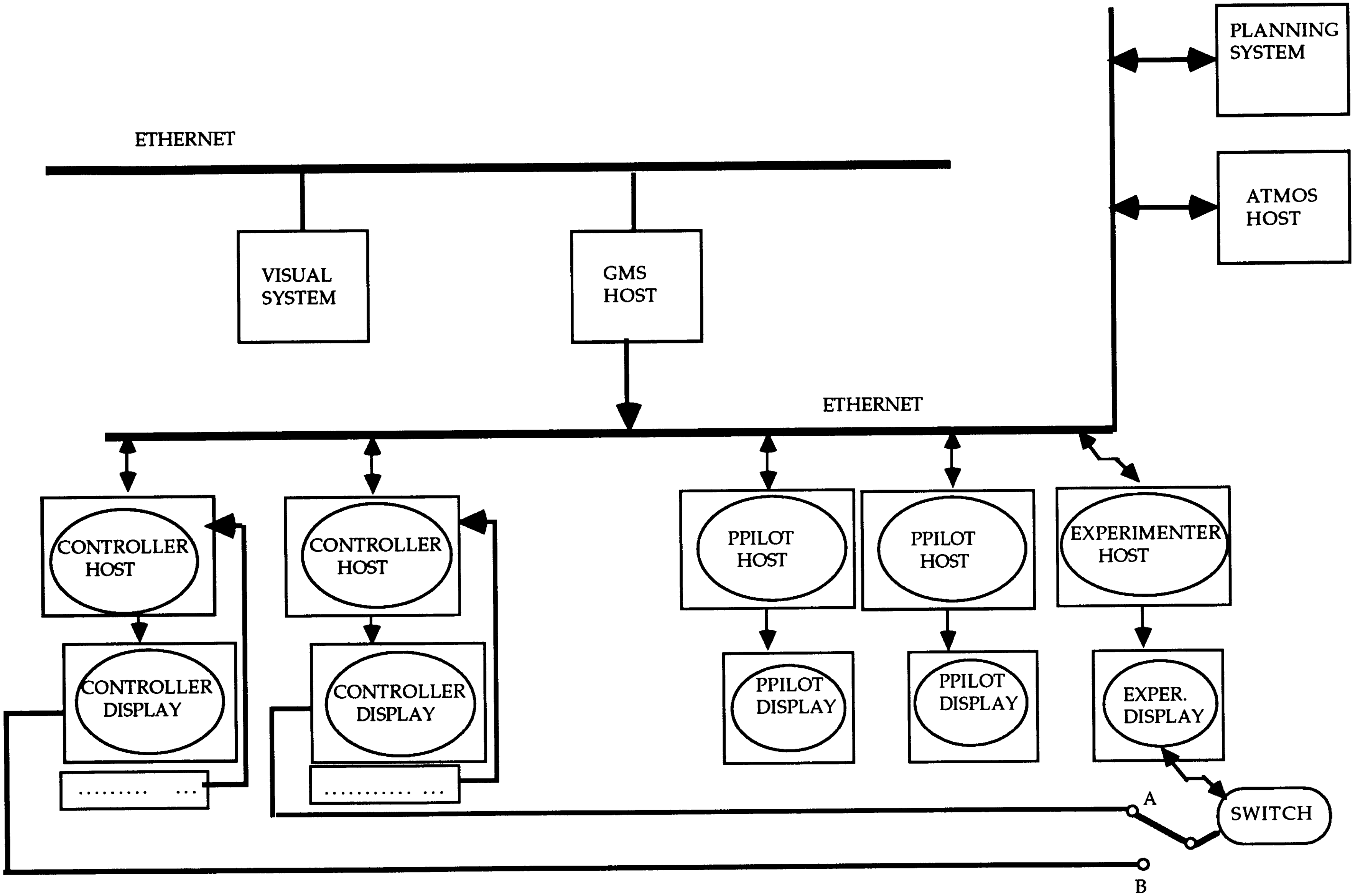
- Stand-alone (conventional) operation. Controllers & pseudopilots manually control traffic

- Automated control: interfaces to planning systems.

- Manage traffic scenarios that insure repeatability of experiments

PLANNING AIRPORT SURFACE TRAFFIC

- Scheduling the Runways
- Scheduling Pushbacks
- Managing taxipaths: Rwy->Gate, Gate->Rwy
- Vehicle traffic
- Challenges
  - Unpredictable traffic behavior
  - Interface to airline operations
  - No "real world" system to simulate.


PLANNING SYSTEM
ETHERNET
ATMOS HOST
VISUAL SYSTEM
GMS HOST
ETHERNET
CONTROLLER HOST
CONTROLLER HOST
PPILOT HOST
PPILOT HOST
EXPERIMENTER HOST
CONTROLLER DISPLAY
CONTROLLER DISPLAY
PPILOT DISPLAY
PPILOT DISPLAY
EXPER. DISPLAY
A
SWITCH
B

OPERATIONAL CHARACTERISTICS

- Aircraft Position Generator
  - Detailed motion models: taxi, turns, takeoff, landing
  - Generates traffic situation broadcasts
  - Accepts messages to perform specific tasks
    - generate new aircraft
    - assign new path to aircraft
    - aircraft status queries

- Controller Stations
  - High resolution plan-view displays of traffic situation
  - "Tower Simulator" provides out-the-window displays similar to cockpit simulators.
  - Voice link to pseudopilots
  - Interfaces to planning systems

- Pseudopilot Stations
  - High Resolution plan view display
  - Path editing capability
  - Menu driven or function key based command generation
  - Voice link to controllers

- **Experimenter's Station**
  - **Plan view display**
  - **Experiment control panel**
  - **Monitoring capability via repeaters**
  - **Traffic scenario control**

SOFTWARE DESIGN CHARACTERISTICS

- Distributed architecture: allows expansion and flexible interfaces to new systems.

- Unix - XWindows - TCP / IP based
  Allows porting to most modern workstation platforms

- Object - oriented approach allows easy adaptations to future needs

FUTURE WORK

- Extension to full mission (air & ground) simulator
- Incorporate voice recognition to alleviate man-power needs for conducting experiments
- Automation Research
    - Landing - takeoff coordination (Runway scheduling)
    - Managing takeoff queues and pushbacks
    - Management of taxiways and intersections.
    - Low visibility traffic management research

Policy Level Decision Support For Airport Passenger Terminal Design

Prepared For The

MIT Cooperative Research Meeting

Tom Svrcek
MIT Flight Transportation Laboratory
May 22, 1992

Outline Of Presentation

Problem Statement

Airport Performance

Passenger/Terminal Types

Estimating Expected Walking Distances

Intelligent Scheduling

Aircraft Effects

Sensitivity Analyses

Conclusions

Research Initiative

To Provide Real-Time Policy Level Decision Support For Airport Passenger Terminal Design.

Current Support Exists In The Form Of Detailed, "Micro" Simulations.

- Presuppose A Given Configuration
- Require Large Amounts Of Detailed Input Data
- Changes Require Lengthy Setup Times
- Design-Simulate-Redesign Process Ultimately Produces "Best" Layout For Given Configuration

  No Guarantee Initial Configuration Was Most Appropriate

Airport Performance

Expected Walking Distances

Congestion

Capacity

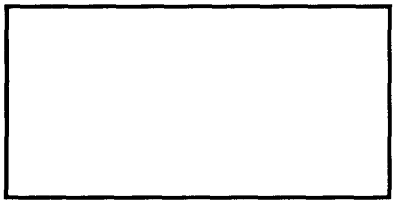
Safety

Signage (Way Finding)

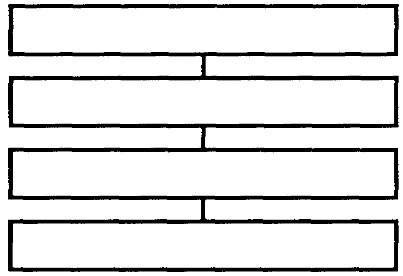
Cost

Concession Revenues

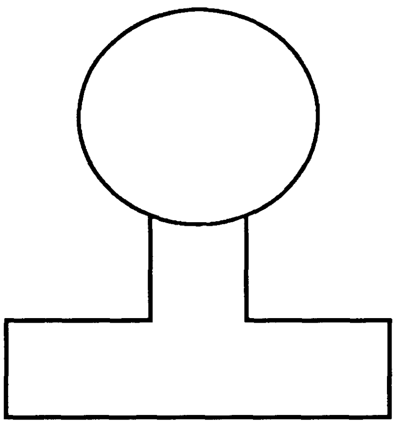
Airport Terminal Types



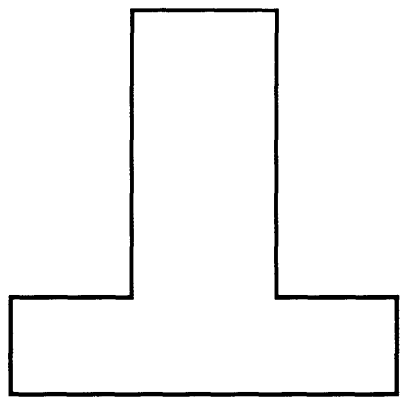
Linear Box Terminal



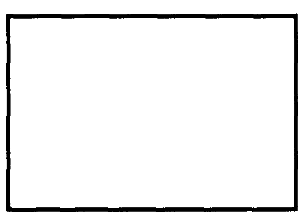
Parallel Box



Satellite



Finger-Pier



Remote Terminal

Passenger Types

1) Originating - Begin Trip At Airport Under Consideration. Distance Walked Is Modeled As Distance From Entrance To Departure Gate.

2) Terminating - Complete Trip At Airport Under Consideration. Distance Walked Is Modeled As Distance From Arrival Gate To Exit.

3) Transfers - Arrive And Depart From Gates Within Airport Under Consideration. (Direct and Indirect)

The Overall Expected Walking Distance Model Is :

$$\overline{D} = p_{ot} \cdot \overline{d_{ot}} + p_{dt} \cdot \overline{d_{dt}} + p_{it} \cdot \overline{d_{it}}$$

Where :

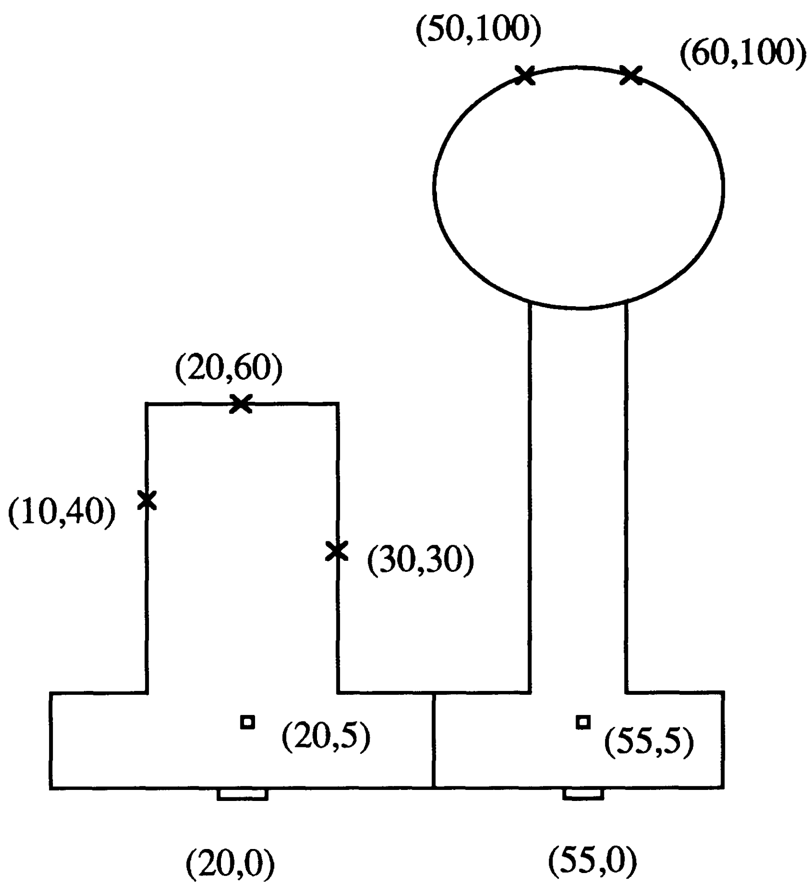
$\overline{D} =$ Overall Expected Walking Distance

$p_i =$ Fraction Of Total Traffic That Is i

$\overline{d}_i =$ Expected Walking Distance For Population i

Estimating Expected Walking Distances

(Direct Transfer Passengers - Terminal 1)



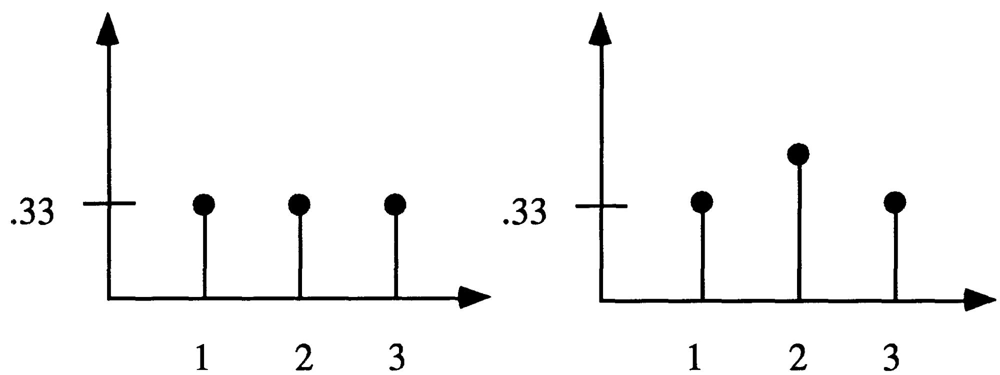
Gate 1 Arrivals Can Depart From Any One Of Three (Terminal 1) Gates. If Assumed That Each Departure Gate Is Equally Likely ...

$$\overline{d}_{dt_1} = (.33)(0) + (.33)(30) + (.33)(20) = 16.7$$

Intelligent Scheduling

Airport Owners/Airlines Have Control Over Flight To Gate Assignments. Thus, We Might Expect Them To Schedule Connecting Flights Closer Together.

(Intelligent Scheduling)



Returning To Our Example ...

$d_{11} = 0$
$d_{12} = 30$
$d_{13} = 20$

Assume 40% Of Passengers Stay On Board (Through Pax)

$t_{11} =$		.40 (Given)
$t_{12} =$	$(.60) * (1 - 30/50) =$	.24
$t_{13} =$	$(.60) * (1 - 20/50) =$	.36

Expected Distance For Gate 1 Arrivals (Terminal 1)

$$\bar{d}_{dt_1} = 14.4$$

Aircraft Effects

Two Universal Truths

- Large Aircraft Carry More Passengers Than Small Aircraft
- Small Aircraft Can Be Turned Around Faster Than Large Aircraft

The Model

Load Factor On All Aircraft = 67%

Type	Size (Seats)	Turnaround Time	Ops/Day	Pax
Large	400	45 min.	32	8576
Med	200	30 min.	48	6432
Small	150	20 min.	72	7236

Two Types Of Gates

Terminal 1 – Medium Gates
Terminal 2 – Large Gates

Gate Utilization

Aircraft Type	Large Gate	Med Gate
Large	0.6	0.0
Med	0.3	0.8
Small	0.1	0.2

Total Pax/Day	7799	6593
Total For Airport		35376

Demand Rate

Large	0.220
Medium	0.186

Combining Gate Affinity With Demand Rate

"Affinity" Transition Matrix

	1	2	3	4	5
1	0.32	0.19	0.29	0.10	0.10
2	0.27	0.32	0.21	0.10	0.10
3	0.32	0.16	0.32	0.10	0.10
4	0.10	0.10	0.10	0.28	0.42
5	0.10	0.10	0.10	0.42	0.28

	Gate Affinity	Demand Rate	Weighted Prob
1	0.32	0.186	0.309
2	0.19	0.186	0.185
3	0.29	0.186	0.278
4	0.10	0.220	0.114
5	0.10	0.220	0.114

Total Combined Transition Matrix

	1	2	3	4	5
1	0.31	0.19	0.28	0.11	0.11
2	0.26	0.31	0.20	0.11	0.11
			...		
5	0.09	0.09	0.09	0.44	0.29

Absolute Transfer Distances

	1	2	3	4	5
1	0	30	20	180	180
2	30	0	40	190	190
			...		
5	180	190	170	10	0

The Complete Model

The Dot Product Of Transition and Distance Matrices

Gate	Distance	P(Arrival)	
1	52.2	0.186	
2	59.3	0.186	<====Demand Rate
3	51.1	0.186	(Symmetric)
4	52.3	0.220	
5	52.3	0.220	

Overall Expected Direct Transfer Distance
= 53.2

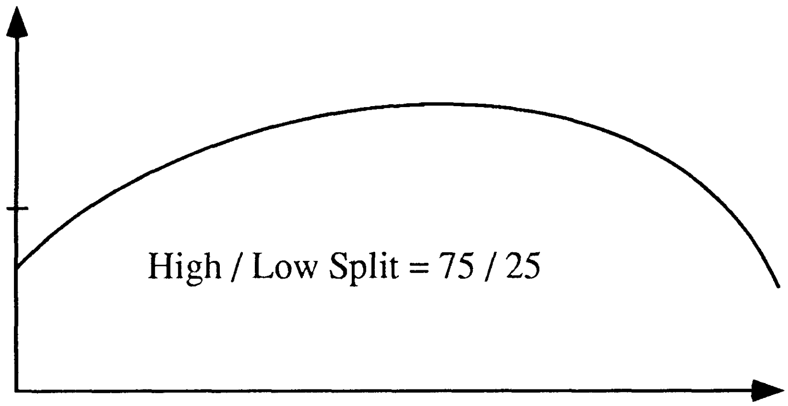
Similar Analysis For Indirect Transfers
= 147.6

Originating/Terminating Passengers
= 74.25

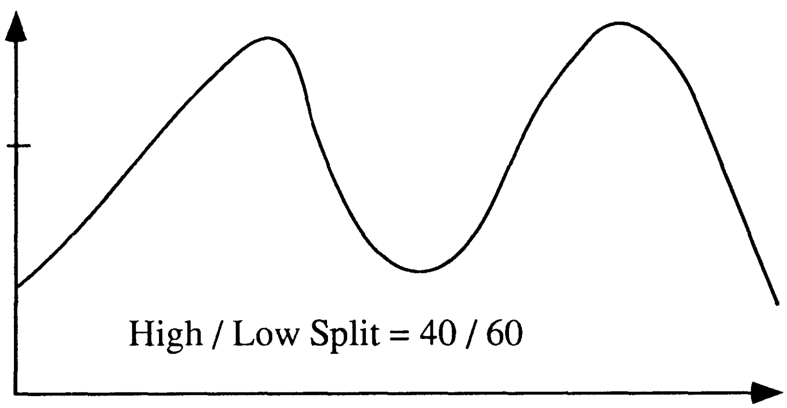
Pax Type	Pax Mix	Dist
Org−Term	60%	74.3
Direct	36%	53.2
Indirect	4%	147.6

Overall Expected Walking Distance
= 69.6

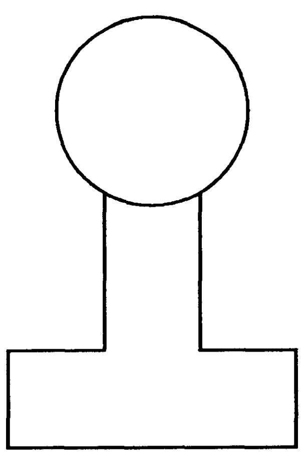
Characteristic Demand Patterns



Demand Pattern A

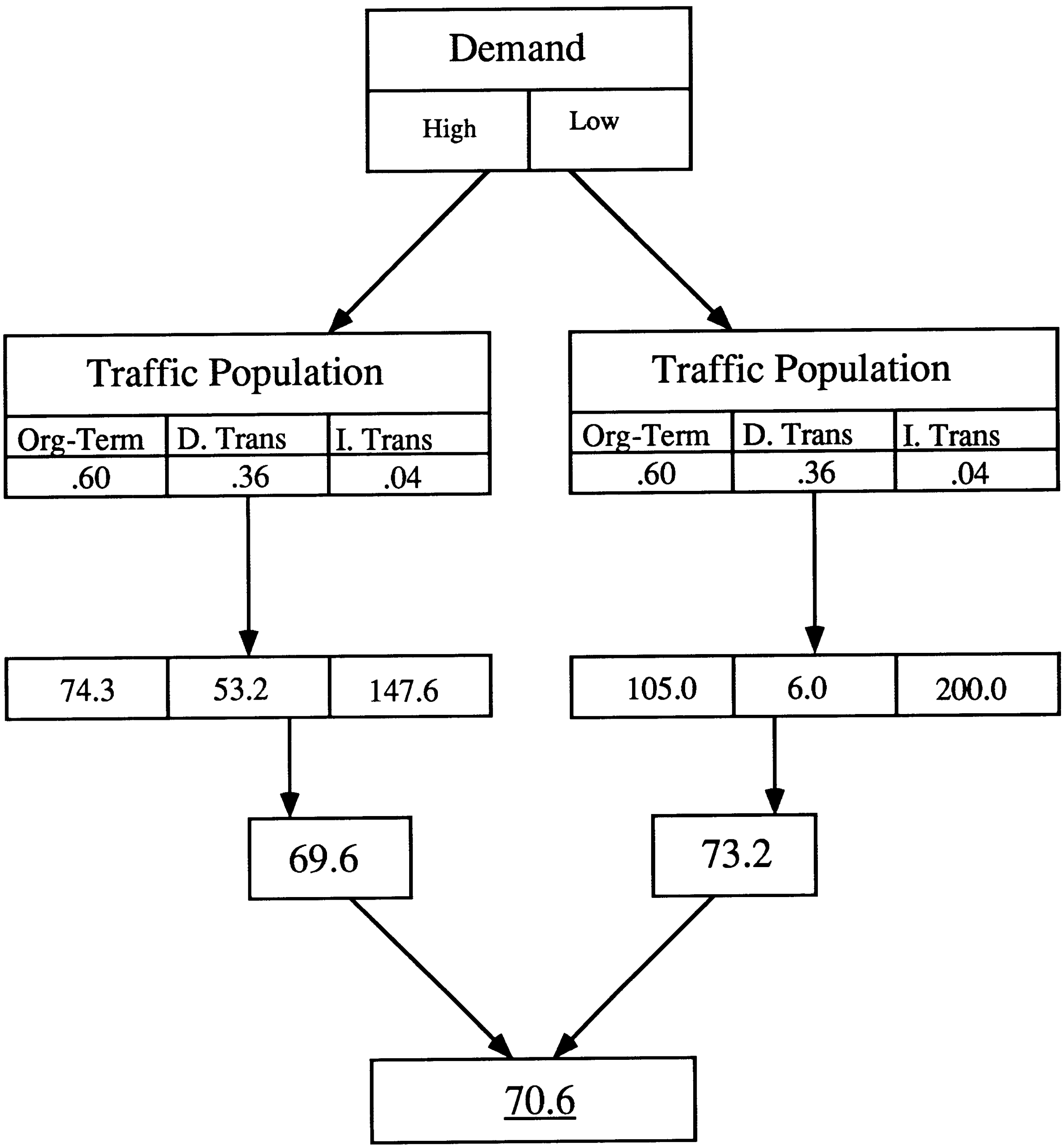


Demand Pattern B



In Periods Of Low Demand, Only Terminal 2 Is Used ...

The Big Picture



Sensitivity To Percent Transfers

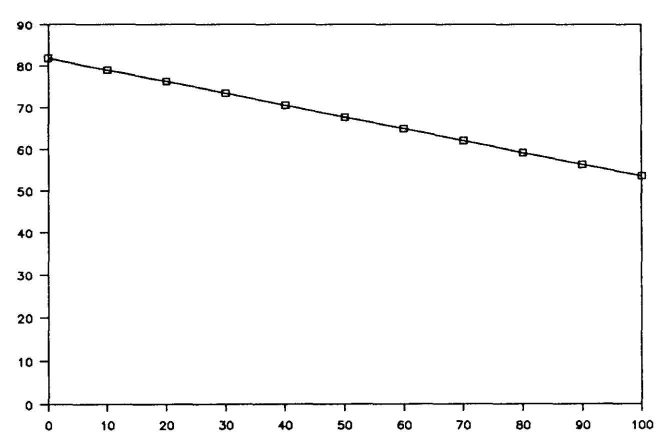
By Varying Percent Transfer Assumption, We Can Assess "Robustness" To Passenger Mix

Percent Transfer	High Config	Low Config
0	74.2	105.0
10	73.1	97.0
20	72.0	89.1
30	70.8	81.1
40	69.7	73.2
50	68.5	65.2
60	67.4	57.2
70	66.2	49.3
80	65.1	41.3
90	63.9	33.4
100	62.8	25.4

Assume Constant Demand Pattern (75/25)

% Trans	Comb
0	81.9
10	79.1
20	76.3
30	73.4
40	70.6
50	67.7
60	64.9
70	62.0
80	59.2
90	56.3
100	53.4

Sensitivity To Transfers



Sensitivity To Demand Pattern

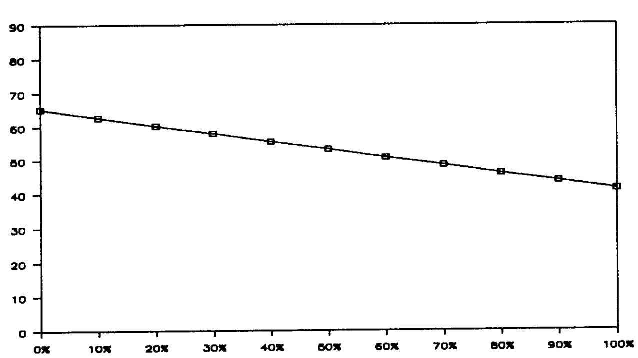
By Varying Percent High/Low Assumption, We Can Assess "Robustness" To Demand Pattern

Assume 80% Transfers

High	65.1
Low	41.3

Percent High	Percent Low	Overall Distance
100%	0%	65.1
90%	10%	62.7
80%	20%	60.3
70%	30%	58.0
60%	40%	55.6
50%	50%	53.2
40%	60%	50.8
30%	70%	48.4
20%	80%	46.1
10%	90%	43.7
0%	100%	41.3

Sensitivity To Demand



Conclusions

- Estimates For Overall Expected Walking Distances Obtained Through A Series Of Simple Calculations

- Calculations Very "Fast", Thus Sensitivity Analyses Very Practical

- Methodology Very General, Any Terminal Configuration Can Be Tested

Further Research

- Extensive Sensitivity Analyses To Determine "Most Robust" Configurations Under Varying Conditions

- Model Can Be Used To Test Several Different Low Demand Policies Under Different Conditions

- Model Can Be Used To Determine Overall Walking Distances For A Particular Airline Or "Passenger Cluster"

Joint Price Level/Seat Allocation Optimization for Airlines

Theodore C. Botimer
MIT Flight Transportation Laboratory
Presentation to the MIT/Industry Cooperative Research Program Annual Meeting
May 22, 1992

Research Motivation

I. Revenue management work has focused on optimal seat allocations with <u>fixed</u> price levels

1) Marginal seat revenue methods
   • Belobaba, 1987

2) Network optimization methods
   • Glover et al., 1984

3) Optimal single leg seat allocations
   • Brumelle et al., 1990
   • Curry, 1990
   • Wollmer, 1990

II. This research seeks to include price level as a decision variable seat allocations

Joint Price Level/Seat Availability Optimization Problem

Simplistic Seat Allocation Optimization

$$\text{Max } R = \sum_{n=1}^{N} P_n Q_n$$

Subject to:

$$\sum_{n=1}^{N} Q_n \leq Cap$$

$$Q_n \geq 0 \qquad \text{for } n = 1,\ldots,N$$

$$Q_n = f(P_n)$$

where
R = total revenue
Q_n = seats allocated to fare class n
P_n= average fare charged to fare class n
N = total number of fare classes
Cap = total aircraft capacity

Joint Optimization (con't)

Optimization Formulation Assumptions

- Single leg/OD pair
- N independent fare classes
- N distinct fare class seat allocations
- Fixed capacity aircraft
- Deterministic demand
- Case I : Separate Linear Demand Curves
- Case II : Single Linear Demand Curve

Two Fare Class Case

Separate Linear Demand Curve Formulation:

$$\text{Max } R = P_Y Q_Y + P_Y Q_B$$

Subject to:

$$Q_Y + Q_B \leq \text{Cap}$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

$$P_Y = P_{Y0} - a_Y Q_Y$$

$$P_B = P_{B0} - a_B Q_B$$

Substituting Demand Curves into Objective :

$$\text{Max } R = P_{Y0} Q_Y - a_Y Q_Y^2 + P_{B0} Q_B - a_B Q_B^2$$

Subject to:

$$Q_Y + Q_B \leq \text{Cap}$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

Two Fare Class Case

Single Linear Demand Curve Formulation:

$$\text{Max } R = P_Y Q_Y + P_Y Q_B$$

Subject to:

$$Q_Y + Q_B \leq Cap$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

$$P_Y = P_0 - aQ_Y$$

$$P_B = P_0 - a[Q_Y + Q_B]$$

Substituting Demand Curve into Objective :

$$\text{Max } R = P_0 Q_Y - aQ_Y^2 + P_0 Q_B - aQ_B^2 - aQ_Y Q_B$$

Subject to:

$$Q_Y + Q_B \leq Cap$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

Two Fare Class Case Optimality Conditions

Separate Linear Demand Curve Formulation

Capacity Constrained Optimal Price Levels :

$$P_Y^* = \left[\frac{(a_Y+2a_B)}{(a_Y+a_B)}\right]\left[\frac{P_{Y0}}{2}\right] + \left[\frac{a_Y}{(a_Y+a_B)}\right]\left[\frac{P_{B0}}{2}\right] - \left[\frac{2a_B}{(a_Y+a_B)}\right]\left[\frac{a_Y Cap}{2}\right]$$

$$P_B^* = \left[\frac{(2a_Y+a_B)}{(a_Y+a_B)}\right]\left[\frac{P_{B0}}{2}\right] + \left[\frac{a_B}{(a_Y+a_B)}\right]\left[\frac{P_{Y0}}{2}\right] - \left[\frac{2a_Y}{(a_Y+a_B)}\right]\left[\frac{a_B Cap}{2}\right]$$

Capacity Unconstrained Optimal Price Levels :

$$P_Y^* = \frac{P_{Y0}}{2}$$

$$P_B^* = \frac{P_{B0}}{2}$$

Two Fare Class Case Optimality Conditions

Single Linear Demand Curve Formulation

Capacity Constrained Optimal Price Levels :

$$P_Y^* = P_0 - \frac{aCap}{2}$$

$$P_B^* = P_0 - aCap$$

Capacity Unconstrained Optimal Price Levels :

$$P_Y^* = \frac{2}{3}P_0$$

$$P_B^* = \frac{1}{3}P_0$$

BOS - LAX Case Study

- Friday, 6:00 P.M. Departure
- 150 & 200 Seat Aircraft

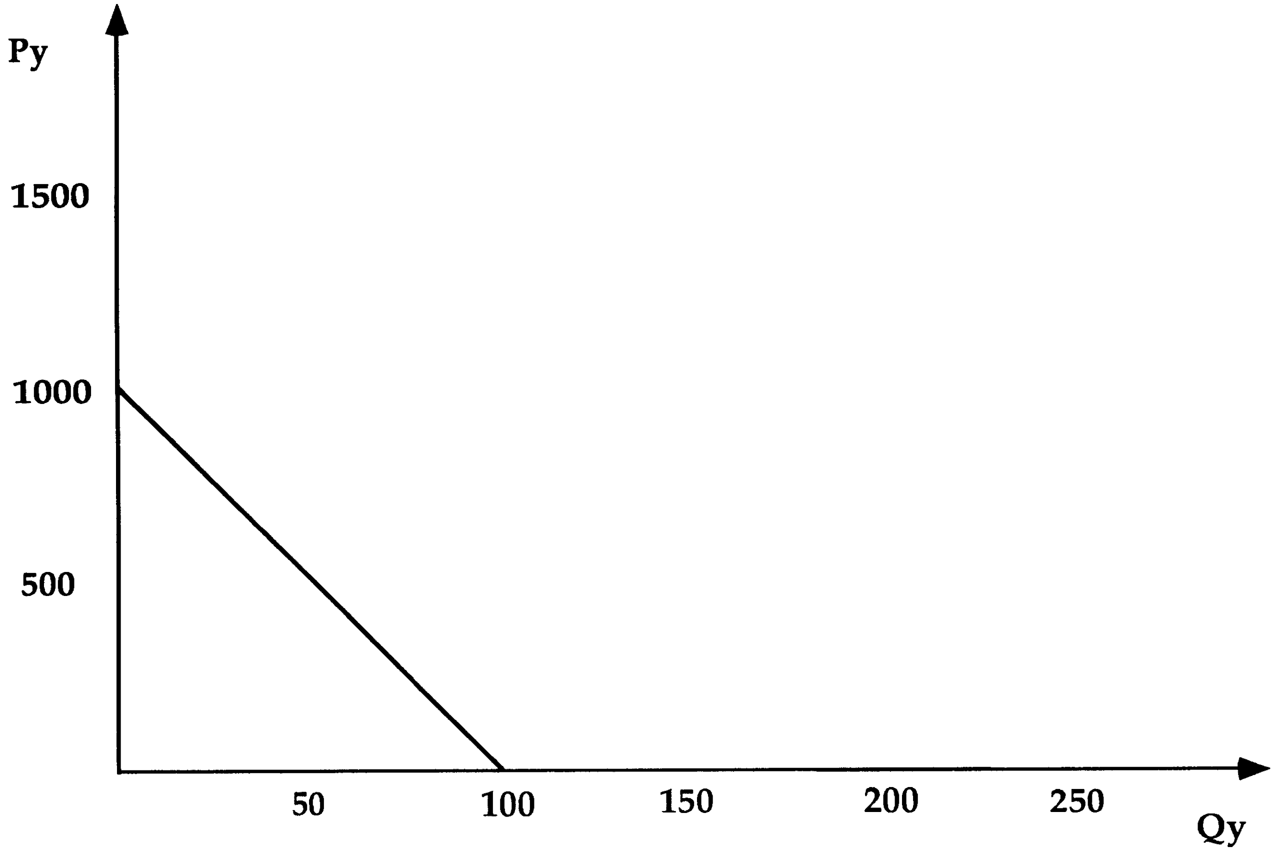
Dual Demand Curve Formulation :

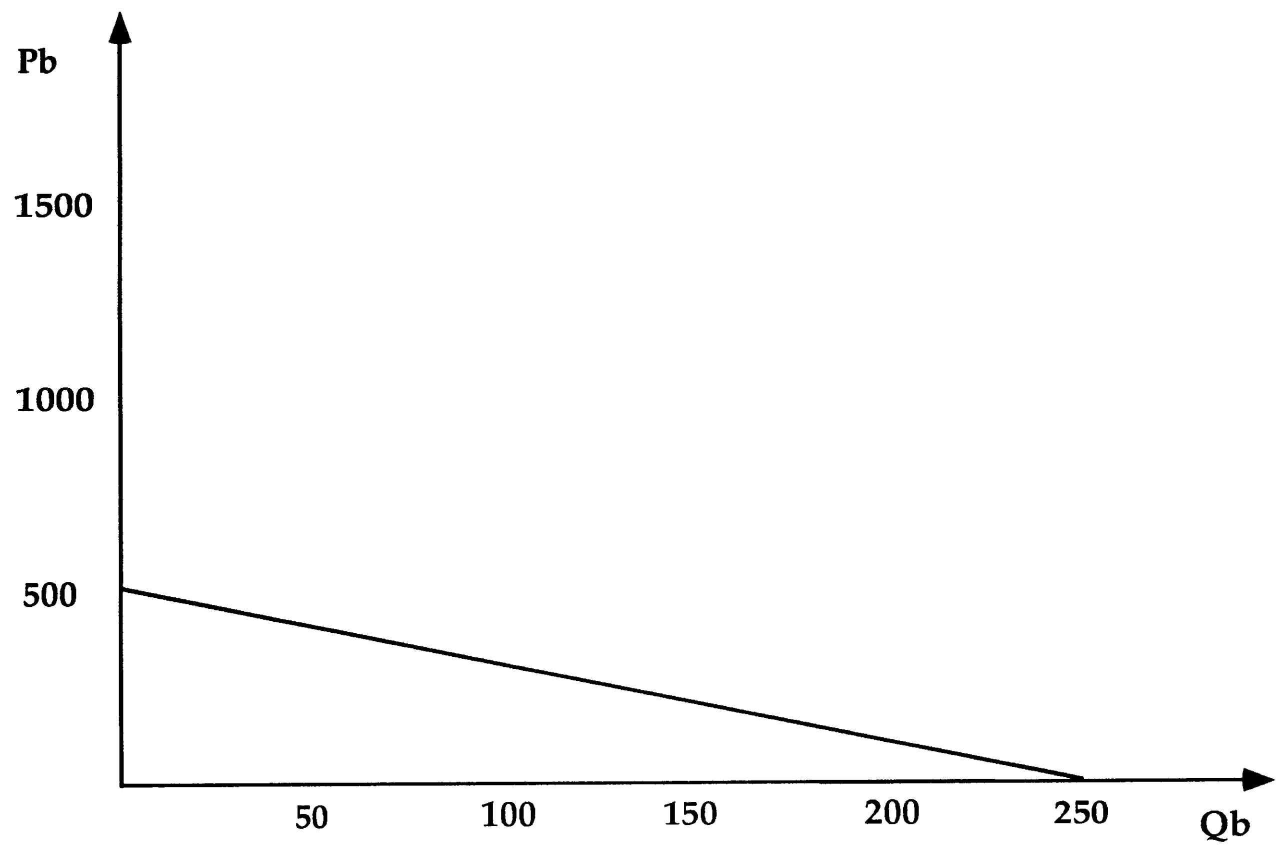
$P_Y = 1500 - 15Q_Y$
$P_B = 500 - 2Q_B$

Single Demand Curve Formulation :

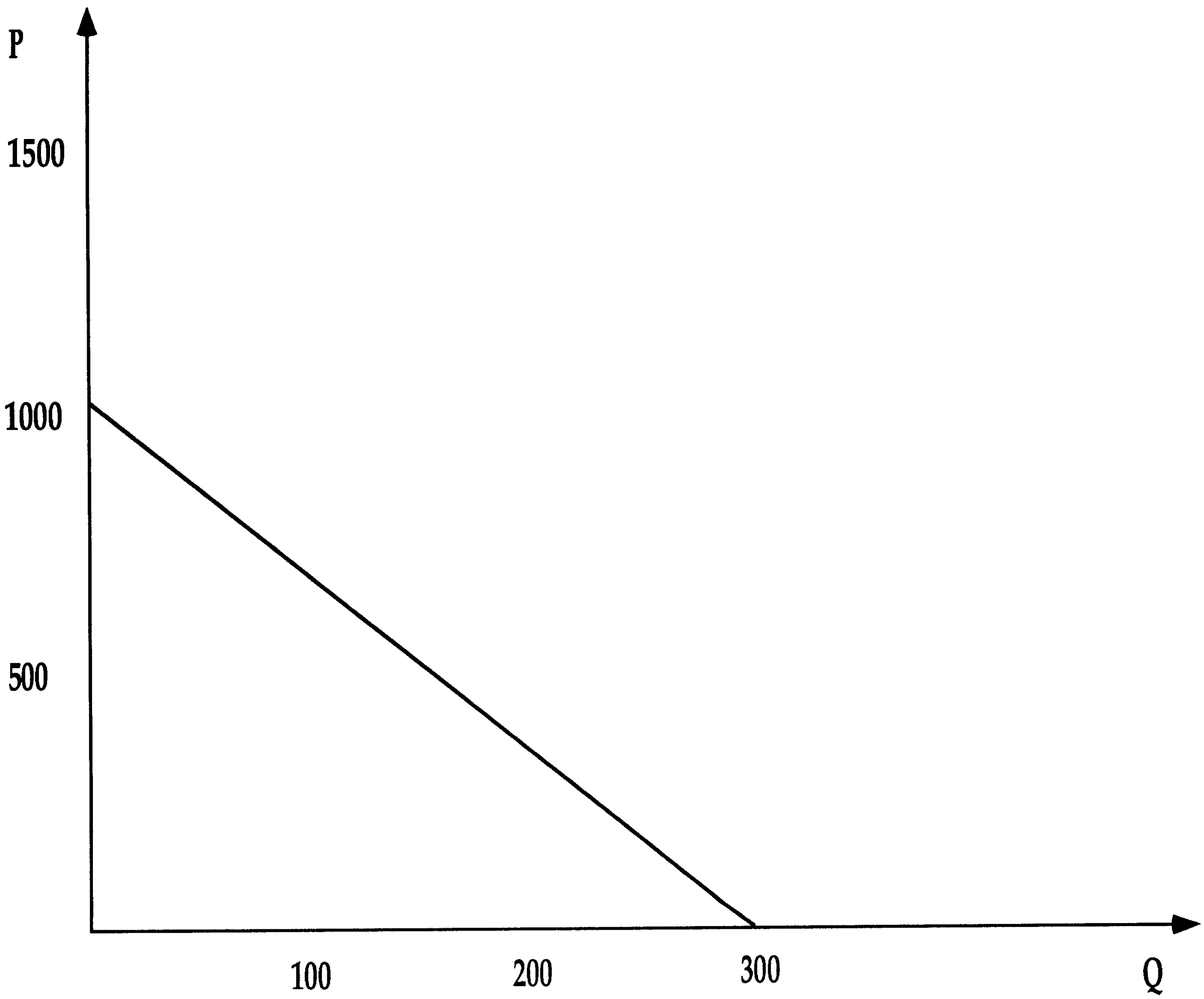
$P_Y = 1000 - 3.33Q_Y$
$P_B = 1000 - 3.33(Q_Y + Q_B)$

BOS - LAX : Dual Demand Curve Formulation





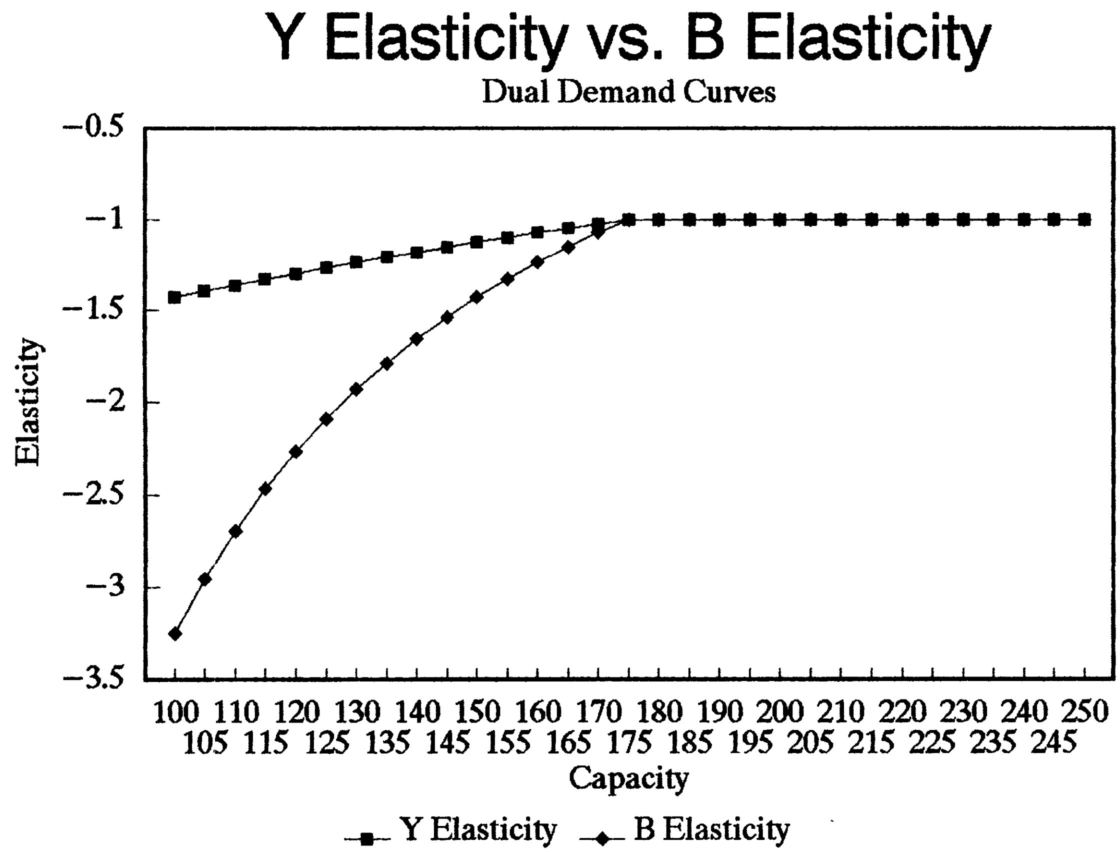
BOS - LAX : Single Demand Curve Formulation



BOS - LAX Example

Single Demand Curve Formulation Results :

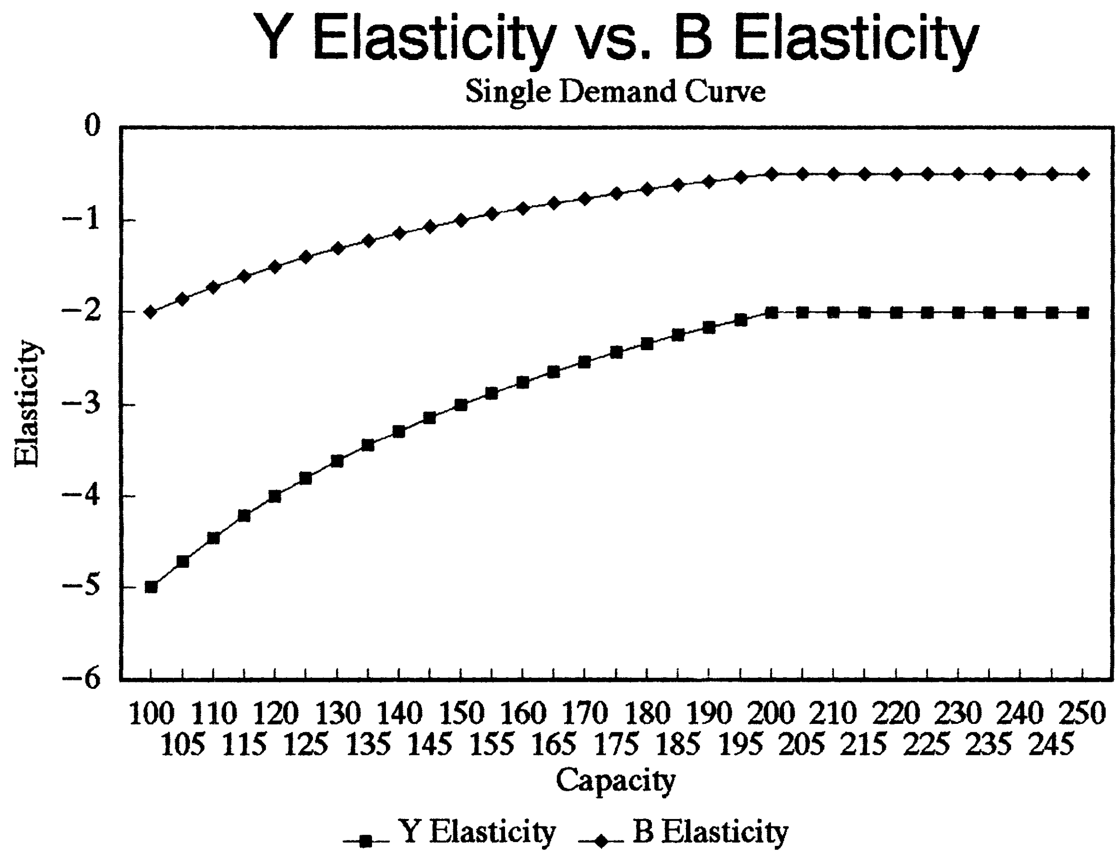
	Capacity Unconstrained	Capacity Constrained
Y Class Fare	666.67	750.00
B Class Fare	333.33	500.00
Y Class Pax	100	75
B Class Pax	100	75
Capacity	200	150
Revenue	100000	93750


Y Elasticity vs. B Elasticity
Dual Demand Curves
Elasticity
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250
105 115 125 135 145 155 165 175 185 195 205 215 225 235 245
Capacity
Y Elasticity
B Elasticity

BOS - LAX Example

Dual Demand Curve Formulation Results :

	Capacity Unconstrained	Capacity Constrained
Y Fare	750	794.12
B Fare	250	294.12
Y Pax	50	47
B Pax	125	103
Capacity	200	150
Revenue	68750	67647



Y Elasticity vs. B Elasticity
Single Demand Curve
0
−1
−2
−3
−4
−5
−6
Elasticity
100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250
105 115 125 135 145 155 165 175 185 195 205 215 225 235 245
Capacity
Y Elasticity
B Elasticity

Sell-Up Probability

Definition:

The probability that a B Class Passenger makes a booking for a seat in Y Class

In this formulation:

$$p_s = P\left[\text{a B Class Pax books in Y Class} \mid Q_B^A \geq Q_B^{BL}\right] \times P\left[Q_B^A \geq Q_B^{BL}\right]$$

where

Q_B^A = # of B Class Pax booked

Q_B^{BL} = B Class Booking Limit

If we assume that the P[a B Class Pax books in Y Class] is not correlated with B Class Pax arrival time, the relationship becomes:

$$p_s = P\left[\text{a B Class Pax books in Y Class}\right] \times P\left[Q_B^A \geq Q_B^{BL}\right]$$

Diversion Probability

Definition:

The probability that a Y Class Passenger makes a booking for a seat in B Class

In this formulation:

$$p_d = P\left[\text{a Y Class Pax books in B Class} \mid Q_B^A < Q_B^{BL}\right] \times P\left[Q_B^A < Q_B^{BL}\right]$$

If we assume that P[a Y Class Pax books in B Class] is not correlated with B Class passenger arrival time, the relationship becomes:

$$p_d = P\left[\text{a Y Class Pax books in B Class}\right] \times P\left[Q_B^A < Q_B^{BL}\right]$$

On Board Percentages

Definition:

Q_Y^{OB} = actual number of Y Class bookings

Q_B^{OB} = actual number of B Class bookings

Sell Up :

$$Q_Y^{OB} = p_s Q_B + Q_Y$$

$$Q_B^{OB} = (1-p_s) Q_B$$

Diversion :

$$Q_Y^{OB} = (1-p_d) Q_Y$$

$$Q_B^{OB} = p_d Q_Y + Q_B$$

Sell Up & Diversion:

$$Q_Y^{OB} = p_s Q_B + (1-p_d) Q_Y$$

$$Q_B^{OB} = p_d Q_Y + (1-p_s) Q_B$$

Variable Definitions Under Sell-Up & Diversion

Q_Y = # of Y Class Pax arrivals expected at the prevailing Y Class Price level ($p_s = p_d = 0$)

Q_B = # of B Class Pax arrivals expected at the prevailing B Class Price level ($p_s = p_d = 0$)

P_Y = Price level for each Y Class seat

P_B = Price level for each B Class seat

Incorporating Sell-Up

Two Fare Class Formulation :

$$\text{Max } R = P_Y Q_Y + P_Y p_s Q_B + P_B(1-p_s)Q_B$$

Subject to:

$$Q_Y + Q_B \leq Cap$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

$$P_Y = P_{Y0} - a_Y Q_Y$$

$$P_B = P_{B0} - a_B Q_B$$

Incorporating Diversion

Two Fare Class Formulation :

$$\text{Max } R = P_Y(1-p_d)Q_Y + P_BQ_B + P_Bp_dQ_Y$$

Subject to:

$$Q_Y + Q_B \leq Cap$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

$$Q_n = f(P_n)$$

Incorporating Sell-Up & Diversion

<u>Two Fare Class Formulation</u> :

$$\text{Max } R = P_Y(1-p_d)Q_Y + P_Y p_s Q_B + P_B(1-p_s)Q_B + P_B p_d Q_Y$$

Subject to:

$$Q_Y + Q_B \leq Cap$$

$$Q_Y \geq 0$$

$$Q_B \geq 0$$

$$P_Y = P_{Y0} - a_Y Q_Y$$

$$P_B = P_{B0} - a_B Q_B$$

Return to BOS - LAX Example with Sell-Up & Diversion

Single Demand Curve Formulation
Capacity Unconstrained Results :

	$P_d = 0$	$P_d = .1$
Y Class Fare	666.67	677.42
B Class Fare	333.33	354.84
Y Class OnBoard	100	87
B Class OnBoard	100	107
Y Class Pax	100	97
B Class Pax	100	97
Capacity	200	200
Revenue	100000	96774

BOS - LAX Example

Single Demand Curve Formulation
Capacity Constrained Results :

	$P_d = 0$	$P_d = .1$
Y Class Fare	750.00	750.00
B Class Fare	500.00	500.00
Y Class OnBoard	75	67.5
B Class OnBoard	75	82.5
Y Class Pax	75	75
B Class Pax	75	75
Capacity	150	150
Revenue	93750	91875

BOS - LAX Example

Dual Demand Curve Formulation Capacity Unconstrained Results :

	$P_s = 0$, $P_d = 0$	$P_s = 0$, $P_d = .1$	$P_s = .1$, $P_d = 0$	$P_s = .1$, $P_d = .1$
Y Fare	750	735.83	861.70	860.62
B Fare	250	255.09	202.13	206.92
Y OnBoard	50	46	57	53
B OnBoard	125	128	134	136
Y Pax	50	51	43	43
B Pax	125	122	149	147
Capacity	200	200	200	200
Revenue	68750	66275	76596	73799

BOS - LAX Example

Dual Demand Curve Formulation
Capacity Constrained Results :

	$P_s = 0$ $P_d = 0$	$P_s = 0$ $P_d = .1$	$P_s = .1$ $P_d = 0$	$P_s = .1$ $P_d = .1$
Y Fare	794.12	779.41	904.41	901.65
B Fare	294.12	296.08	279.41	279.78
Y OnBoard	47	43	51	47
B OnBoard	103	107	99	103
Y Pax	47	48	40	40
B Pax	103	102	110	110
Capacity	150	150	150	150
Revenue	67647	65309	73621	71140

Future Directions

- Further Computational Testing
- Generalized Sensitivity Analysis
- Test Different Booking Limit Control Policies
- Test Model With Varying Pax Arrival Pattern