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SOLVING THE SCHEDULE TRANSITION PROBLEM USING OPTIMIZATION TECHNIQUES

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Solving the Schedule Transition Problem Using Optimization Techniques

by

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Abstract

A new algorithm is introduced for effectively solving the airline schedule transition problem, which involves efficiently re-routing aircraft in order to balance the number and the types of aircraft at each station at the beginning of a new schedule with minimum cost.

An extensive study was performed on using "pre-switches" - changing aircraft types of certain flights on the last day of the current schedule - and "post-switches" - changing aircraft types of certain flights on the first day of the new schedule - to balance the types of aircraft at each station for a pair of aircraft types. Several possibilities for extension to fleets of more than two aircraft types were examined.

Airlines may use this algorithm in order to transition smoothly to the new schedule instead of relying on instincts of schedule analysts.

Thesis Supervisor: Dr. Robert W. Simpson

Title: Professor of Aeronautics and Astronautics and Director, Flight Transportation Laboratory

Acknowledgements

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Last but not least, for two people in this entire world, I have been one of two children to whom their unmatched, endless love has been devoted. They gave me life in whatever I have tried. I dedicate, with my thesis, the entire MIT experience to my parents.

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Preface

Have you ever wondered how the daily flight schedules for today's major airlines with hundreds of airplanes come to life each month ? Then, have you ever asked yourself what happens between the end of one flight schedule and the beginning of the next flight schedule which requires both different number and types of aircraft at most stations ? Are there any "rough edges" during this period ? Obviously there are. If your new schedule requires five 747s to fly out from a station the next morning when there are only three 757s flying in to that station on the previous night according to the old schedule, you find yourself with an interesting problem to solve.

This paper is intended to introduce an algorithm for smoothing these "rough edges" that do exist between two consecutive flight schedules. The algorithm is named the "Pre-switch/Post-switch Algorithm" for the reasons you shall discover later.

Neither obscure mathematical statements nor complicated mathematical proofs are included in this paper in order to present this material to anyone with enough interest in air transportation and flight scheduling. Also, each section title will be followed by a list of prior required readings from this paper to enhance comprehension and continuity.

Suggestions and comments are always highly appreciated at the Flight Transportation Lab, Room 33-412, MIT, Cambridge, MA 02139.

Tsuneo Fujiwara
April 22, 1989

1. Definitions

Prerequisites: None.

In this paper, the following terms are used frequently. Since they may have a special meaning here, the definition of each term is given below[1].

AAOG: Aircraft available on ground; the number of aircraft which have completed its unloading and are available for loading prior to departure. See "Cluster".

Aircraft Tail Number:
A label assigned to each aircraft in the fleet. Synonymous with tail number. See "Fleet".

Aircraft Type: A type of aircraft is defined by having common aircraft characteristics - engines, crew requirements, seating capacities, cruising speed, etc. Synonymous with type.

Aircraft Rotation:
A sequence of flight segments which forms a daily itinerary of a particular aircraft. Synonymous with rotation. A rotation starts with the first departure of the day and ends with the last arrival (perhaps extending into the next day).

[1]Simpson, Robert W., "Definition of Entities Used in Airline Scheduling", FTL Memorandum 88-7, Flight Transportation Laboratory, MIT, Cambridge, MA, November 1988.

Cancellation Problems:

When a pre-switch/post-switch is performed, the original rotation numbers are no longer valid. So for a schedule transition period, if we perform two or more pre-switches/post-switches that involve the same rotation(s), problems arise such as removal of the effects of the pre-switches/post-switches and mismatch of rotation numbers in terms of aircraft types. Therefore, we do not advise to perform two or more pre-switches/post-switches using the same rotation(s). For a daily flight schedule, in addition to the above, a post-switch of the previous day may overlap or cause problems to a pre-switch of the current day if both are to be performed and the rotations involved are not independent. See "Independent Rotation Pairs".

Cluster: A period of time when there are one or more available aircraft of a certain type on ground. Before and after a cluster, there is no available aircraft of that type on ground. See "AAOG".

Connection: Flights are defined as "connecting" if it is advertised that passengers can change from one flight to another at a "connecting station".

Crew: A unit consisting of pilots (flight crew) and flight attendants (cabin crew). Each flight segment requires one crew.

Deficit Station: The station which has more originating flights of certain aircraft type in the new schedule than terminating flights of that aircraft type in the old schedule. See "Flight", "Surplus Station", "New Schedule", and "Old Schedule".

Fleet: All aircraft of the same airline, or of one type at the same airline.

Flight: A sequence of one or more flight segments to be flown by the same aircraft under the same flight number. Passengers can remain on-board at intermediate stops. See "Flight Segment".

Flight Schedule: A daily list of flights flown by the fleet effective for a "schedule period", whose duration is usually one month. Synonymous with daily flight schedule.

Flight Segment: A nonstop flight between a pair of stations, composed of an origin station, a destination station, a departure time from the origin station, and an arrival time at the destination station. Synonymous with segment and flight leg.

Hub: A station at which the flight schedule is coordinated to have a large number of convenient connections available for passengers. Hubs generally have many intersections. See "Intersection".

Imbalance: One imbalance refers to the following condition at a station with physical balance: (1) one surplus aircraft type A rotation and one deficit aircraft type B rotation (surplus imbalance of A), or (2) one surplus aircraft type B rotation and one deficit aircraft type A rotation (deficit imbalance of A), for the period being examined, for a schedule with two aircraft types. A pre-switch/post-switch eliminates exactly one pair of imbalances, usually an instance of (1) and an instance of (2) above. This definition is extended to the multiple aircraft type case to refer to a station with one surplus aircraft type rotation and one deficit aircraft type rotation.

Independent Rotation Pairs:

Two rotation pairs are independent if all four rotations involved are distinct. There could be "cancellation" problems when performing pre-switches/post-switches unless the rotation pairs are independent. See "Cancellation Problems".

Intersection: Two rotations by aircraft of different types are said to intersect at a station if the respective aircraft can be pre-switched/post-switched. Two clusters are said to intersect if one or more aircraft of both types are available on ground at the same station at the same time, plus the departures following and the arrivals preceding the above time until the AAOG for that aircraft type reaches zero.

Market: An origin-destination pair. For example, Boston to Los Angeles is a market.

New Schedule: The daily flight schedule effective after the old schedule is discontinued. See "Flight Schedule".

Old Schedule: The daily flight schedule preceding the new schedule. After the last day of the old schedule comes the first day of the new schedule. See "Flight Schedule".

Overnighting Station:
The station at which an aircraft overnights (after the end of one day's rotation until the beginning of the next day's rotation).

Physical Balance:
The total number of originating flights at a station by all types equals the total number of terminating flights at that station, but their types may be different (e.g. two aircraft type A terminations and one each of aircraft types A and B originating the next morning). However, across all stations

served by the fleet, the total number of each terminating aircraft type must match the total number of the corresponding originating aircraft type. There can be imbalances at a station when there is physical balance. See "Fleet" and "Imbalance".

Post-switch: A post-switch occurs on the first day of a new schedule if aircraft A and B, flying portions of a rotation scheduled to be flown by the other type, arrive at a station from which they fly the remaining portions of their originally scheduled rotations.

Pre-switch: A pre-switch occurs at a station on the last day of an old schedule if aircraft A and B fly the remaining portions of each other's rotation.

Pre-switch/Post-switch Algorithm:

An algorithm that scrutinizes the schedule transition period and finds the "best" set of pre-switches and/or post-switches to accomplish the transition.

Schedule Transition Period:

The period consisting of n days during which pre-switches and/or post-switches are performed; generally n=2, consisting of the last day of the old schedule and the first day of the new schedule.

Schedule Transition Problem:

The problem of finding the least cost set of pre-switches and post-switches between aircraft types during the schedule transition period in order to execute the new schedule with the correct type of aircraft.

Surplus/Deficit Imbalance Pair or Surplus/Deficit Instance:

This is the condition necessary (but not sufficient) for pre-switch/post-switch. If at one station there is a surplus of aircraft type A and a deficit of aircraft type B after the last day of the old schedule, and at another station there is a surplus of aircraft type B and a deficit of aircraft type A, a surplus/deficit imbalance pair exists for the two aircraft types. For multiple aircraft types, if there is a station with surplus of an aircraft type and another station with deficit of the same aircraft type, a surplus/deficit instance exists for that aircraft type.

Surplus Station:

The station which has more terminating flights of certain aircraft type in the old schedule than originating flights of that aircraft type in the new schedule. See "Deficit Station".

Total Balance: If there are no imbalances at all stations for the period being examined, then the schedule is in total balance. See

"Imbalance".

Transition Flights:

Flights that operate due to physical imbalance during schedule transition period to balance the number and the type of aircraft needed for the new schedule. These flights are usually expensive and should be avoided, if possible. There also can be cancellations of flights to remove physical imbalance.

Turn: The connection of a specific aircraft tail number from one flight to another.

2. Organization

Prerequisites: None.

Most airlines publish a monthly schedule of all the flights in their fleet. Since flight schedules usually change from one month to the next (in terms of the required number and the types of aircraft needed to execute the flight schedule), the problem of correctly matching the number and the types of aircraft arises at all stations during the schedule transition period. We will call this the schedule transition problem[1], which usually consists of two stages: (1) match the number of aircraft to execute the new schedule at all stations, and (2) match the types of aircraft to execute the new schedule at all stations. In this paper, a new algorithm for solving the second stage of the schedule transition problem for an airline is developed and presented in detail.

This section summarizes the organization of the presentation. In the following page, a pictorial representation of the flow of this paper is shown. Solid arrows indicate strong dependence of the latter section to the former, and dashed arrows indicate weak connections between the two sections.

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[1]Simpson, Robert W., "The Schedule Transition Problem", FTL Memorandum 88-9, Flight Transportation Laboratory, MIT, Cambridge, MA, December 1988.

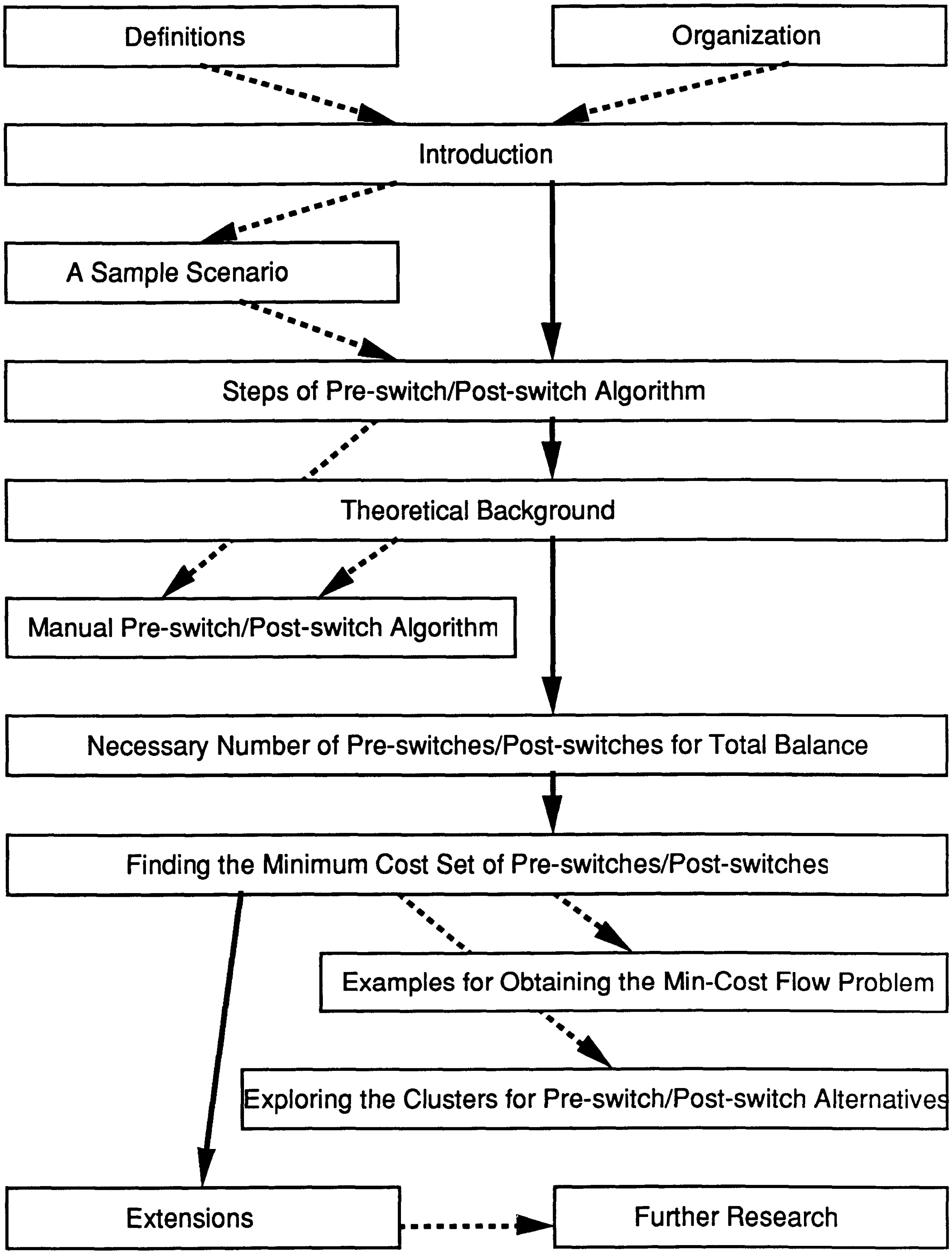

Figure 1 Summary Chart of Context Organization

3. Introduction

Prerequisites: Definitions. Organization.

Each month, airlines that operate scheduled air services are faced with the construction of the following month's flight schedule. Considering their fleet size, aircraft types in their fleet, seasonality of demand for their flight segments, operational constraints at their hubs and other stations, availability of certain aircraft, and/or various other constraints, there is usually a difference in the number and the types of aircraft required to run one monthly flight schedule from the next. So certain flight segments must be added, canceled, and/or flown by a different aircraft type than the one originally scheduled during the schedule transition period in order to adjust to the necessary number and the types of aircraft needed to execute the new schedule at all stations. Throughout this paper, the assumption is made that a monthly flight schedule is complete and has total balance every day at all stations during its effective period. It is also assumed that the schedule transition period consists only of the last day of the old schedule and the first day of the new schedule.

Since the new flight schedule may require a different number as well as types of aircraft in service from the previous flight schedule, it makes sense to divide the problem into two parts: (1) obtain physical balance at all stations, and (2) remove imbalances at all stations.

First, only the number of terminating flight segments for all types of aircraft is matched at all stations on the last day of the old schedule to the number of originating flight segments at all stations on the first day of the new schedule. Since the difference among aircraft types is neglected, this is just a matter of adding and/or cancelling flights at the stations with different number of originating flights from terminating flights. We must be careful to incorporate all aircraft newly purchased and/or aircraft going into and/or coming out of extended maintenance to adjust the number at the stations involved. This may be done by scheduling some late night flights to stations with deficits and out of stations with surpluses. We will not go into detail on the finer aspects of this first stage of schedule transition problem in this paper.

Once this problem is resolved, we have a physically balanced schedule transition period. Now the types of the terminating flights must be matched to the types of the originating flights. This is when the Pre-switch/Post-switch Algorithm to be described will be useful. It identifies the rotations to/from surplus and/or deficit stations and lists all possible pre-switches and/or post-switches between aircraft types that can be made to eliminate these surplus and deficit stations. Let's start with a fleet consisting only of a pair of aircraft types. A pre-switch can be done at any station between two rotations by aircraft types A and B. To reduce imbalances, we want to pre-switch a rotation which ends at a surplus A/deficit B station (colored GREEN) with one that ends at a surplus B/deficit A station (colored RED). Thus, the remaining flight segments for the two rotations affected will be flown by different aircraft types from the ones originally scheduled. Note that a pre-switch reduces the surpluses of the two aircraft types affected at the two terminating stations by one. A post-switch, on the other hand, exchanges an aircraft type A rotation originating at a surplus B/deficit A station (colored YELLOW) with an aircraft type B rotation originating at a surplus A/deficit B station (colored BLUE) at some station where both aircraft are present at the same time on the first day of the new schedule. Thus, the first few flight segments on the first day of the new schedule for the two rotations affected will be flown by different aircraft types from the ones originally scheduled. Note again that a post-switch reduces the deficits of the two aircraft types affected at the two originating stations by one.

If the pre-switches and the post-switches do not resolve all the imbalances at the stations, transition flights must be flown or certain flights cancelled in order to balance the schedule transition period as a last resort. Since these flights are sporadic and generate low revenues as mentioned before, they are to be avoided if possible. We will not go deeply into this situation because given enough intersections (large hubs where flights congregate provide many), we are safe to assume that all possible imbalances can be removed without the use of transition flights.

At the end of steps 1 through 3 of the Pre-switch/Post-switch Algorithm which will be described later, the aircraft types, rotations, stations and the times required in order to perform the pre-switch and/or post-switch are displayed for the schedule transition period requested. This will enable the flight scheduler to quickly perform the necessary pre-switches and/or post-switches manually for the schedule transition period with minimal (preferrably no) transition flights. We will call this the Manual Pre-switch/Post-switch Algorithm. Also, given the cost of each of the pre-switch and post-switch possibilities, we develop an algorithm that can automatically find the minimum cost set of pre-switches/post-switches that will result in the most resolutions of pairs of imbalances. We will call this the Automatic Pre-switch/Post-switch Algorithm, which is only an extension of the Manual Pre-switch/Post-switch Algorithm.

We will first introduce the Automatic Pre-switch/Post-switch Algorithm for the two aircraft type case. Then we will introduce possible approaches to solving the schedule transition problem for multiple aircraft types applying the concepts of Automatic Pre-switch/Post-switch Algorithm.

Later it will be shown that the daily flight schedule can be made totally balanced (after physical balance has been attained) by using the Pre-switch/Post-switch Algorithm with some care for the entire effective period of the flight schedule.

It can easily be noticed that the Pre-Switch/Post-switch Algorithm can also be applied to holidays, including Thanksgiving and Christmas, which requires a different schedule for every single day for a certain period of time. In this case, we will be performing the algorithm between each consecutive two-day period. Each consecutive two-day period from the last day of the incumbent schedule before the holidays begin until the first day of the schedule that becomes effective after the holidays finish behaves as a separate schedule transition period here.

Before we describe the algorithm for the two aircraft case, let us illustrate how this algorithm could be a part of a flight scheduler's job for an airline.

4. A Sample Scenario

Prerequisite: Introduction.

In order to motivate the subject, let us assume that you are a flight schedule coordinator for an airline with a fleet consisting of the following two aircraft types: 757s and MD11s. You have just completed the daily flight schedule effective September 1. The flight schedule effective August 1 which runs until August 31 was approved a couple of weeks ago. Due to the high demand of summer travel in August, you have fully utilized all of your fleet during this month, but in September, lower demand projections for air travel dictates that not all of your aircraft are necessary. Thus, you constructed a schedule requiring two fewer aircraft for September by systematically eliminating flight segments with low revenue projections. You have informed management of this intention, and two 757s have been contracted to go on lease on September 1 to a charter airline which will keep those aircraft during the fall/winter months. Since those two aircraft are going to be taken out of your rotations at your main hub station, you cleverly constructed a September schedule requiring two fewer overnighting aircraft at your main hub station and left the number of overnighting aircraft the same at all other stations to avoid confusion. The ground personnel will rest easy knowing the right (same) number of aircraft came back to their stations each night in September as did in August. No new gates are required at the stations, either. You will not hear another major complaint from the crew schedulers because they do not have to worry about getting extra crews over to some stations just for the first day of the new schedule due to an imbalance of the number of originating and terminating flights there.

Finally you got the September schedule approved by both management and other personnel. You say to yourself, "Super ! My job is done. I can take it easy until the time comes for October schedule." Right ? Usually not.

Remember that after all the hours, hassles, troubles, and meetings that you had to go through this week, life of a flight schedule coordinator is a tough one. That is why a hard-working fellow like yourself has been hired to do the job. What can be wrong with the September schedule ? You ask yourself. You did not use more aircraft than you have in your fleet, all rotations are continuous, and you have the same total number of terminating flights as the total number of originating flights. But have you looked more carefully into the schedule transition period ? Does the <u>correct type</u> of aircraft overnight during that period ?

Look at New York's JFK station, for instance. There is exactly one overnighting aircraft during both August and September, but in August it is a MD11 and in September it is a 757. What happens during the schedule transition period ? You say, "Right. That one flies to BDA from there, and in September, I decided to fly a 757 on that flight segment instead of a big MD11 because fewer people are expected fly to BDA in September than did in August according to the demand forecasting people. So what are you telling me to do here ?"

This is an example in which we have a surplus MD11 at JFK on the last day of the old schedule because the MD11 terminating at JFK on that day is not scheduled to fly to BDA the next morning due to a change in aircraft type requirement in the new schedule. JFK also has a deficit of 757 because the 757 that is scheduled to fly on the first day of the new schedule will not be there because there is no terminating 757 at JFK in the old schedule.

Notice that this only occurs during the schedule transition period. After September 1, the daily September flight schedule is in effect and there will be a 757 terminating at JFK each night (otherwise the September schedule is not balanced).

The question now is, what shall be done to avoid this aircraft type mismatch during the schedule transition period ? You do not want to rearrange and reshuffle the entire September schedule to spare a MD11 at JFK.

You may say that if it only happens on the night going into September 1, just forget it and deadhead a 757 from another station that has excess 757 to JFK and also deadhead the MD11 from JFK to another station that has deficit of MD11 during this period. There are some problems with this. It is not economical at all. First of all, you must pay the crew to fly them. That means each month you must look for those stations with excess or shortage of some aircraft type and bother the crew schedulers to find someone to fly these transition flights. Secondly, you cannot expect much revenue, if any, from a sporadic flight such as this. Therefore, an effective method to solve this schedule transition problem may save a lot of money for the airline.

The Pre-switch/Post-switch Algorithm addresses these problems and tries to overcome the costly approach of transitioning empty aircraft to balance the number and the type of aircraft on ground at the beginning of a new schedule by cleverly extracting the aircraft that can be pre-switched (on the last day of the old schedule) or post-switched (on the first day of the new schedule) at certain stations during the schedule transition period. This means that a different aircraft type will fly certain flight segments just during the schedule transition period rather than the one the daily flight schedule dictates. This may either cost more or less than flying the originally scheduled aircraft depending upon the load factor of the affected segments during that period. If the load factor of the flight segment being interchanged to the smaller aircraft is low so that the smaller aircraft can accommodate all passengers, and if the load factor of the flight segment being interchanged to the larger aircraft is high so that only the larger aircraft can handle the demand, then it may be better overall to have the aircraft interchanged. So your airline may actually save money doing this. Moreover, no flight segments will be canceled or transition flights added to correct these imbalances. And this is exactly what you want in order to optimally solve the problem at hand.

5. Steps of Pre-switch/Post-switch Algorithm

Prerequisites: **Introduction.** A Sample Scenario.

Now we are ready to describe the simplest Pre-switch/Post-switch Algorithm. This algorithm works for fleets consisting of exactly two aircraft types. We will later show that a simple extension of the concept described in the next few sections will enable us to solve the problem for any number of aircraft types in the fleet. Suppose we have a completed old and new schedule in front of us for both aircraft types. We will look at the schedule transition period.

Step 0: Physically balance all stations by cancelling/adding flight segments at stations with surpluses/deficits (stage 1 of schedule transition problem) of some aircraft types.

Step 1: Find all stations with a surplus of aircraft type A and a deficit of aircraft type B (surplus A/deficit B stations). Color terminating rotations by type A at these stations GREEN. Color originating rotations by type B at these stations BLUE.

Step 2: Find all stations with a surplus of aircraft type B and a deficit of aircraft type A (surplus B/deficit A stations). Color terminating rotations by type B at these stations RED. Color originating rotations by type A at these stations YELLOW.

Step 3: Look for intersections on the last day of the old schedule between flight segments of GREEN rotation(s) and RED rotation(s) or for intersections on the first day of the new schedule between flight segments of BLUE rotation(s) and YELLOW rotation(s). The identified intersections are candidates for pre-switch or post-switch, respectively.

Step 4: By selectively pre-switching and/or post-switching aircraft at the intersections identified, eliminate aircraft imbalances at the lowest possible overall cost.

Step 4 can be automated after some amount of work (if the costs pertaining to the pre-switches/post-switches are calculated from given data), which is the crux of the research. At the beginning, we leave Step 4 a manual task to be performed by trial and error looking at the output of Steps 1 through 3. The Manual Pre-switch/Post-switch Algorithm will have the user pick the pre-switch or the post-switch to take next, and then the user starts the Pre-switch/Post-switch Algorithm over to find the next pre-switch or post-switch to take. Later, the Automatic Pre-switch/Post-switch Algorithm will be introduced to automatically find the answer to Step 4. But first, theoretical justification is given in order to show that by some sequence of pre-switches/post-switches, the schedule transition problem will be improved after each pre-switch/post-switch to result in possible elimination of imbalances to achieve total balance. Note that we will concentrate on schedules with exactly a pair of aircraft types until the section entitled **Extensions**. In that section, we will show how all of the concepts introduced to solve the schedule transition problem for schedules with a pair of aircraft types extend to fleets consisting of more aircraft types.

6. Theoretical Background

Prerequisite: **Steps of Pre-switch/Post-switch Algorithm.**

The time has come for some justification to the preceding discussion. A series of simple proofs are given below to show that by correctly performing the pre-switches between a GREEN and a RED rotation and the post-switches between a BLUE and a YELLOW rotation as identified in the previous section, a pair of imbalances will be eliminated for each pre-switch/post-switch.

Suppose there is just one pair of aircraft types in your fleet. If the total number of rotations for those aircraft types is fixed and if each station has physical balance during the schedule transition period, then for a station that has a surplus of one aircraft type A, there exists another station with a deficit of one aircraft type A.

Proof: If station X has m+1 aircraft type A's terminating and m aircraft type A's originating during the schedule transition period, then since the total number of terminating and originating aircraft type A's must be equal across all stations from the definition of physical balance, (otherwise, further cancellations/additions of flights are necessary to achieve physical balance first) there must be some station Y at which the number of terminating aircraft type A's is less than the number of originating aircraft type A's. Let K' be the total number of terminating aircraft type A's except station X on the last day of the old schedule. Let K be the total number of originating aircraft type A's except station X on the first day of the new schedule. Then,

$$m + 1 + K' = m + K$$

So K' < K. Thus, we have some station Y at which the number of terminating aircraft type A's is less than the number of originating aircraft type A's.

Also, if there is physical balance between aircraft types A and B, for every surplus of aircraft type A at a station, there is a deficit of aircraft type B at the same station. Similarly, for every surplus of aircraft type B at some station, there is a deficit of aircraft type A at the same station.

Proof: Physical balance implies that the total number of terminating flight segments equals the total number of originating flight segments between aircraft types A and B. Let this total be T. So if m aircraft type A flight segments terminate then T-m aircraft type B flight segments terminate at the same station, and if m-k aircraft type A flight segments originate, then T-m+k aircraft type B flight segments originate from the same station. This shows that for every k surplus of aircraft type A at the station, there is k deficit of aircraft type B at that station.

If we identify a pair of stations with one having surplus aircraft type A (and deficit aircraft type B) and the other having surplus aircraft type B (and deficit aircraft type A), then we have found a surplus/deficit imbalance pair at those stations between the two aircraft types.

Now we show how the Pre-switch/Post-switch Algorithm correctly identifies the candidates to be pre-switched or post-switched. Look at the following figure.

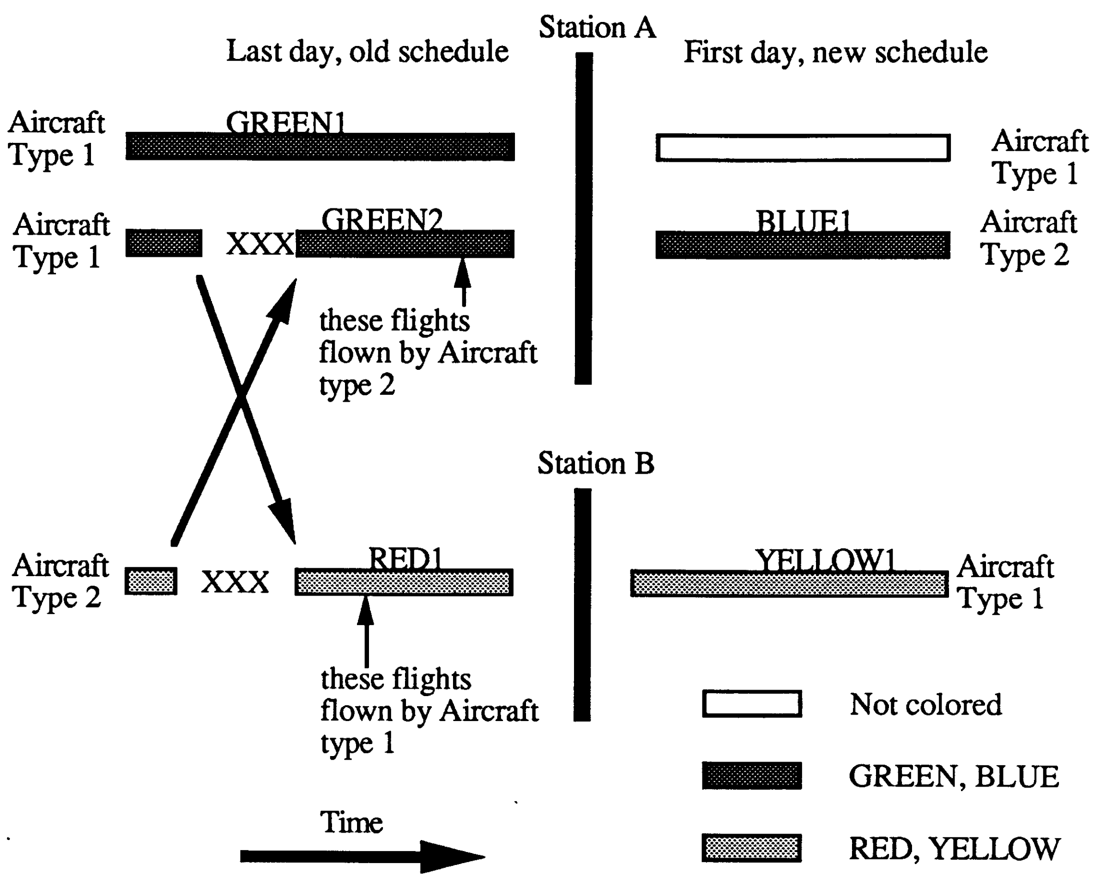

Figure 2 Illustration of Pre-switch with Two Aircraft Types

We see that at Station A there is a surplus of aircraft type 1 and a deficit of aircraft type 2, and at Station B there is a surplus of aircraft type 2 and a deficit of aircraft type 1. Thus, a surplus/deficit imbalance pair exists at stations A and B. The Pre-switch/Post-switch Algorithm first colors the aircraft type 1 rotations in the old schedule terminating at Station A GREEN, and the aircraft type 2 rotation in the new schedule originating at Station A is colored BLUE. Also, the aircraft type 2 rotation in the old schedule terminating at Station B is colored RED, and the aircraft type 1 rotation in the new schedule originating at Station B is colored YELLOW. If there is an intersection between the GREEN and the RED rotations, or between the BLUE and the YELLOW rotations, it will be identified. In our figure, the second aircraft type 1 rotation in the old schedule terminating at Station A intersects at station XXX with the aircraft type 2 rotation in the old schedule terminating at Station B. This is a candidate for pre-switch because the intersection occurs in the old schedule on the last day. If this pre-switch is performed, the remaining flights from XXX to Station A originally flown by aircraft type 1 will be flown by aircraft type 2, and the remaining flights from XXX to Station B scheduled to be flown by aircraft type 2 will be flown by aircraft type 1. Hence at both Stations A and B, one imbalance will be eliminated and the schedule transition problem improved.

Notice that all rotations of all colors (GREEN, RED, BLUE, and YELLOW) would disappear from the previous problem once it is solved. In general, a pre-switch removes at least one rotation of each color. This is because, by performing the pre-switch, which removes a surplus aircraft type 1 rotation terminating at some station A and a surplus aircraft type 2 rotation terminating at some station B, one deficit of aircraft type 2 originating from station A and one deficit of aircraft type 1 originating from station B the following morning will also automatically be removed by the nature of the involved stations (surplus

1/deficit 2 and surplus 2/deficit 1). With just a pair of aircraft types in the fleet for a physically balanced schedule, the number of surpluses of one aircraft type equals the number of deficits of the other aircraft type at the same station, as shown earlier. Thus, removing a surplus of one aircraft type also removes a deficit of the other aircraft type at the same station, depicted by two different colored rotations. Since two such stations are involved in a pre-switch, one depicting the rotations not in balance by GREEN and BLUE, the other depicting the rotations not in balance by RED and YELLOW, at least one rotation of each color will be removed by a pre-switch.

As an example of post-switch, look at figure 3. Here we will interchange the aircraft types of flights between Station A and YYY and between Station B and YYY on the first day of the new schedule. Therefore, the originating flights will be flown by a different aircraft from the ones originally scheduled. But by doing so, both stations A and B have one of their imbalances removed, with the same effect of removing at least one rotation of each color, which can be proven in a similar manner. An intersection of GREEN and RED rotations introduces a candidate for pre-switch, and an intersection of BLUE and YELLOW rotations introduces a candidate for post-switch.

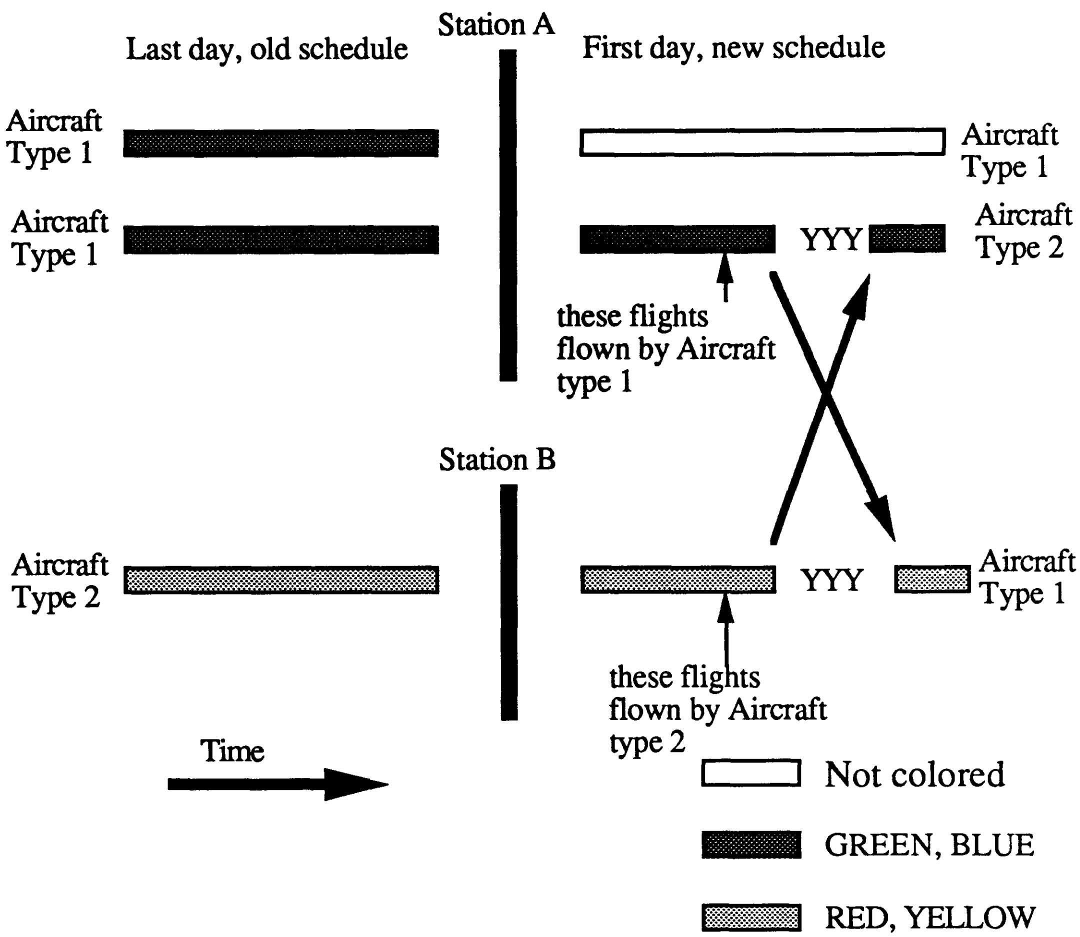

Figure 3 Illustration of Reswitch with Two Aircraft Types

Now we give a proof that no matter which of the possible pre-switch/post-switch you take, it will always result in an improvement (resolution of exactly one pair of imbalances) for the schedule transition problem with a fleet of two aircraft types.

Proof: Assume physical balance exists at all stations during the schedule transition period. For every surplus of aircraft type A at a station, we have shown that there is a deficit of aircraft type A at a different station. Moreover, we have proved that at the station where there is a surplus of aircraft type B, there is also a deficit of aircraft type A.

Every pre-switch leaves the first day of the new schedule unchanged; it exchanges exactly one aircraft type A rotation that was originally scheduled to fly to a surplus station of aircraft type A, thus reducing the surplus at that station by one, with one aircraft type B rotation that was originally scheduled to fly to a deficit station of aircraft type A, thus reducing the deficit of aircraft type A by one. This, at the same time reduces the surplus and the deficit of aircraft type B by one due to the surplus/deficit imbalance pair that exists between the pair of stations being pre-switched.

A post-switch will also remove a pair of imbalances in a similar fashion.

Therefore, for each pre-switch/post-switch performed one pair of imbalances is removed from the previous schedule transition period, thereby improving the problem.

However, this does not imply that random applications of the Pre-switch/Post-switch Algorithm can always eliminate the most number of pairs of imbalances between a pair of aircraft types. Even though pre-switch/post-switch improves the balance, to be able to perform this, we must find an intersection (as well as a surplus/deficit imbalance pair). Some pre-switch/post-switch selections may lead to no further intersections even though surplus/deficit imbalance pair exists. The question now arises: which of the possible pre-switches/post-switches should be performed when there are more than one possibility ?

Consider the example in figure 4 to illustrate the complication.

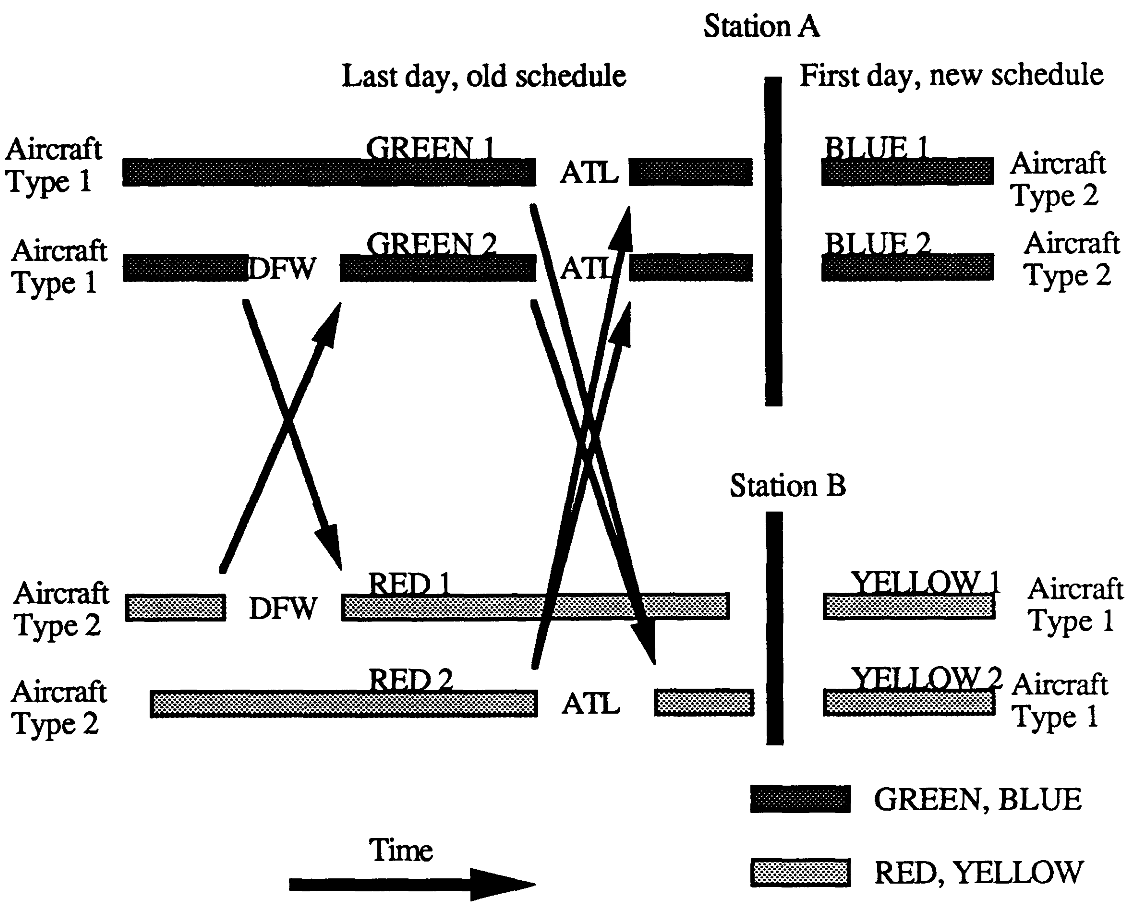

Figure 4 Example to Illustrate the Significance of Order

Notice that there are three separate pre-switch/post-switch possibilities - (1) between GREEN rotation 1 and RED rotation 2 at ATL, (2) between GREEN rotation 2 and RED rotation 1 at DFW, and (3) between GREEN rotation 2 and RED rotation 2 at ATL. Assume we first decide to perform a pre-switch described by (3). Then the schedule is improved as follows:

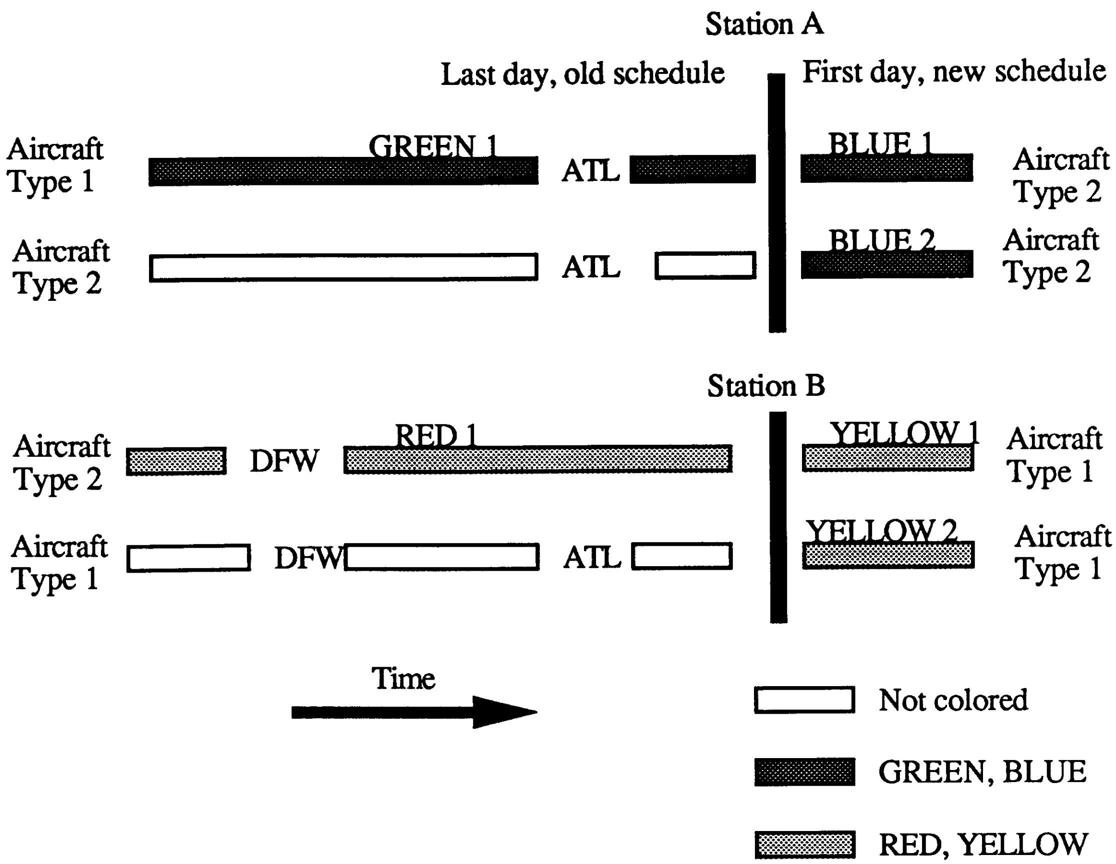

Figure 5 Example After a Bad Selection of Switch

Notice that we do not have a further possible pre-switch/post-switch now due to the absence of any intersections between two colored rotations (GREEN and RED or BLUE and YELLOW). Therefore, even though we have a surplus/deficit imbalance pair, we cannot improve the schedule further. If, instead, we had performed pre-switch (1) first, we can further improve the schedule by another pre-switch (2) because the schedule would look as follows after the first pre-switch.

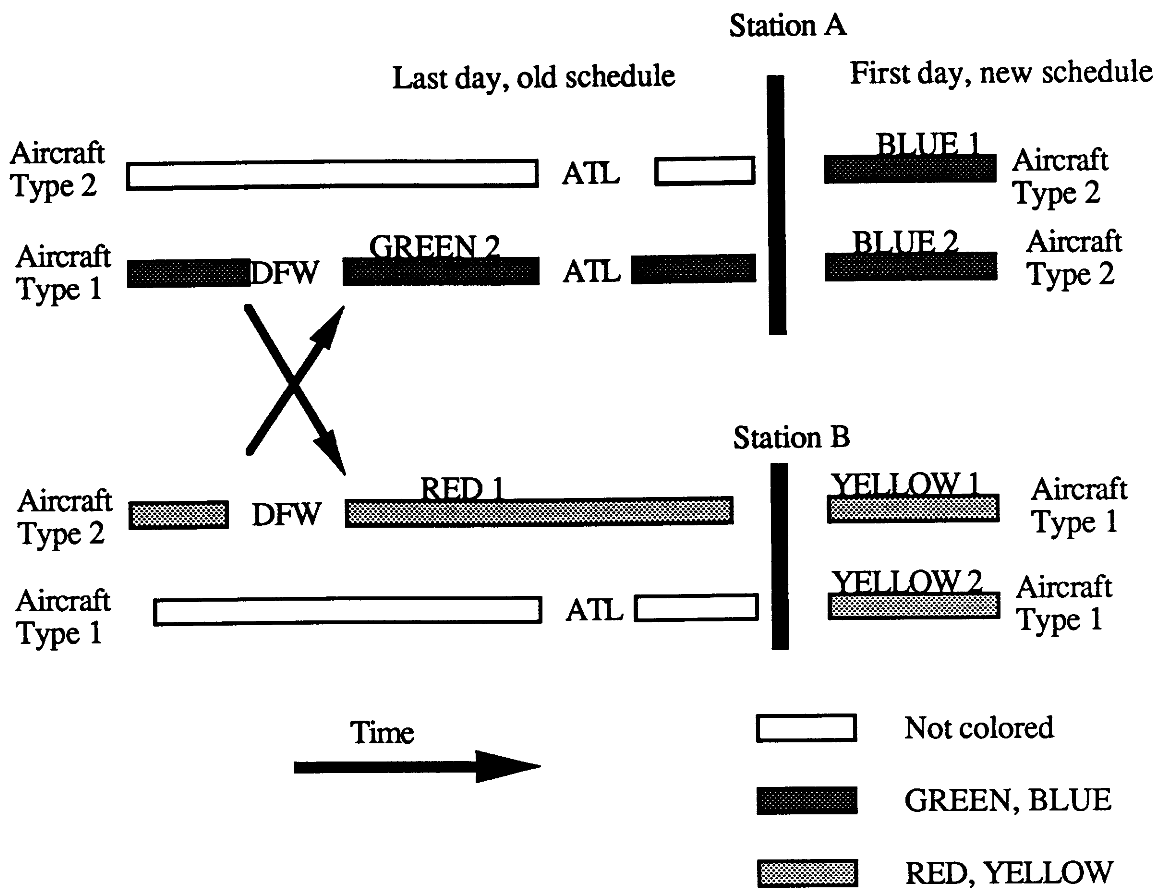

Figure 6 Example After a Good Selection of Switch

This shows that depending upon the pre-switches/post-switches chosen, the number of improvements to the schedule that can be made differs. In order to get the best improvement (preferrably total balance), we must choose the pre-switches and post-switches wisely.

This leads to the automation of Step 4 of the Pre-switch/Post-switch Algorithm. The two sections following the next section will develop an automated algorithm for optimizing both the number of pre-switches/post-switches to be performed and the total cost of pre-switches/post-switches in order to achieve the elimination of as many pairs of imbalances as possible. The first section describes a method for finding the necessary number of pairs of imbalances that should be removed from the schedule transition period to achieve total balance. This number gives a lower bound on the number of pre-switches/post-switches that is required. The second section describes how to find the minimum cost set of pre-switches/post-switches that removes the imbalances using the smallest number of pre-switches/post-switches. But first, let us give an example of how the Manual Pre-switch/Post-switch Algorithm aids in removing pairs of imbalances at stations in the next section.

7. Manual Pre-switch/Post-switch Algorithm

Prerequisites: Theoretical Background. Steps of Pre-switch/Post-switch Algorithm.

In this section, an example will be given to illustrate the steps taken for the Manual Pre-switch/Post-switch Algorithm described earlier. Appendix A at the end of the paper shows the pre-switch/post-switch report generated at the end of each iteration of the algorithm. Notice that between two reports, you are performing one (or more) of the pre-switches/post-switches generated in the report. One pre-switch/post-switch at a time is recommended for this procedure. In Appendix A, we see how the report changes as we continue to perform the pre-switches/post-switches.

All pre-switch and/or post-switch possibilities to improve the schedule transition problem will be identified and listed in a report after the first three steps of the Pre-switch/Post-switch Algorithm. The user merely selects the most desirable pre-switch/post-switch to be performed from the report and manually pre-switches/post-switches the affected flights on the schedule. A new flow balance report is then generated, and if there are still pairs of imbalances in the revised schedule, a new report listing all the pre-switch/post-switch possibilities at the current point will be generated, and the user repeats the selection of the next pre-switch/post-switch from the new report. The Manual Pre-switch/Post-switch Algorithm is available on the ISS (Interactive Scheduling System being developed at the Flight Transportation Lab, MIT) software.

Since the selection of the pre-switch/post-switch to be performed is random (by the user), there is no guarantee that the highest possible number of resolutions will result. This was shown at the end of the previous section. Automatic Pre-switch/Post-switch Algorithm which will be developed starting with the next section will solve for the minimum cost set of pre-switches/post-switches that removes the required number of pairs of imbalances from the schedule transition period to achieve total balance, given the appropriate costs for the pre-switches/post-switches.

8. Necessary Number of Pre-switches/Post-switches For Total Balance

Prerequisite: **Theoretical Background.**

In this section, we will calculate the number of pre-switches/post-switches necessary in order to remove all pairs of imbalances. This step will lead us to finding the minimum cost set of pre-switches/post-switches in the next section.

Remember that we showed earlier that a pre-switch or a post-switch removes a pair of imbalances from the schedule transition problem. Therefore, what we want is the number of pairs of imbalances in the schedule transition period.

First, let us consider only the pre-switches. Remember that a pre-switch between a GREEN rotation and a RED rotation removes one pair of imbalances. Note that we may only perform pre-switches between independent pairs of rotations; we cannot, for instance, pre-switch rotation 1 with rotation 2 and also pre-switch rotation 1 with rotation 3, because when the first pre-switch is performed, the rotation numbers are changed and the subsequent pre-switch may not be valid at all. This leads to cancellation problems. Therefore, once we have identified the set of all intersections between the pair of aircraft types of interest, the problem of finding the necessary number of pre-switches to achieve total balance becomes that of finding as many distinct pairs of GREEN and RED rotations that have intersections as possible.

If we draw a graph with vertices corresponding to GREEN and RED rotation numbers and edges of capacity one as intersections (there exists an edge between two vertices if that pair of rotation numbers intersect) and if we align the vertices so that the GREEN rotations will form one column and the RED rotations another, the problem of finding the necessary number of pre-switches to achieve total balance becomes an assignment problem; assign as many GREEN rotations column to the RED rotations column.

As an example, the graph corresponding to the example depicted in figure 4 is as follows.

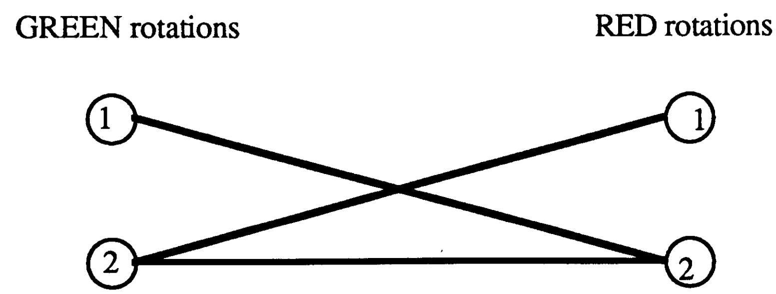

Figure 7 Assignment Problem for Example from Figure 4

Notice that the first GREEN rotation is the top rotation terminating in station A in figure 4, and the first RED rotation is the top rotation terminating in station B. Since there exists an intersection between GREEN1 rotation and RED2 rotation (at ATL), between GREEN2 rotation and RED1 rotation (at DFW), and between GREEN2 rotation and RED2 rotation (at ATL), there are edges between those corresponding vertices. The most number of pre-switches possible can be obtained when we choose the assignments given by GREEN1 - RED2 and GREEN2 - RED1. Thus, we pre-switch GREEN1 rotation with RED2 rotation (at ATL) and GREEN2 rotation with RED1 rotation (at DFW), which removes two

GREEN and two RED rotations if we just concentrate on the pre-switches, ignoring the post-switch possibilities.

This is the necessary number of pre-switches in this example to remove all imbalances because there are only two rotations of each color.

In general, assume that there are m GREEN rotations, out of which x rotations have matching aircraft types to turn into the next morning, and n RED rotations, out of which y rotations have matching aircraft types to turn into the next morning in the schedule. $x < m$ and $y < n$; otherwise, there is no surplus at these stations and the rotations will not be colored GREEN and RED, respectively. Since x out of the m GREEN and y out of the n RED rotations can already turn into rotations of the same aircraft types the next morning, there are $m - x$ surpluses of aircraft type 1 and $n - y$ surpluses of aircraft type 2 across all stations. For a physically balanced schedule, $m - x = n - y$ because for each surplus of aircraft type 1, we must have a surplus of aircraft type 2, since both are equal to the number of deficit of aircraft type 1 at the surplus stations of aircraft type 2, as we proved earlier. So $m - x$ or $n - y$ is the number of imbalance pairs we want to remove.

In a similar manner, if we now concentrate on the first day of the new schedule, when post-switches are performed, the problem of finding the necessary number of post-switches to achieve total balance can be solved by an assignment problem between BLUE rotation columns and YELLOW rotation columns. Again, if there are k BLUE and h YELLOW rotations, of which z and w have matching aircraft types to turn into at the stations involved ($z < k$, $w < h$), then k - z or h - w is the number of imbalance pairs that should be removed to achieve total balance for a physically balanced schedule.

Since both a pre-switch and a post-switch removes one pair of imbalances, if the schedule is physically balanced, then we want to remove m - x = n - y = k - z = h - w pairs of imbalances to achieve total balance.

Up to now, we have ignored the fact that one rotation may turn to one of several choices of rotations at the end of one day's schedule. Generally, if there are p aircraft type 1's overnighting in one station, there are up to p ! = p x (p-1) x ... x 2 x 1 ways to perform these turns. In case of the example in figure 4, GREEN1 rotation may turn to BLUE1 or BLUE2 rotation instead of being pre-switched with RED2 rotation (maybe to be post-switched later). Similarly, RED1 rotation may turn into YELLOW1 or YELLOW2 rotation at the end of the last day of the old schedule. We are assuming that if an aircraft flies the last segment scheduled to be flown by RED1 rotation, that aircraft's rotation is, in effect, RED1, and if it then flies the first segment of the YELLOW2 rotation the next morning, that aircraft turned from RED1 rotation to YELLOW2 rotation. So if there is a pre-switch of aircraft, RED1 rotation will be flown by aircraft type 1 instead of the originally scheduled aircraft type 2. By incorporating the turns, we can enhance our problem and also give a sense of timing to the problem.

Let's go back to the point at which we identified the problem of finding the necessary number of possible pre-switches to be an assignment problem. The problem of finding the necessary number of post-switches was another assignment problem. If we generalize the problem to one of finding the maximum number of possible pre-switches or post-switches, we can see that there are now four vertex columns because the BLUE and the YELLOW rotations will each represent a vertex column. If we now try to draw the graph for this problem, assuming that time runs from the left side of the graph to the right, we can order the columns so that the left two vertex columns would be the GREEN and the RED, and the right two vertex columns would be the BLUE and the YELLOW, since post-switches occur on the next day after the pre-switches.

By starting the graph this way, we can incorporate the turns easily. Note that if we assume any GREEN (RED) rotation can turn to any BLUE (YELLOW) rotation the next morning because a GREEN (RED) rotation not pre-switched may turn into any of the BLUE (YELLOW) rotations on the first day of the new schedule to be post-switched, then we include all possible edges between GREEN vertices and BLUE vertices, and between RED vertices and YELLOW vertices to show these permissible turns (Note: If a particular turn is not feasible, the edge corresponding to that turn will not be drawn.). They will be called "turn" edges to distinguish themselves from the edges depicting pre-switches and post-switches. We must be careful, however, not to have more than one flow through any vertices representing rotations because if we do, then we will have cancellation problems; the same rotation is utilized more than once for pre-switches, post-switches, and/or turns.

Now we have a graph with 4 columns (corresponding to GREEN, RED, BLUE, and YELLOW rotations, in that order) which incorporates the turns. If we add a source before the GREEN rotation(s) and a sink after the YELLOW rotation(s), the problem of finding the pre-switches/post-switches that can be done to resolve all pairs of imbalances is a "special" feasibility problem from the source to the sink, where each edge has capacity one except for the back edge, which has capacity equal to the number of pairs of imbalances that must be removed from the schedule to achieve total balance, and which requires an additional flag to check that there will not be more than one flow going through the vertices except the source and the sink. Because a flow from the source to the sink represents the resolution of one pair of imbalances using a pre-switch or post-switch with the given turn, a flow equal to the number of pairs of imbalances calculated earlier should remove all pairs of imbalances. Figure 8 depicts this feasibility problem graph obtained for the example in figure 4.

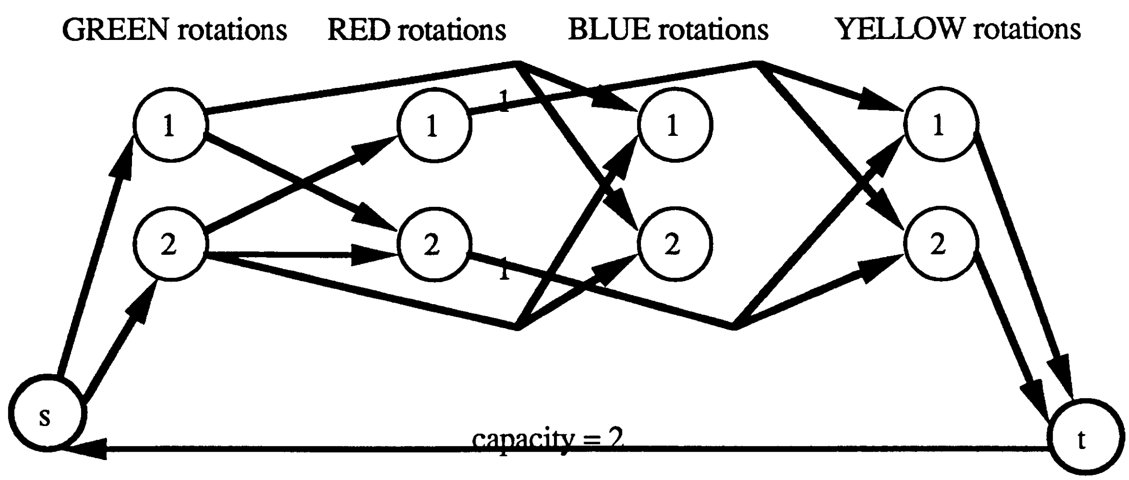

Figure 8 Special Feasibility Problem for Example from Figure 4

Note that solving this problem is as difficult as solving some max-flow problem because the additional flag required for this problem can be driven out be using an extra edge and an extra vertex for each flag. See figure 9 for the regular max-flow representation of the example from figure 4.

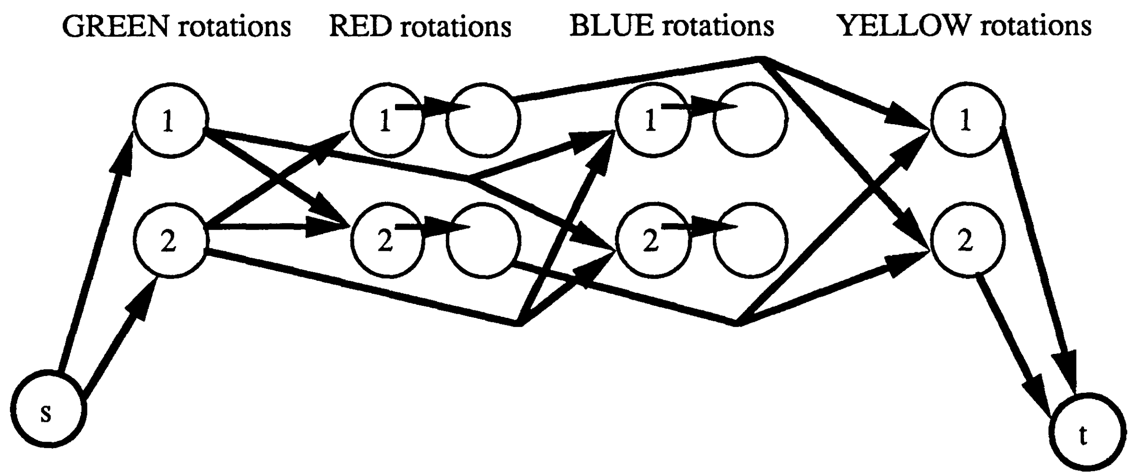

Figure 9 Max-flow Problem for Example from Figure 4

This representation has all pre-switch/post-switch/turn information from the feasibility problem and also guarantees that there will not be more than one flow going through each vertex depicting a rotation. Applying any available max-flow algorithm on this graphical representation will yield the maximum number of pre-switches/post-switches possible. If this number is greater than or equal to the number of pairs of imbalances we have to remove to achieve total balance, which was determined earlier, then we have a set of pre-switches/post-switches that will resolve all imbalances.

9. Finding the Minimum Cost Set of Pre-switches/Post-switches

Prerequisite: **Necessary Number of Pre-switches/Post-switches for Total Balance.**

Now that you know the necessary number of pre-switches/post-switches you must make to achieve total balance, you would like to perform the minimum cost set of pre-switches/post-switches to get that number of resolutions. How can we find such a set of pre-switches/post-switches ?

First, we must contemplate what kind of costs are necessary to model this problem. What is the cost of performing a certain pre-switch/post-switch ? And what cost should be assigned to a turn ?

Suppose that a flight segment originally scheduled to be flown by aircraft type 1 will be flown by aircraft type 2 after some pre-switch we may undertake on the last day of the old schedule. The cost of flying that flight segment using aircraft type 2 will be different from the cost using aircraft type 1 due to the difference in (1) fuel efficiency of aircraft, (2) number of seats available, (3) requirements for crew, etc. If we assume that the cost is higher (therefore, the net income is lower) when we fly the flight segment using the substitute aircraft type (in this case aircraft type 2), then the cost associated with that pre-switch/post-switch will be positive; otherwise, the cost is negative. Moreover, the cost of a pre-switch/post-switch is the difference between the cost of flying all the affected flight segments using the substitute aircraft type and the cost of flying all the flight segments to be affected with the one originally scheduled. In order to calculate this, we need, for each aircraft type, the cost of flying each flight segment it is able to fly in the route system plus the load factors for these flight segments during the schedule transition period. This information is assumed to be readily available from forecast studies and/or past data. Remember that the cost of a pre-switch/post-switch could also be negative. If the load factor of the flight segment being interchanged to the smaller aircraft is low so that the smaller aircraft can accommodate all passengers, and if the load factor of the flight segment being interchanged to the larger aircraft is high so that only the larger aircraft can handle the demand, then it may be better overall to have the aircraft interchanged. We assign an infinite cost for a flight segment flown by some aircraft type if the particular aircraft type cannot fly that segment due to the limit in range of the aircraft type or some other reason(s). Once these costs of pre-switches/post-switches are calculated, they will be assigned to the corresponding edges on the feasibility problem graph from the previous section.

The cost associated with each turn should measure the relative economy in choosing a certain turn. Let the "best" turn have a cost equal to zero. Due to the ground times at overnighting stations, total hours flown by an aircraft since the last maintenance check, etc., the cost of the other turns may be higher. This will give a measure of which turns are favored. These costs will be associated with the "turn" edges in our graphical representation from the previous section. At this point, we merely assume that most of the time these "turn" edges have costs of zero (we favor all possible turns equally) because these costs are difficult to calculate.

Now look at the graph we generated in order to solve the "special" feasibility problem from the previous section with the above cost additions. Each path from the source to the sink corresponds to a resolution of one pair of imbalances as discussed earlier. The cost of the path is readily available by adding the cost of each pre-switch/post-switch/turn edge associated with the path. Since we already know the necessary number of pre-switches/post-switches that are required, at the source vertex we make available that magnitude of supply, and at the sink vertex we put a demand of that magnitude to resolve that number of pairs of imbalances. The other vertices have zero supply or demand. Once we enter the cost of pre-switches/post-switches and turns into the corresponding edges in this graph, make the source a supply vertex with supply of the maximum possible number of resolutions of pairs of imbalances, and make the sink a demand vertex with the same magnitude of demand, the minimum cost set of pre-switches/post-switches resulting in the required number of resolutions of pairs of imbalances can be found by solving a "special" min-cost flow problem on this graph. The "special" part comes, again, from the flag that is needed for each vertex that is not a source or a sink to ensure at most one flow will go through each of them to avoid cancellation problems.

We can use a slightly modified cost-scaling algorithm by Ahuja and Orlin (1987)[1] to solve this min-cost flow problem. It is slightly modified because we must accommodate for the additional flag. There will be no negative cost cycles, since all edges have finite capacity limits. Since the capacity of each edge is one, the same pre-switch/post-switch will not be redundantly performed; moreover, because of the flag that is added at the vertices, it ensures that the same rotation will not be interchanged more than once. Thus, a feasible minimum cost set of

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[1]Orlin, James B., "15.082 Class Notes", Sloan School of Management, MIT, Cambridge, MA, February 1988.

pre-switches/post-switches and turns will be obtained.

In the figure below, we illustrate the "special" min-cost flow problem for the example given in figure 4. Note that the cost in each edge is the cost of performing a pre-switch calculated from a set of data pertaining to the costs of flying each affected flight segment using different aircraft types from the ones originally scheduled.

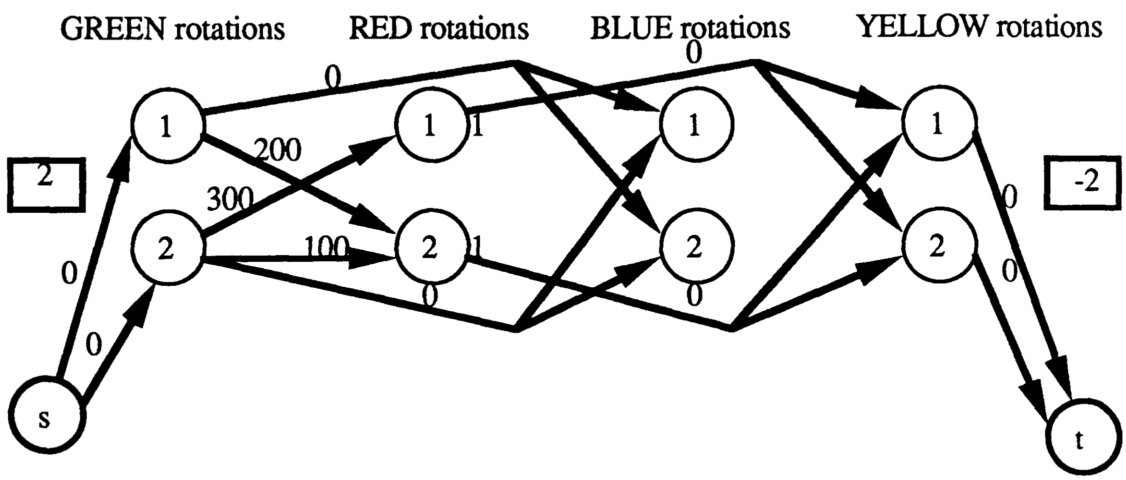

Figure 10 Special Min-cost Flow Problem for Example from Figure 4

Note that the equivalent min-cost flow graph, shown in figure 11, can be obtained by the addition of a vertex and an edge for each vertex that denotes a rotation, as before. The cost-scaling algorithm for the min-cost flow problem developed by Ahuja and Orlin (1987) may be used to solve this min-cost flow problem. Now that there are no special flags, this is a regular min-cost flow problem from the source to the sink.

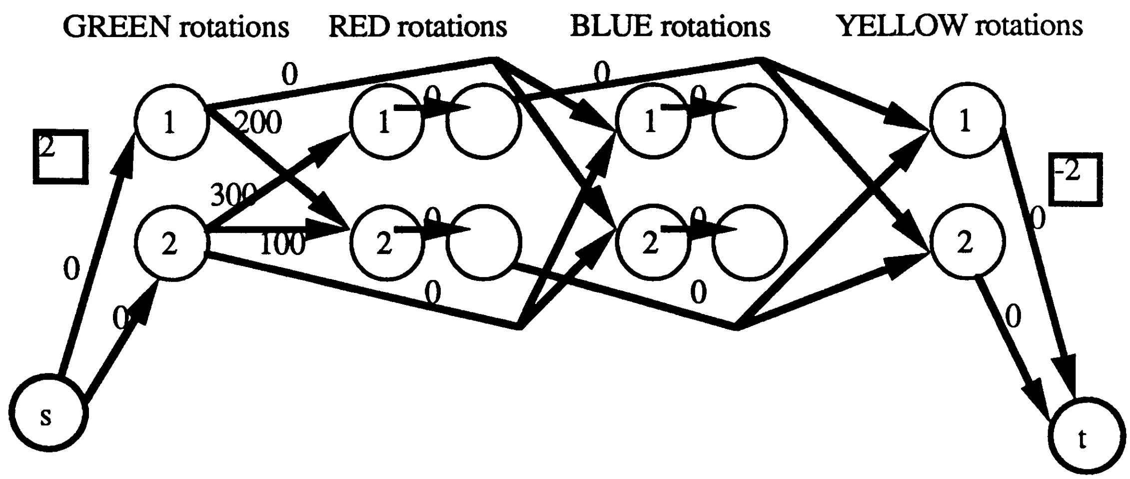

Figure 11 Min cost-flow Problem for Example from Figure 4

10. Examples for Obtaining the Min-Cost Flow Problem

Prerequisite: Finding the Minimum Cost Set of Pre-switches/Post-switches.

Now that we have thoroughly discussed the theory of the Pre-switch/Post-switch Algorithm and how it can be automated by finding a corresponding min-cost flow problem, we will give two examples illustrating graphically the conversion from the schedule transition problem to the min-cost flow problem. Assume that no cost is associated with the turns in both examples.

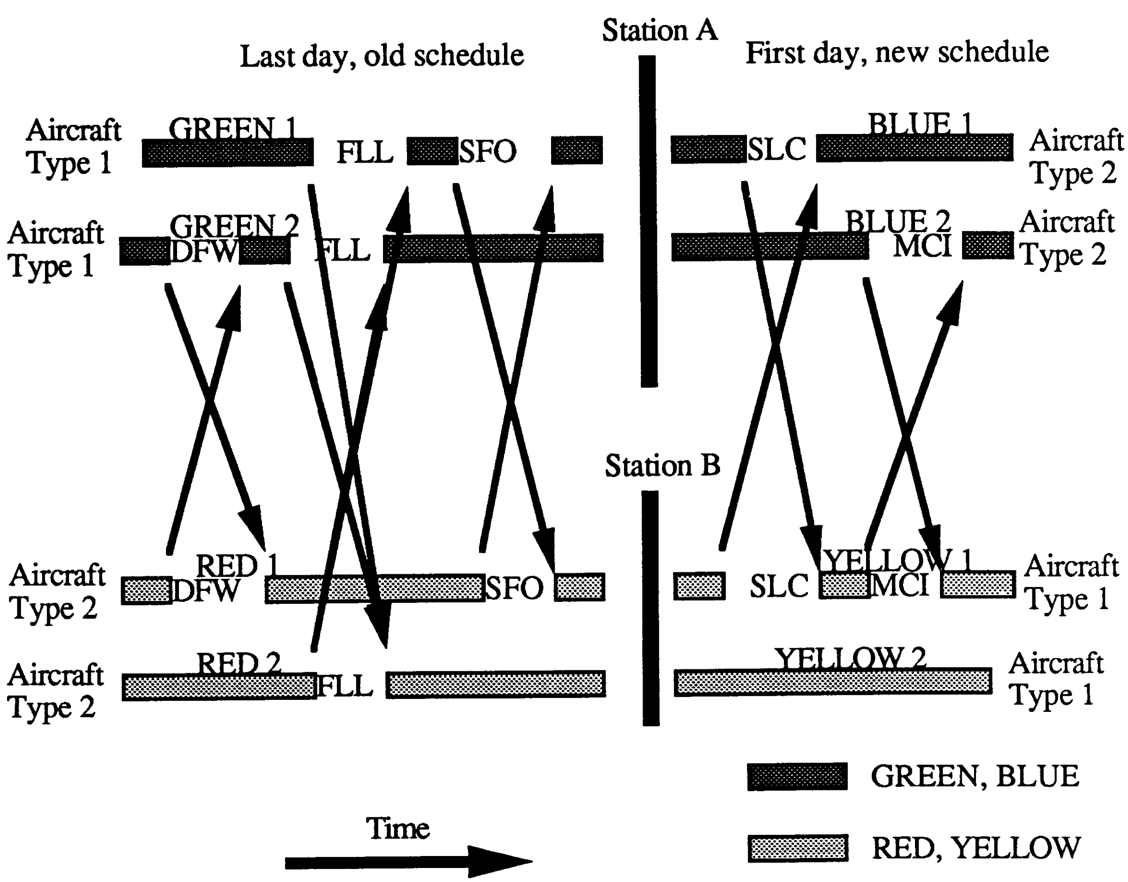

Figure 12 Example (1)

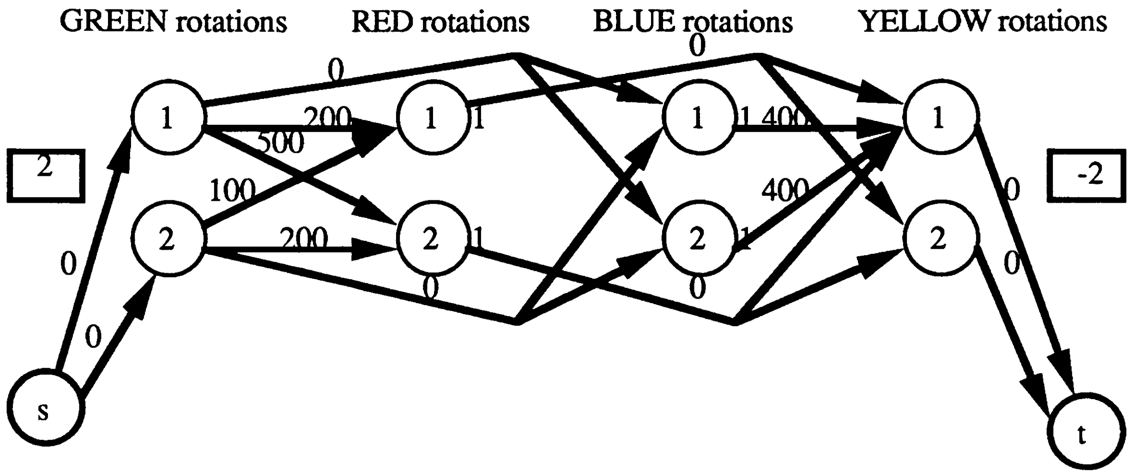

Figure 13 Special Min-cost Flow Problem for Example (1)

In figure 12, there are two GREEN, two RED, two BLUE, and two YELLOW rotations. Notice that all turns are assumed feasible in this case. Since the intersections are between (1) GREEN1 and RED1, (2) GREEN1 and RED2, (3) GREEN2 and RED1, (4) GREEN2 and RED2, (5) BLUE1 and YELLOW1, and (6) BLUE2 and YELLOW1, the edges corresponding to these are drawn in figure 13. Two of these intersections that minimize the total cost of pre-switches/post-switches will be found by solving the "special" min-cost flow problem.

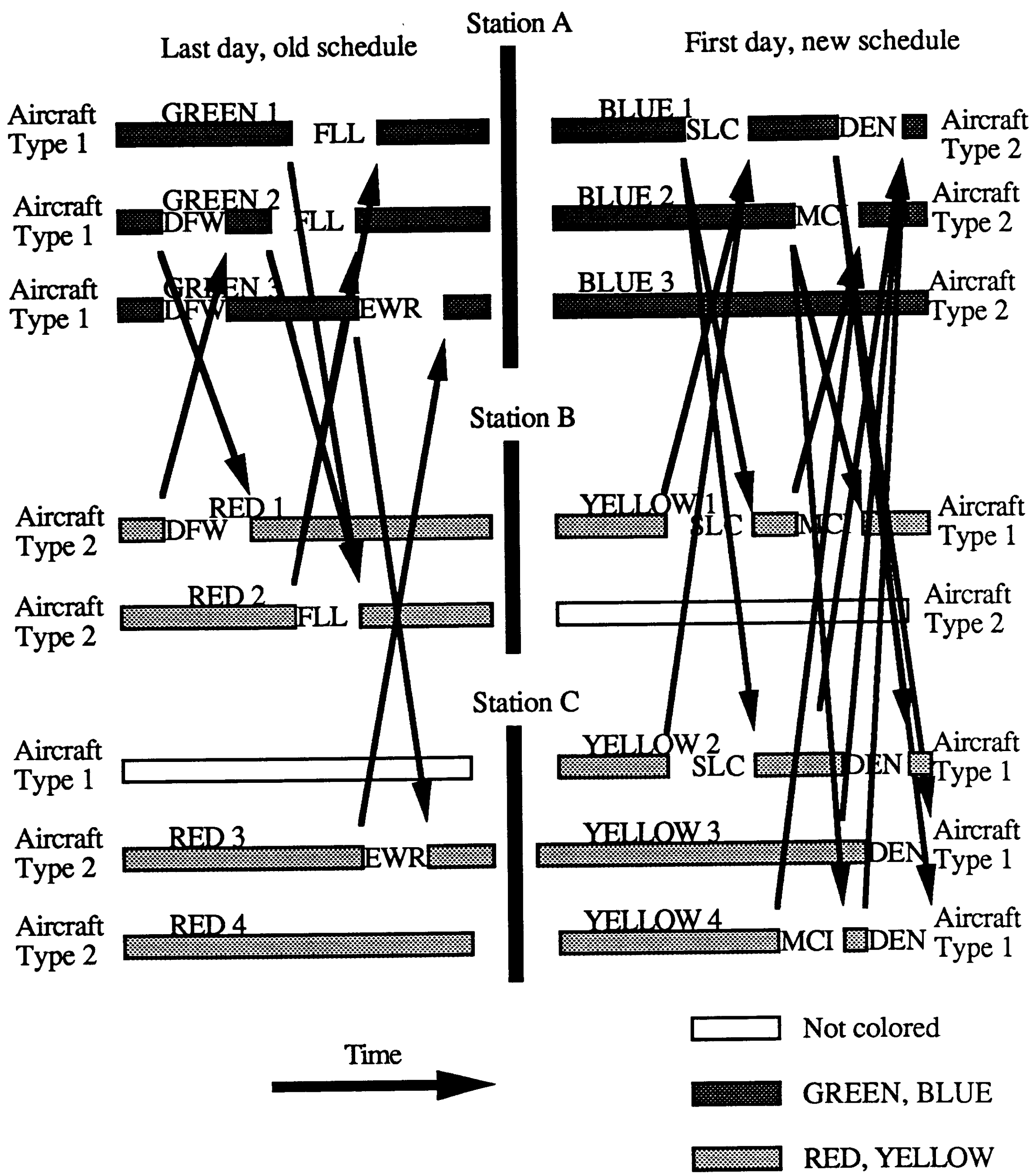

Figure 14 Example (2)

In figure 14, there are three GREEN, four RED, three BLUE, and four YELLOW rotations, of which zero, one, zero, and one rotations have matching aircraft types to turn to/from, respectively (These are the non-colored rotations in the figure.). Let us try to find the min-cost flow representation here. The intersections are (1) GREEN1 and RED2, (2) GREEN2 and RED1, (3) GREEN2 and RED2, (4) GREEN3 and RED3, (5) BLUE1 and YELLOW1, (6) BLUE1 and YELLOW2, (7) BLUE1 and YELLOW2, (8) BLUE1 and YELLOW3, (9) BLUE1 and YELLOW4, (10) BLUE2 and YELLOW1, and (11) BLUE2 and YELLOW4. Note that (6) and (7) above are separate edges even though they connect the same vertices. This is so since the intersection at SLC is separate from the intersection at DEN, usually requiring different costs, even though they are both between the same rotations. Figure 15 shows emphasis on the available turns and simplifies the previous figure for this example, and figure 16 represents the correct "special" min-cost flow problem graph that requires a slightly modified min-cost flow algorithm to solve.

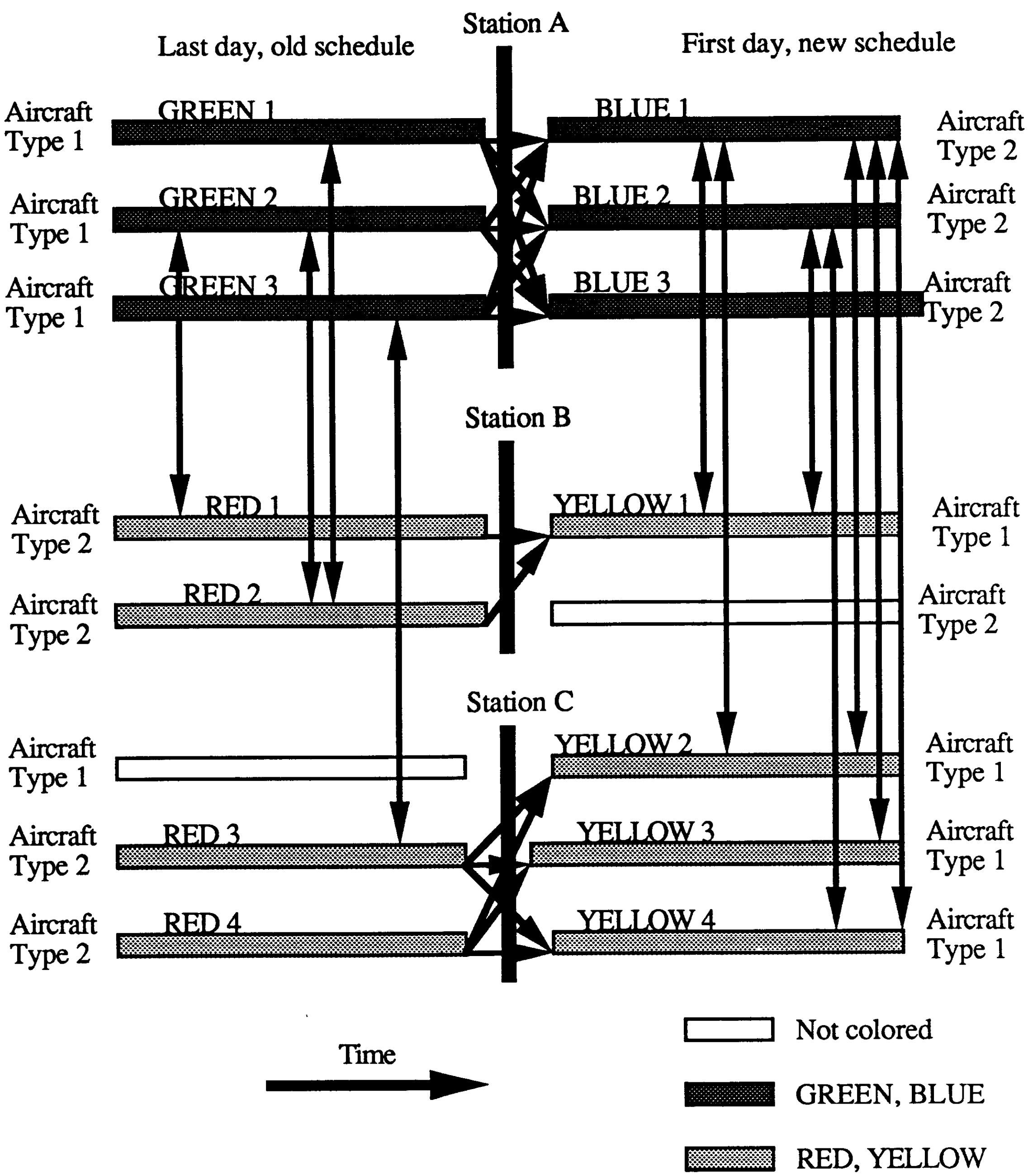

Figure 15 Pre-switch/Post-Switch/Turn Edges for Example (2)

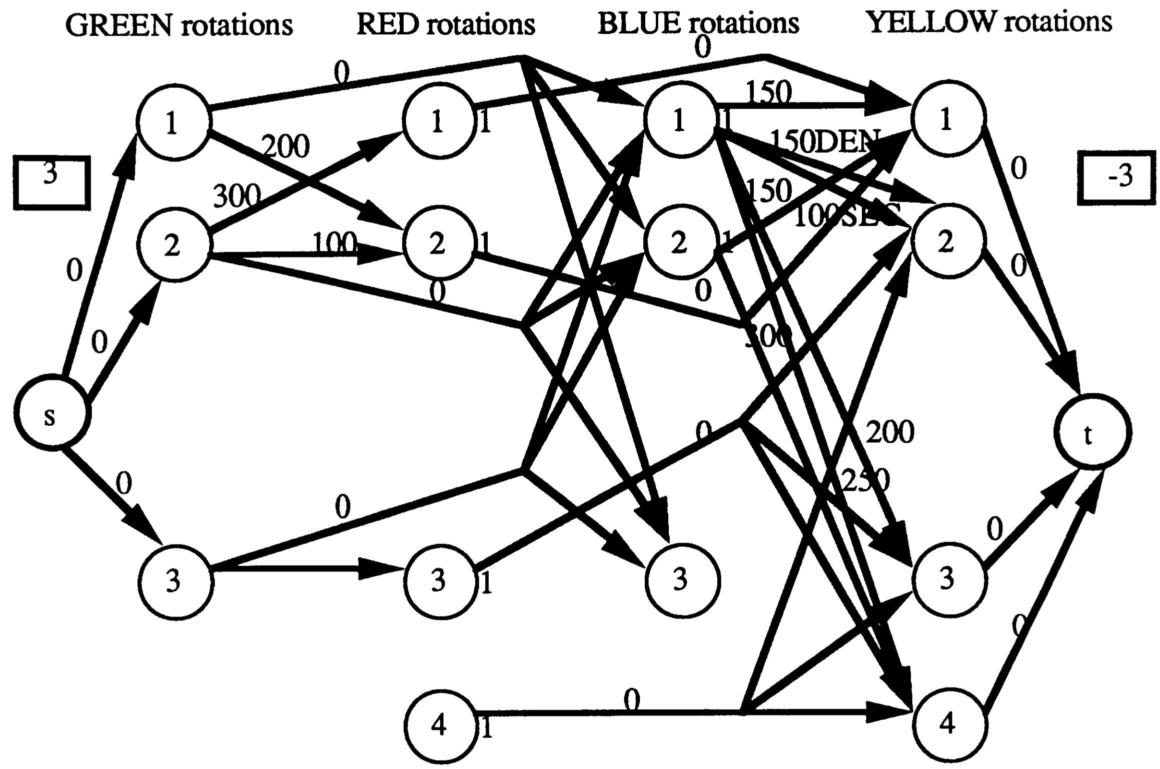

Figure 16 Special Min-cost Flow Problem for Example from Figure 14

11. Exploring the Clusters for Pre-switch/Post-switch Possibilities

Prerequisite: Finding the Minimum Cost Set of Pre-switches/Post-switches.

In this section, one of the necessary and sufficient conditions for a pre-switch/post-switch will be relaxed for a cluster with many aircraft available on ground. An intersection of two rotations was a necessary condition for a pre-switch/post-switch up to now. But look at the following figure.

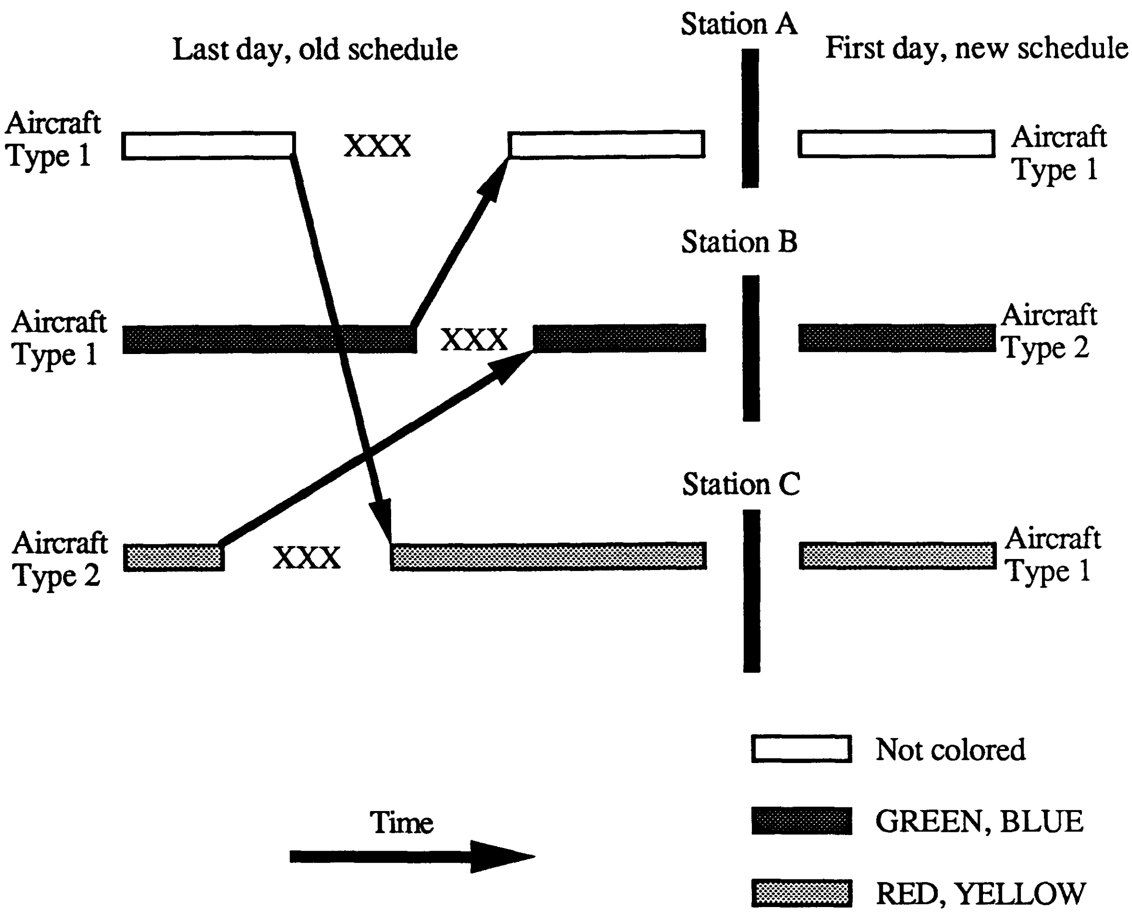

Figure 17 Relaxation to Intersection of Clusters

We see that there is no intersection between the second aircraft type 1 rotation (terminating in station B) and the aircraft type 2 rotation (terminating in station C) on the last day of the old schedule. So conventionally we cannot pre-switch the second aircraft type 1 rotation with the aircraft type 2 rotation at XXX. However, if we look more carefully, it is easy to see that a "three-way pre-switch" is feasible that is depicted in figure 17. This is due to a cluster of aircraft type 1 in XXX. Whenever there is a cluster of an aircraft type, it is possible to perform a pre-switch/post-switch of any rotation within the same cluster as long as the rotation that we would like to pre-switch/post-switch "intersects" with the cluster of the other aircraft type we are examining. Therefore, if there are clusters for both aircraft type 1 and aircraft type 2, we may perform a pre-switch/post-switch as long as the two clusters intersect. To illustrate this, look at the station activity chart below.

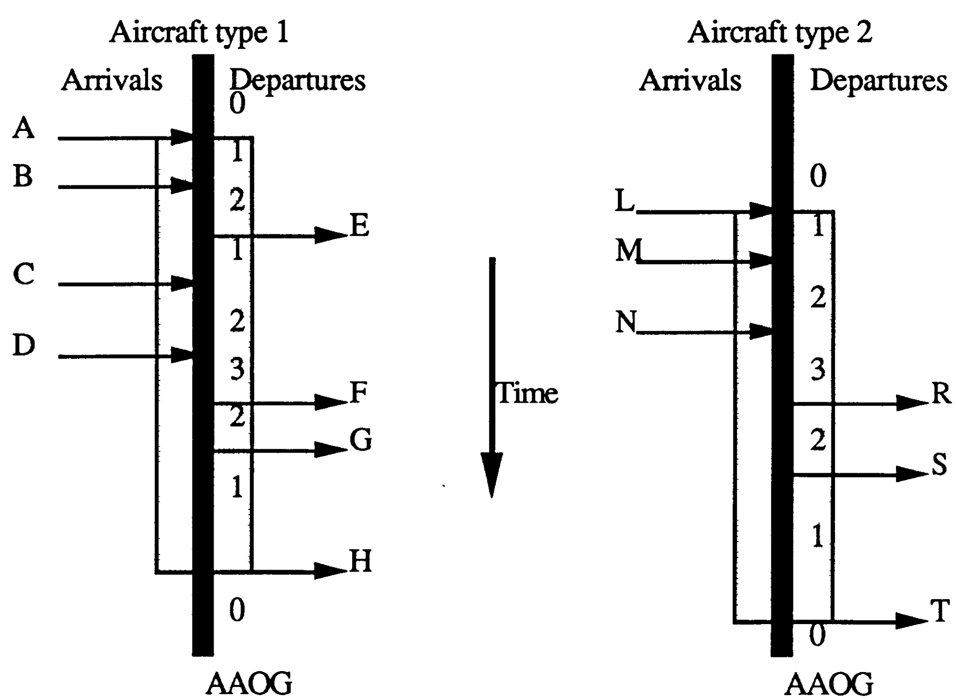

Figure 18 Intersection of Clusters

It is easy to see that there exists an intersection of the two clusters (shaded regions) between the aircraft type 1 arrival A and aircraft type 2 departure T. We claim that any combinations of pre-switches/post-switches are possible during this intersection of clusters. For instance, any of the departures E, F, G, or H by aircraft type 1 may be interchanged with any of the departures R, S, or T by aircraft type 2 as the reader can easily verify by looking at possible effects of each of the changes[1]. Therefore, we include this entire set in the intersection of clusters. For instance, if departure E should be flown by aircraft type 2, then arrival L can turn into departure E, and one of departure R, S or T can be flown by aircraft type 1. If departure S should be flown by aircraft type 1, any of departures E, F, G, or H can be interchanged with it to be flown by aircraft type 2. The reason for including the arrivals and departures outside of the "intersection" is simple. If the arrival to be interchanged occurs before the intersection of clusters begins (arrivals A and B in figure 18), then it can be interchanged with any arrival during the intersection of clusters (arrivals L, M or N in figure 18), possibly breaking up a cluster; similarly, if the departure to be interchanged is scheduled to depart after the intersection of clusters (departure T in figure 18), then it can be interchanged with any departure during the intersection of clusters (departures E, F, G or H in figure 18), again possibly breaking up a cluster. If the clusters are broken, it is a good sign because we have created a shorter ground time and a longer ground time for flights, which may enable us to increase the utilization of aircraft by possible additional flight segments.

[1]Simpson, Robert W., "Canceling and Switching Flights within Clusters of Aircraft in a Schedule", FTL Memorandum 88-8, Flight Transportation Laboratory, MIT, Cambridge, MA, November 1988.

A generalization of this concept occurs when a pre-switch/post-switch is performed between rotation pairs that consist of one colored (GREEN, RED, BLUE, or YELLOW) and one non-colored rotations. Up to now, we did not consider these rotation pairs as possible rotation pairs to be pre-switched/post-switched. However, performing such a pre-switch/post-switch may lead to producing an overall lowest cost set of pre-switches/post-switches.

In this type of a pre-switch/post-switch (which we will call an intermediate pre-switch/post-switch), the schedule will not be improved by elimination of a pair of imbalances; instead, the pre-switch/post-switch possibilities afterwards will be changed without creating additional new imbalances. This can easily be demonstrated as shown below.

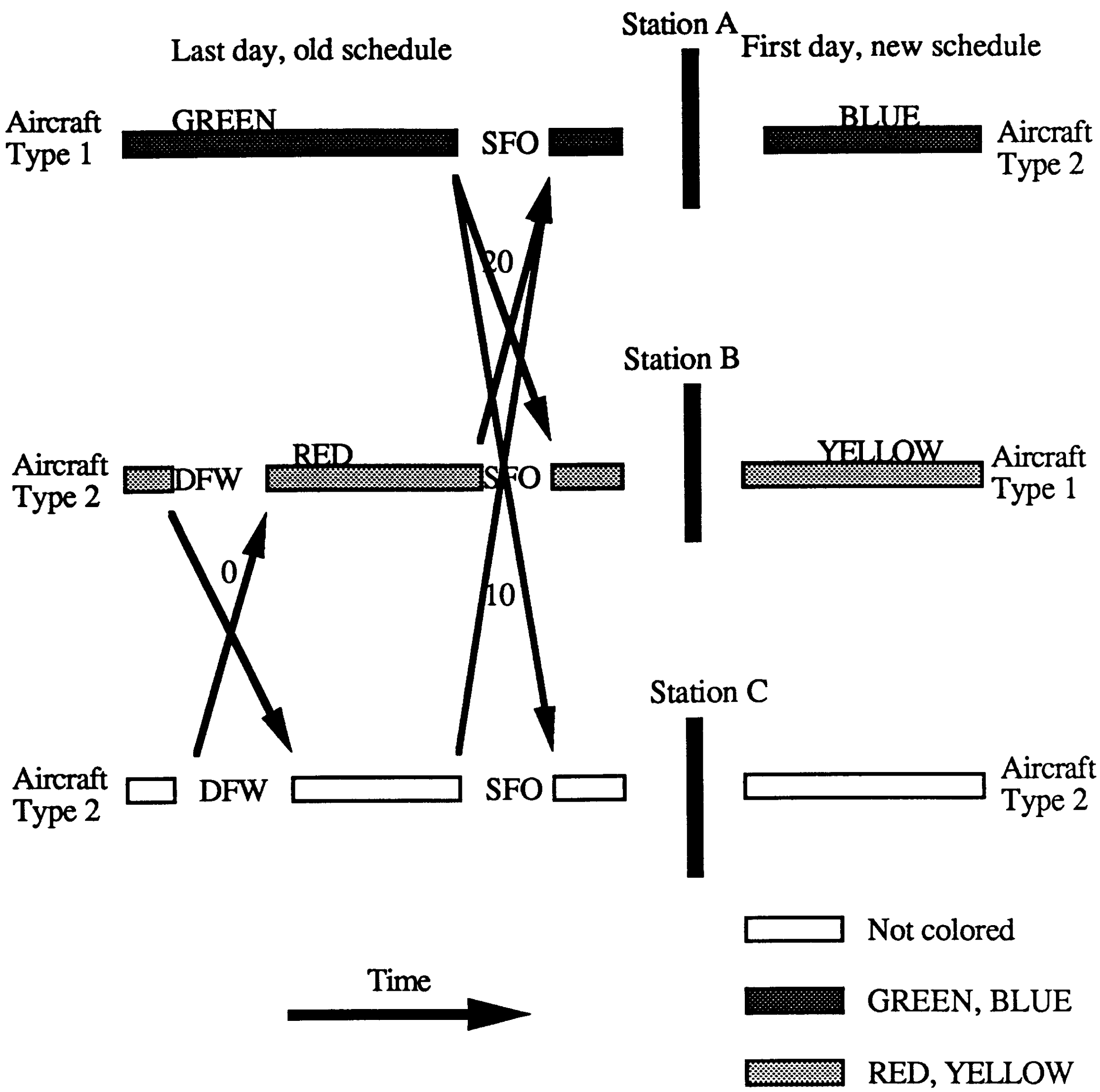

Figure 19 Intermediate Pre-switch

By pre-switching the GREEN rotation with the rotation that terminates at station C (rotation not colored), you have just removed the surplus of aircraft type 1 at station A; however, you also have created a new surplus of aircraft type 1 at station C. In general, the station at which the non-colored rotation involved in the pre-switch (post-switch) terminated (originated) may get an additional surplus (deficit) of the aircraft type of the colored rotation because the colored rotation upsets the balance which existed at that station before the pre-switch (post-switch). But when this occurs, the station that originally had the colored rotations will have one surplus (deficit) removed. Now, a different set of colored rotations arise, but the total number of pairs of imbalances in the schedule remains the same because we removed one imbalance from one station and created one imbalance at another.

Note that the pre-switch depicted in figure 19 would have been productive if the pre-switch denoted by 10 would have been less expensive than the pre-switch denoted by 20; since we merely pre-switch two rotations flown by the same aircraft type, the cost of the pre-switch between the RED and the non-colored rotation may be assumed to be zero here.

Also, note that any colored rotation may be pre-switched/post-switched with any non-colored rotation in this manner regardless of the aircraft types without increasing or decreasing the number of imbalances in the schedule transition period as long as the rotations intersect. So an endless possible number of them are available. The problem of extracting the best (or the helpful) ones is left as an open problem here.

12. Extensions

Prerequisite: **Finding the Minimum Cost Set of Pre-switches/Post-switches.**

12.1 Multiple Aircraft Types

Up to now, we could only use the Pre-switch/Post-switch Algorithm for an interchange of two aircraft types. Most major airlines have more than two aircraft types in their fleet. One important extension will be to develop a Pre-switch/Post-switch Algorithm for multiple aircraft types. We will discuss four possible approaches to accomplish this. Only the first one will use the Pre-switch/Post-switch Algorithm, as in the two aircraft case, to solve the multiple aircraft type schedule transition problem.

12.1.1 Priority List Approach

A simple approach would be to have a priority list of the aircraft type pairs. Seldom do we want to pre-switch or post-switch a big aircraft such as 747 with an aircraft too small such as MD80. Therefore, we order the aircraft types in decreasing order of passenger seats. From the biggest aircraft down, we will perform Pre-switch/Post-switch Algorithm for two aircraft types in pretty much the same way starting with adjacent pairs of aircraft types. For instance, if the fleet consists of (in decreasing order of passenger seats) 747-MD11-757-MD80, we will run Pre-switch/Post-switch Algorithm with 747-MD11 first, then MD11-757, then 757-MD11. Next, we will systematically compare the closest (in terms of passenger capacity) pair of aircraft types at each turn. In this case, 747-757, then MD11-MD80, then finally, 747-M80 will therefore follow. Note that physical balance in this case means physical balance across all aircraft types, not just the pair which you are comparing. The by-product of this sequence of selecting the aircraft type pair to compare is that a flight will seldom be flown by an aircraft type that has a big difference in passenger capacity to the originally scheduled aircraft type because we always compared the two closest aircraft types in terms of passenger capacity first.

One problem which arises with this simple approach is that we find physical balance across all aircraft types but we have surplus (or deficit) of both aircraft types that we are currently comparing. In this case, just skip that pair. There will be no intersections if all stations that have surplus aircraft type A also have surplus aircraft type B, and if all stations that have deficit aircraft type A also have deficit aircraft type B, since all GREEN and RED rotations will have no rotations to turn to the next morning due to absence of deficit aircraft types A and B at those stations, and since all BLUE and YELLOW rotations will have no terminating surplus aircraft types A and B at their originating stations the previous day. If there was physical balance, however, eventually all the pairs of imbalances will disappear after the final pair of aircraft types are compared if we are lucky to have intersections for the surplus/deficit imbalance pairs. This is because physical balance at all stations implies that if there is a surplus of some aircraft type, there must be a deficit of some other aircraft type at that station.

This scheme, in the worst case, will result in performing pairwise Pre-switch/Post-switch Algorithm (n choose 2) times where n is the number of different aircraft types. An example with three aircraft types is illustrated in figure 20. Here we would have compared aircraft type 1 with aircraft type 2 first, and then with aircraft type 3 to resolve the imbalances using the simple extension of the original Pre-switch/Post-switch Algorithm described above. We do not have to perform Pre-switch/Post-switch Algorithm between aircraft types 2 and 3 due to the lack of surplus/deficit imbalance pair. Also note that we do not have to compare the same pair of aircraft types more than once because there will be no surplus/deficit imbalance pairs left after the first comparison between that pair of aircraft types and we do not create any new ones by comparing other aircraft type pairs later. However, this scheme does not guarantee a global optimal solution to the problem. In the example given by figure 20, if the cost of pre-switches are as shown and if there are no post-switch possibilities and no costs for turns, then when we first compare aircraft types 1 and 2, we will pre-switch the third aircraft type 1 rotation with the aircraft type 2 rotation and then we will pre-switch the second aircraft type 1 rotation with the aircraft type 3 rotation when we next compare aircraft types 1 and 3, for a total cost of 30 + 40 = 70. We can see that if we had started by comparing aircraft types 1 and 3 first, then we can achieve total balance with a cost of 10 + 50 = 60. So depending upon the order of comparison of aircraft types, the outcome changes. Since we cannot specify the correct order of comparison to get the min-cost set of pre-switches/post-switches, we cannot guarantee the solution that gives the minimum cost set for all aircraft types.

Appendix B illustrates a sample set of reports generated as the approach described above was used to resolve imbalances for a flight schedule with multiple aircraft types. Note that both the flow balance report as well as the pre-switch/post-switch Report must be updated after each pre-switch/post-switch.

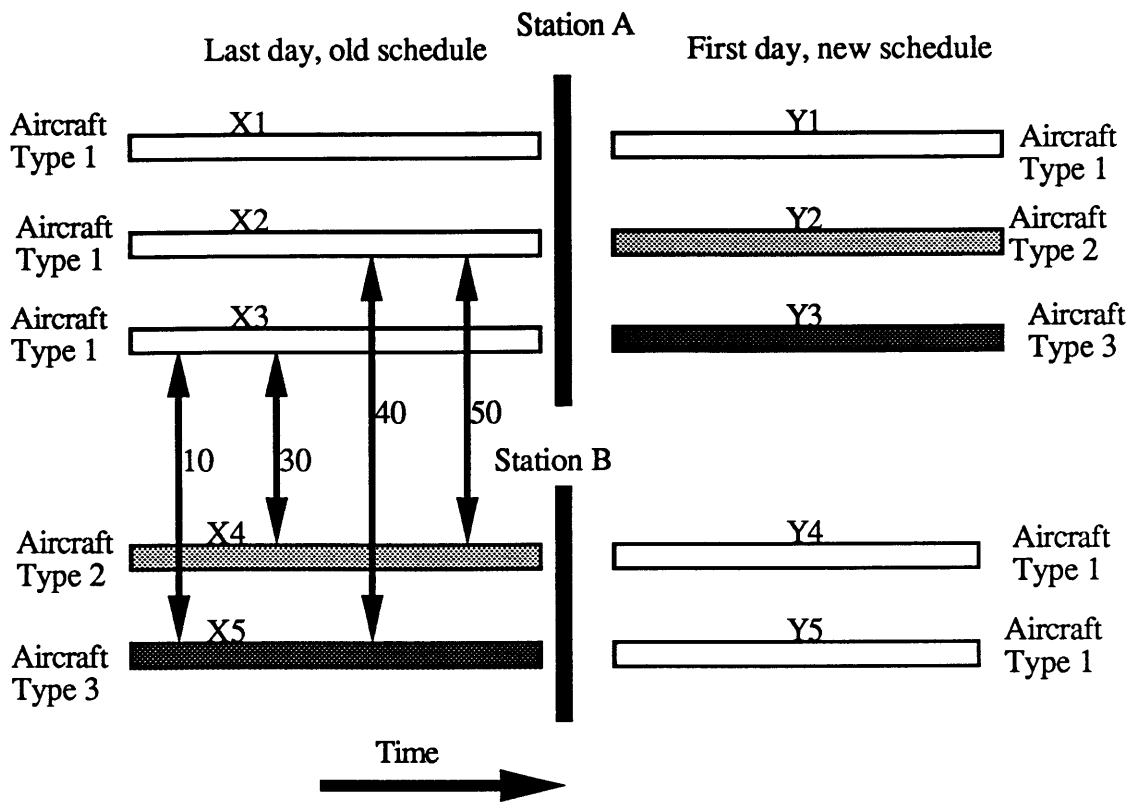

Figure 20 Schedule Transition Problem with Three Aircraft Types

12.1.2 Greedy Approach

We will now introduce a "greedy" approach that will require at most n-1 different applications of a slightly modified Pre-switch/Post-switch Algorithm for two aircraft types in order to remove imbalances from schedules with n different aircraft types. It is called "greedy" because during each application of this slightly modified Pre-switch/Post-switch Algorithm, the lowest cost pre-switches/post-switches that are available at that point are taken. This slightly modified algorithm will successively eliminate imbalances from a particular aircraft type chosen. So after n-1 iterations, n-1 aircraft types (or all n aircraft types) would be totally balanced.

Remember that we needed a surplus/deficit imbalance pair in order to perform a pre-switch or a post-switch in the two aircraft type case. In general, if we concentrate on one of the aircraft types, we need a rotation terminating at its surplus station and another rotation of the same aircraft type originating the next morning at its deficit station. This condition is called the surplus/deficit instance. The example in figure 21 illustrates the surplus/deficit instance and the coloring of the rotations. Note that we are currently balancing aircraft type 2 and this is the first iteration. All aircraft type 2 rotations terminating at its surplus station(s) are colored GREEN; all aircraft type 2 rotations originating at its deficit station(s) are colored YELLOW. Rotations of other aircraft types terminating at stations where YELLOW rotations originate and rotations of other aircraft types originating at stations where GREEN rotations terminate are colored RED and BLUE, respectively. Now we can perform pre-switches and post-switches between GREEN and RED and between BLUE and YELLOW rotations exactly the same way as we have done for two aircraft type case. We eliminate all aircraft type 2 rotations that give imbalances in this way.

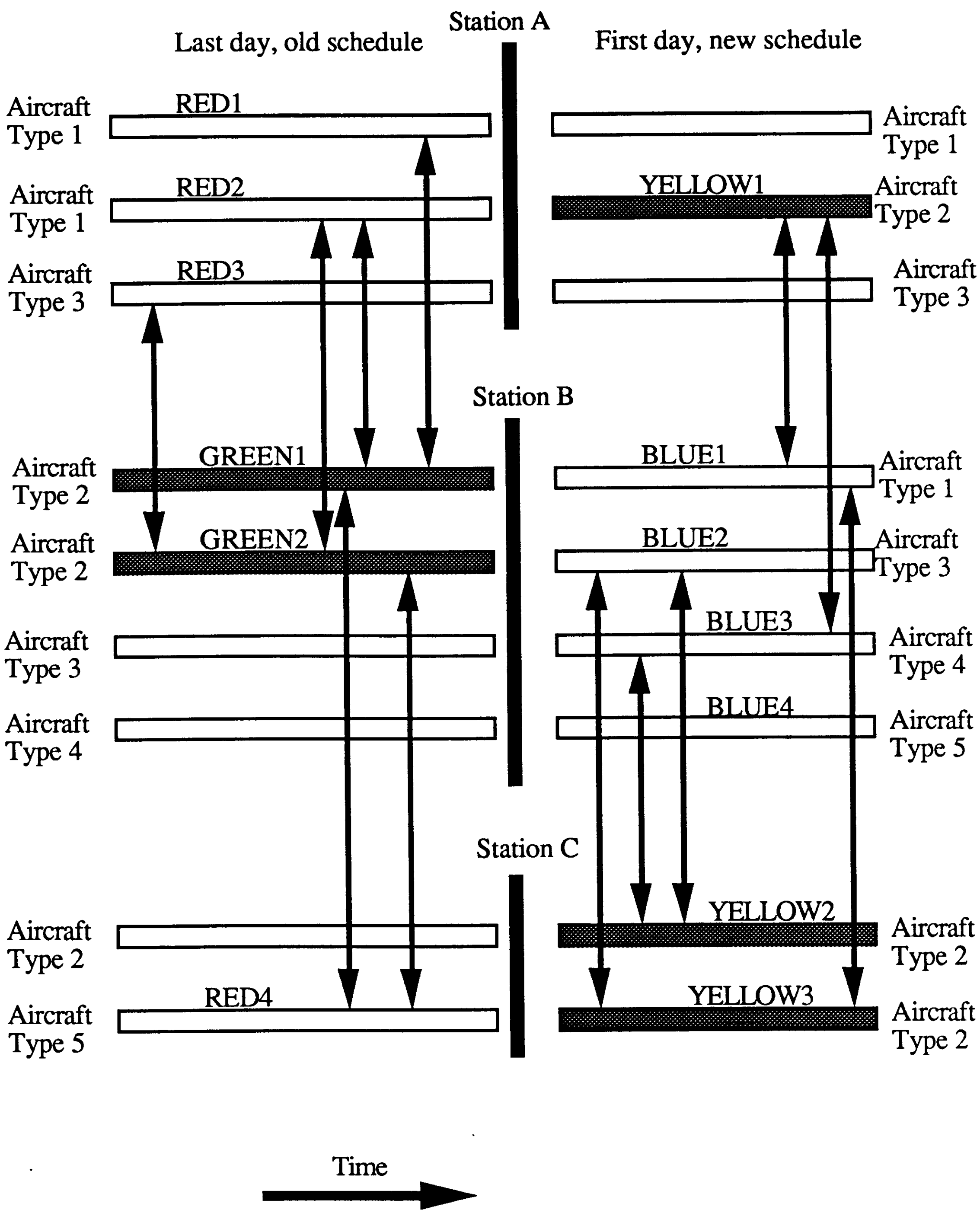

Figure 21 Example with Multiple Aircraft Types

Let's go back again to the example in figure 21. Aircraft type 2 rotations terminating at its surplus station B are colored GREEN following the old coloring scheme and aircraft type 2 rotations originating at its deficit stations A and C are colored YELLOW. All other aircraft type rotations are treated the same way; there is no differentiation between the other aircraft types. So rotations of other aircraft types terminating at stations with YELLOW rotations are colored RED, and rotations of other aircraft types originating at stations with GREEN rotations are colored BLUE. Also, since this is the first iteration, even though aircraft type 4 rotations are balanced here, we incorporate them into pre-switch/post-switch possibilities, which may result in an imbalance of aircraft type 4 later. In this example, two imbalances exist for aircraft type 2, and two pre-switches/post-switches can balance this type. The min-cost flow representation of this example is given in figure 22. The construction of this graph is exactly the same as for the two aircraft type case. Remember that this is only the first iteration; we must perform a total of n - 1 = 5 - 1 = 4 iterations before the schedule transition period can be totally balanced. In this graph, all pre-switches and post-switches possible between aircraft type 2 rotations and other aircraft type rotations have been depicted as before. The super-sink and super-source show a demand and a supply of the number of pairs of imbalances that need to be removed.

Notice also that due to the lack of knowledge of the correct order to choose the next aircraft type, we cannot guarantee the minimum cost solution. So we will start with the largest capacity aircraft type and work downwards as we have done with the previous approach.

12.1.3 "All-at-once" Approach

The third approach, the "all-at-once" approach, described below, will try to solve for the global optimal solution of the schedule transition problem by using only one iteration of a specialized min-cost flow problem.

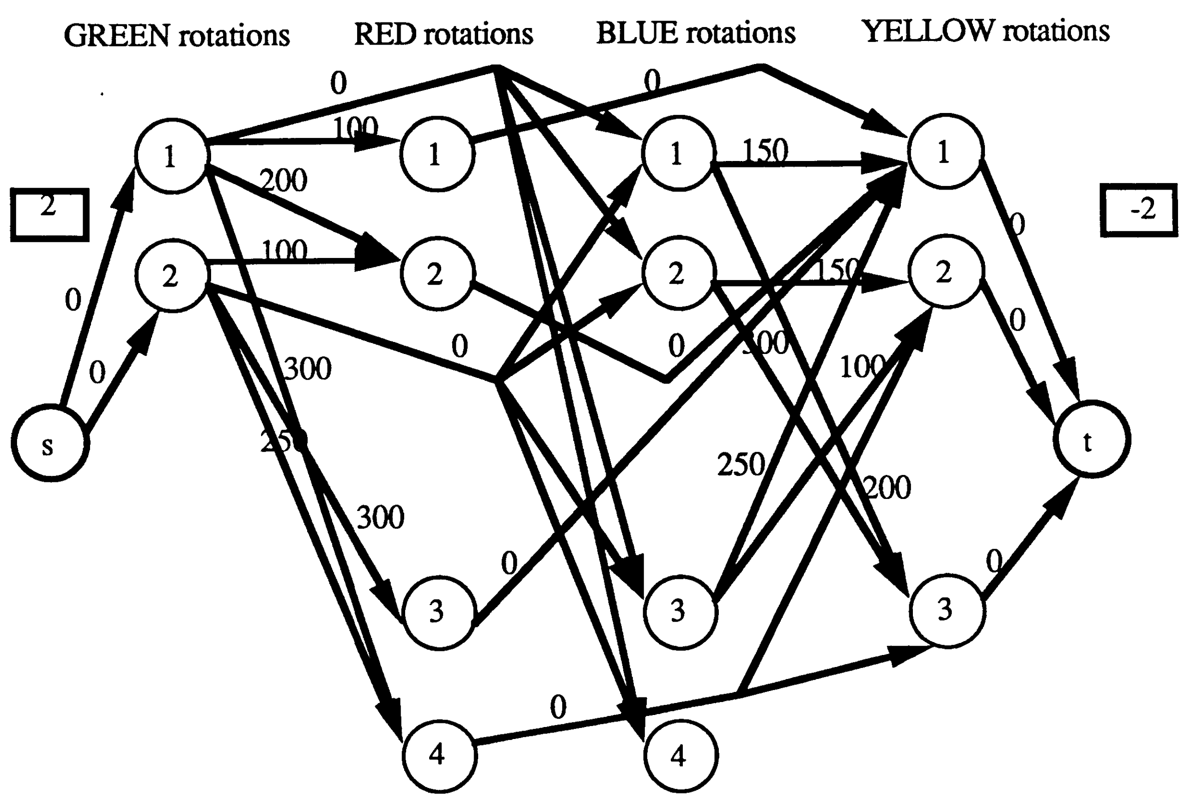

Figure 22 Greedy Min-cost Flow Problem for Example from Figure 21

This approach incorporates the schedule transition problem for multiple aircraft types in one graph to avoid the problem of selecting aircraft types to be compared in the next iteration. Note that the graph should contain two types of edges - the edges designating pre-switches/post-switches and the edges representing turns. Suppose we start by drawing all vertices representing the

GREEN and the RED rotations which have more terminating flights of that aircraft type than originating flights the next morning and another denoting the BLUE and the YELLOW rotations which have more originating flights of that aircraft type than terminating flights the previous evening.

Once we find all surplus/deficit imbalance pairs, we can easily draw in the edges onto the set of vertices in exactly the same way as we did for the two aircraft type case. This will make us consider only the pre-switches/post-switches that resolve a pair of imbalances at a time when performed. So, the number of pre-switches/post-switches will be minimized. But we can only solve a very limited number of problems using this approach because we must find a surplus/deficit imbalance pair instead of a surplus/deficit instance.

If we now construct a super-source and a super-sink vertex and draw edges between the super-source and the first vertex column and between the second vertex column and the super-sink vertex, then we have a special min-cost flow problem, as shown in figure 23 for the example from figure 21. Remember that all edges have capacity equal to one. Notice, however, that if we perform a pre-switch/post-switch on this graph, flows are needed in both directions which came in to the vertices at both ends through separate paths. Therefore, one path from the super-source to the super-sink denotes a resolution of one imbalance, not a pair of imbalances. We cannot pre-switch and/or post-switch the same rotation more than once, and we cannot turn one rotation to another unless the aircraft types match. Cancellation problems arise if we try to perform more than one pre-switch/post-switch involving the same rotation, as discussed before.

We can take care of this complication by using a flag, as before. First, we set the flag equal to zero. If we push flows (must be in both directions) on a pre-switch edge, we set it to one at both ends of the edge. If the flag is one, we cannot push any more flows on any of the pre-switch edges. We may push on the turn edges regardless of the value of the flag as long as the aircraft types match. By the time we come to the second vertex column, only if the flag is still zero, we want to push flows (in both directions) on a post-switch edge, at which time the flag is set to one. If the flag of the current path is one, we can push a flow towards the super-sink from the second vertex column. This will make sure a path from the super-source to the super-sink will give a resolution of one imbalance.

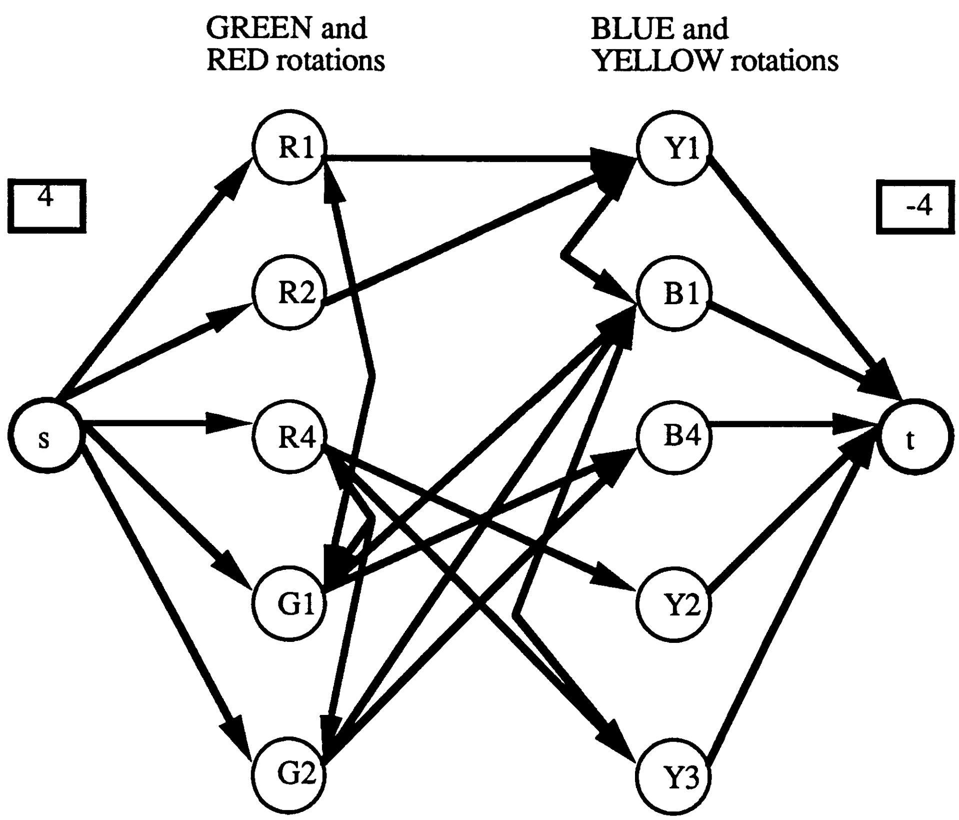

Figure 23 All-at-once Min-cost Flow Problem for Example from Figure 21

subtours", which are closed tours consisting of a proper subset of vertices. If these subtours are formed, we do not have one tour visiting all vertices but many tours which, when combined, visits all vertices. If we can tell whether or not we are completing a subtour beforehand, then we can avoid these subtours. In order to accomplish this, we must be sure that the edge that completes a subtour never gets added to the solution of the Traveling Salesman Problem. Thus, we must know the vertices that have already been visited and also the edges that have already been used in the current tour. This information is exactly what we need to solve this specialized min-cost flow problem.

If we have an efficient way to keep track of this information, we will be able to solve the multiple aircraft type case of the schedule transition problem from this specialized min-cost flow problem. Being able to solve this special min-cost flow problem means finding an efficient solution for the Traveling Salesman Problem, of which none exists.

12.1.4 "Set-covering" Approach

The fourth approach looks more carefully at the fact that you cannot use the same rotation for more than one distinct pre-switch/post-switch. We will call this the "set-covering" approach, because it is similar to a set-covering problem[1]. We observe that there are three phases in a schedule transition period - pre-switch phase, turn phase, and the post-switch phase. We list all the rotation numbers that are colored for each phase in a column; and we can describe a path

[1]Larson, Richard C., and Odoni, Amedeo R., Urban Operations Research, Prentice Hall, Inc., Englewood Cliffs, NJ, 1981, pp. 446-449.

from the super-source to the super-sink of the previous approach in a row as follows. There will be a 1 in the row if the path uses that rotation and a 0 if the rotation is not used for the particular phase. For instance, a pre-switch between rotations G1 and R2 which turn into rotation W1 will have a 1 under those columns denoting the rotations for the appropriate phase. Once we list all possible paths from the super-source to the super-sink in this manner, we pick out as many "independent rotation pairs" as the number of pre-switches/post-switches necessary to remove the imbalances that result in the minimum cost overall. Two rotation pairs are "independent" if all four rotations are different. This translates into not having more than one "1" in a column for the listing we have constructed. Note that if two rotation pairs that denote pre-switches/post-switches are independent, both of the pre-switches/post-switches can be performed; otherwise, we cannot use both of the pre-switches/post-switches due to possible "cancellations" discussed earlier. The problem of finding the "best" set of pre-switches/post-switches is now reduced to solving a set-covering problem on the list: find as many independent rows as the necessary number of pre-switches/post-switches at the lowest possible cost. A sample listing for the example in figure 20 is given below in figure 24. Note that this is not a very attractive approach as more rotations are introduced due to the combinatorial nature of the growth of the listing.

	Pre-switches					Turns										Post-switches					Cost
Rotation	X1	X2	X3	X4	X5	X1	X2	X3	X4	X5	Y1	Y2	Y3	Y4	Y5	Y1	Y2	Y3	Y4	Y5	
	0	0	1	0	1	0	0	1	0	1	0	0	1	0	1	0	0	0	0	0	10
	0	0	1	0	1	0	0	1	0	1	0	0	1	1	0	0	0	0	0	0	10
	0	0	1	1	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	30
	0	0	1	1	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	30
	0	1	0	0	1	0	1	0	0	1	0	0	1	0	1	0	0	0	0	0	40
	0	1	0	0	1	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	40
	0	1	0	1	0	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0	50
	0	1	0	1	0	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	50

Figure 24 Set-covering Problem for Example from Figure 20

For the sample listing in figure 24, we see, after looking at all possibilities, that the first and the last rows are independent and gives two pre-switches for a total cost of 10 + 50 = 60. Note that for each column no more than one "1" is present, meeting the requirement of independence. Also, since the necessary number of pre-switches/post-switches for total balance is two for this example, two independent rows that give the lowest total cost give the best set of pre-switch/post-switch possibilities. So we perform pre-switches between rotations X3 and X5 and between X2 and X4, and turn X3 to Y3, X5 to Y5, X2 to Y2, and X4 to Y4. Similarly, the second from the top and the second from the bottom rows give a solution that has the same total cost. The only difference here is the turns; rotation X4 turns to Y5 and X5 turns to Y4.

In order to obtain the minimum total cost solution, however, you must exhaustively search all possibilities. No efficient algorithm is available today. As the listing grows, both horizontally and vertically, the problem becomes extremely difficult to solve. Notice that this approach does give the optimal solution for the multiple aircraft type schedule transition problem. Remember that the priority list approach given earlier resulted in a solution that had a total cost of 70, which is higher than the 60 we obtained from the "set-covering" approach. In fact, the priority list approach only looked at a proper subset of the rows given in the listing; each iteration of the approach reduced the list to include only those rows that are independent of the previous iterations' results. A more practical way to solve the problem with multiple aircraft type case is left open for further research.

12.2 Balancing a Daily Flight Schedule

In our prior discussions, we utilized the Pre-switch/Post-switch Algorithm only for schedule transition periods. The same algorithm can also be used to balance the daily flight schedule effective usually for a duration of a month. Assume that the flight schedule is physically balanced but not totally balanced. Then for every station with a surplus of aircraft type A on a certain day, there is also a deficit of aircraft type B, and there is another station with a deficit of aircraft type A and surplus of aircraft type B, as we have shown earlier. Since this is a surplus/deficit imbalance pair, which will exist for the entire effective period of the flight schedule, by using the Pre-switch/Post-switch Algorithm for the entire effective period of the flight schedule, we can identify rotations that can be pre-switched/post-switched in order to achieve total balance of the daily flight schedule. The only complication here is that a pre-switch may "cancel" a post-switch from the previous day. This is possible, for instance, if a pre-switch identified for day r and a post-switch identified for day r - 1 denote the same exchange and are both performed. In a daily flight schedule, a rotation may have two distinct colors. For instance, if some rotation originates from a deficit station for that aircraft type and terminates at a surplus station for that aircraft type, then this rotation will be colored with two colors, according to our colorization scheme. In order not to cancel any of the pre-switches/post-switches, measures should be taken to check that a pre-switch and a post-switch involving the same rotations will not both be performed. The simplest way to handle this is to eliminate the vertex corresponding to the duplicate rotation from the min-cost flow problem graph from all except one of the vertex columns. Then, each rotation will only appear at most once. Remember that for totally balancing the schedule, pre-switches/post-switches must be performed for the entire effective period of the schedule instead of just on one of the schedule transition period days.

Even when each daily flight schedule is different, when each schedule transition period is physically balanced, then the Pre-switch/Post-switch Algorithm can be utilized to totally balance the entire period, one schedule transition period at a time. Therefore, holiday schedules may also be totally balanced using this algorithm. In fact, pre-switches/post-switches on the sample outputs given in Appendices A and B were done for a period of four days.

13. Further Research

Prerequisite: Extensions.

This paper is intended to introduce the schedule transition problem for airlines and the first approach at automatically solving the problem. There remains many open problems. A possible application of the schedule transition problem may exist in the area of changing job schedule for workers with different types of qualities with minimum interruption. Another application may exist in the area of scheduling transportation vehicles when certain routes become unusable for certain time period. The general idea of the Pre-switch/Post-switch Algorithm should aid in finding the best deviation during some transition period when the preceding and the succeeding schedules are complete. These examples are by no means complete. But we believe that Pre-switch/Post-switch Algorithm is a useful way of solving the schedule transition problem for major airlines, who today do this task without the aid of the computer.

Appendix A

Time starting current report : 2/19/1989 at 14:57:34

{This sample report is the output from ISS Check_flow_balance report option.}

MISSING FLIGHT SEGMENTS IN ROTATION

FLOW BALANCE FOR : 881001 TO 881004

L10

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
ATL	2	2	2	2	2	2	6	6
BDL	1	1	1	1	1	1	3	3
BOS	2	2	2	2	2	2	6	6
DFW	1	1	1	1	1	1	3	3
EWR	2	2	2	2	2	2	6	6
FLL	1	3	1	3	1	3	3	9
JFK	1	1	1	1	1	1	3	3
LAX	2	2	2	2	2	2	6	6
LGA	1	1	1	1	1	1	3	3
MCO	2	1	2	1	2	1	6	3
ORD	2	2	2	2	2	2	6	6
PBI	1	1	1	1	1	1	3	3
SEA	1	1	1	1	1	1	3	3
SFO	1	0	1	0	1	0	3	0

767

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
FLL	2	0	2	0	2	0	6	0
MCO	0	1	0	1	0	1	0	3
SFO	0	1	0	1	0	1	0	3
TPA	1	1	1	1	1	1	3	3

757

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O

PHYSICAL IMBALANCE CHECK

STATION	TOT TERM	TOT ORIG

No entries below this line.

Time starting current report : 1/ 9/1989 at 14:21:35

{This is an extract from a report generated from the ISS pre-switch/post-switch report option.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : 767 a/c 2 : L10

FLL has surplus a/c 1 and deficit a/c 2.
Rotation 2 will be colored GREEN. Stations visited :
FLL visited after 2740
LGA visited between 2355 and 2455
ATL visited between 2042 and 2155
MCO visited between 1820 and 1925
ATL visited between 1637 and 1707
SFO visited before 1231
Rotation 7 will be colored GREEN. Stations visited :
FLL visited after 2826
BOS visited between 2420 and 2520
ATL visited between 2103 and 2205
BDA visited between 1555 and 1755
BOS visited between 1325 and 1355
MCO visited before 1056
Rotation 12 will be colored BLUE. Stations visited :
FLL visited before 934
ATL visited between 1109 and 1139
MIA visited between 1315 and 1415
ATL visited between 1558 and 1650
PBI visited between 1820 and 1920
ATL visited between 2100 and 2155
MCO visited between 2312 and 2412
EWR visited after 2625
Rotation 13 will be colored BLUE. Stations visited :
FLL visited before 946
MCO visited between 1031 and 1101
DFW visited between 1330 and 1528
SFO visited between 1855 and 2220
DFW visited between 2518 and 2619
MCO visited after 2830
Rotation 14 will be colored BLUE. Stations visited :
FLL visited before 1142
ATL visited between 1320 and 1350
MCO visited between 1503 and 1605
ATL visited between 1722 and 1752
BDA visited between 2012 and 2050
BOS visited between 2257 and 2344
BDL visited after 2425

MCO has surplus a/c 2 and deficit a/c 1.
Rotation 4 will be colored RED. Stations visited :
MCO visited after 2950
ATL visited between 2718 and 2835
EWR visited between 2405 and 2510

ATL visited between 2141 and 2213
MCO visited between 1954 and 2024
BOS visited between 1635 and 1705
PBI visited before 1400
Rotation 7 will be colored YELLOW. Stations visited :
MCO visited before 1056
BOS visited between 1325 and 1355
BDA visited between 1555 and 1755
ATL visited between 2103 and 2205
BOS visited between 2420 and 2520
FLL visited after 2826
Rotation 13 will be colored RED. Stations visited :
MCO visited after 2830
DFW visited between 2518 and 2619
SFO visited between 1855 and 2220
DFW visited between 1330 and 1528
MCO visited between 1031 and 1101
FLL visited before 946

SFO has surplus a/c 2 and deficit a/c 1.
Rotation 2 will be colored YELLOW. Stations visited :
SFO visited before 1231
ATL visited between 1637 and 1707
MCO visited between 1820 and 1925
ATL visited between 2042 and 2155
LGA visited between 2355 and 2455
FLL visited after 2740
Rotation 23 will be colored RED. Stations visited :
SFO visited after 2833
DFW visited between 2357 and 2459
FLL visited between 2005 and 2110
ATL visited before 1826

Pre-switch rotation 2 between 2042 and 2155
with rotation 4 between 2141 and 2213 at station ATL.

Pre-switch rotation 2 between 1637 and 1707
with rotation 23 between 0 and 1826 at station ATL.

Pre-switch rotation 7 between 2103 and 2205
with rotation 4 between 2141 and 2213 at station ATL.

Pre-switch rotation 7 between 0 and 1056
with rotation 13 between 1031 and 1101 at station MCO.

Post-switch rotation 12 between 2100 and 2155
with rotation 7 between 2103 and 2205 at station ATL.

Post-switch rotation 12 between 1558 and 1650
with rotation 2 between 1637 and 1707 at station ATL.

Post-switch rotation 12 between 2100 and 2155 with rotation 2 between 2042 and 2155 at station ATL.

Post-switch rotation 13 between 1031 and 1101
with rotation 7 between 0 and 1056 at station MCO.

No entries below this line.

Time starting current report : 1/ 9/1989 at 14:24:14

{This report was generated after rotations 2 and 4 have been pre-switched at ATL.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : 767 a/c 2 : L10

FLL has surplus a/c 1 and deficit a/c 2.
Rotation 7 will be colored GREEN. Stations visited :
FLL visited after 2826
BOS visited between 2420 and 2520
ATL visited between 2103 and 2205
BDA visited between 1555 and 1755
BOS visited between 1325 and 1355
MCO visited before 1056
Rotation 12 will be colored BLUE. Stations visited :
FLL visited before 934
ATL visited between 1109 and 1139
MIA visited between 1315 and 1415
ATL visited between 1558 and 1650
PBI visited between 1820 and 1920
ATL visited between 2100 and 2155
MCO visited between 2312 and 2412
EWR visited after 2625
Rotation 13 will be colored BLUE. Stations visited :
FLL visited before 946
MCO visited between 1031 and 1101
DFW visited between 1330 and 1528
SFO visited between 1855 and 2220
DFW visited between 2518 and 2619
MCO visited after 2830
Rotation 14 will be colored BLUE. Stations visited :
FLL visited before 1142
ATL visited between 1320 and 1350
MCO visited between 1503 and 1605
ATL visited between 1722 and 1752
BDA visited between 2012 and 2050
BOS visited between 2257 and 2344
BDL visited after 2425

SFO has surplus a/c 2 and deficit a/c 1.
Rotation 2 will be colored YELLOW. Stations visited :
SFO visited before 1231
ATL visited between 1637 and 1707
MCO visited between 1820 and 1925
ATL visited between 2042 and 2213
EWR visited between 2405 and 2510
ATL visited between 2718 and 2835
MCO visited after 2950
Rotation 23 will be colored RED. Stations visited :
SFO visited after 2833
DFW visited between 2357 and 2459

FLL visited between 2005 and 2110
ATL visited before 1826

Post-switch rotation 12 between 1558 and 1650
with rotation 2 between 1637 and 1707 at station ATL.

Post-switch rotation 12 between 2100 and 2155
with rotation 2 between 2042 and 2213 at station ATL.

Post-switch rotation 13 between 2830 and 4800
with rotation 2 between 2950 and 4800 at station MCO.

No entries below this line.

Time starting current report : 1/ 9/1989 at 14:35:29

{This report has been generated after post-switching rotations 2 and 13 at MCO.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : 767 a/c 2 : L10

No entries below this line.

Appendix B

Time starting current report : 3/21/1989 at 17: 9:22

{This sample report is an extract from ISS Check_flow_balance report option.}

MISSING FLIGHT SEGMENTS IN ROTATION

FLOW BALANCE FOR : 881001 TO 881004

72R

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
FAR	1	0	1	0	1	0	3	0
YYZ	0	1	0	1	0	1	0	3

M80

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
DCA	1	2	1	2	1	2	3	6
FAR	0	1	0	1	0	1	0	3
TYS	1	0	1	0	1	0	3	0
YYZ	1	0	1	0	1	0	3	0

D95

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
DCA	1	0	1	0	1	0	3	0
TYS	0	1	0	1	0	1	0	3

PHYSICAL IMBALANCE CHECK
STATION TOT TERM TOT ORIG

No entries below this line.

Time starting current report : 3/21/1989 at 17:11:40

{This is a sample report generated from ISS pre-switch/post-switch report option.}
{72Rs are being compared to M80s.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : 72R a/c 2 : M80

FAR has surplus a/c 1 and deficit a/c 2.
Rotation 152 will be colored BLUE. Stations visited :
FAR visited before 1225
MSP visited between 1319 and 1400
SNA visited between 1735 and 1825
MSP visited between 2152 and 2245
GRB visited after 2346
Rotation 154 will be colored GREEN. Stations visited :
FAR visited after 2452
MSP visited between 2258 and 2355
MCI visited between 2044 and 2140
MSP visited between 1839 and 1925
PHL visited between 1515 and 1600
MSP visited before 1250

YYZ has surplus a/c 2 and deficit a/c 1.
Rotation 151 will be colored RED. Stations visited :
YYZ visited after 2623
DTW visited between 2310 and 2520
SFO visited between 1645 and 1855
MSP visited between 1203 and 1300
RST visited before 1130
Rotation 155 will be colored YELLOW. Stations visited :
YYZ visited before 1130
DTW visited between 1240 and 1325
PDX visited between 1750 and 1825
SEA visited between 1909 and 2015
ANC visited after 2335

Pre-switch rotation 154 between 0 and 1250
with rotation 151 between 1203 and 1300 at station MSP.

The result from automatic Pre-switch/Post-switch Algorithm :

No entries below this line.

Time starting current report : 3/21/1989 at 17:13:11

{This sample report is an extract from ISS Check_flow_balance report option.}
{The pre-switch identified above has been performed prior to this run.}
{Notice the removal of imbalances from the 72Rs.}

MISSING FLIGHT SEGMENTS IN ROTATION

FLOW BALANCE FOR : 881001 TO 881004

72R

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O

M80

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
DCA	1	2	1	2	1	2	3	6
TYS	1	0	1	0	1	0	3	0

D95

	881001	881002	881002	881003	881003	881004		
STATION	TERM	ORIG	TERM	ORIG	TERM	ORIG	TOT-T	TOT-O
DCA	1	0	1	0	1	0	3	0
TYS	0	1	0	1	0	1	0	3

PHYSICAL IMBALANCE CHECK
STATION TOT TERM TOT ORIG

No entries below this line.

Time starting current report : 3/21/1989 at 17:14:32

{This report was generated after the pre-switch between rotations 154 and 151.}
{Now 72Rs and M80s are balanced. }

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : 72R a/c 2 : M80

The result from automatic Pre-switch/Post-switch Algorithm :

No entries below this line.

Time starting current report : 3/21/1989 at 17:17: 8

{This report was generated right after the most recent ISS Check_flow_balance report.}
{Now we are comparing M80s with D95s.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : M80 a/c 2 : D95

DCA has surplus a/c 2 and deficit a/c 1.
Rotation 143 will be colored RED. Stations visited :
DCA visited after 2647
DTW visited between 2441 and 2525
IND visited between 2255 and 2340
DTW visited between 2051 and 2150
MSP visited between 1710 and 1915
BIS visited between 1525 and 1600
MSP visited between 1322 and 1415
EWR visited before 1040
Rotation 149 will be colored YELLOW. Stations visited :
DCA visited before 1420
MEM visited between 1626 and 1730
BOS visited between 2023 and 2105
MEM visited between 2405 and 2500
DCA visited after 2655
Rotation 150 will be colored YELLOW. Stations visited :
DCA visited before 1050
MSP visited between 1324 and 1410
DLH visited between 1454 and 1535
MSP visited between 1619 and 1700
SNA visited between 2035 and 2130
MSP visited between 2455 and 2640
RST visited after 2717

TYS has surplus a/c 1 and deficit a/c 2.
Rotation 144 will be colored BLUE. Stations visited :
TYS visited before 1140
MEM visited between 1248 and 1355
MCI visited between 1515 and 1555
MSP visited between 1710 and 1805
YWG visited between 1931 and 2020
MSP visited between 2136 and 2230
STL visited between 2400 and 2440
MSP visited after 2606
Rotation 148 will be colored GREEN. Stations visited :
TYS visited after 2614
MEM visited between 2358 and 2509
MCO visited between 2110 and 2159
MSP visited between 1715 and 1804
SNA visited before 1350

Pre-switch rotation 148 between 1715 and 1804 with rotation 143 between 1710 and 1915 at station MSP.

Post-switch rotation 144 between 2606 and 4800
with rotation 150 between 2455 and 2640 at station MSP.

The result from automatic Pre-switch/Post-switch Algorithm :

Post-switch rotations 144 and 150 at 2606 and 2455

No entries below this line.

Time starting current report : 3/21/1989 at 17:21:33

MISSING FLIGHT SEGMENTS IN ROTATION

{This report was generated after the suggested post-switch.}
{Notice how the automatic pre-switch/post-switch Algorithm achieved total balance.}

FLOW BALANCE FOR : 881001 TO 881004

72R

STATION	881001 TERM	881002 ORIG	881002 TERM	881003 ORIG	881003 TERM	881004 ORIG	TOT-T	TOT-O

M80

STATION	881001 TERM	881002 ORIG	881002 TERM	881003 ORIG	881003 TERM	881004 ORIG	TOT-T	TOT-O

D95

STATION	881001 TERM	881002 ORIG	881002 TERM	881003 ORIG	881003 TERM	881004 ORIG	TOT-T	TOT-O

PHYSICAL IMBALANCE CHECK

STATION	TOT TERM	TOT ORIG

No entries below this line.

Time starting current report : 3/21/1989 at 17:23:33

{This report was generated after the previously suggested post-switch.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : M80 a/c 2 : D95

The result from automatic Pre-switch/Post-switch Algorithm :

No entries below this line.

Time starting current report : 3/21/1989 at 17:24:38

{Comparison of 72Rs with D95s reveal no imbalances.}
{We are done.}

PRE-SWITCH POST-SWITCH FOR : 881001 TO 881002

a/c 1 : 72R a/c 2 : D95

The result from automatic Pre-switch/Post-switch Algorithm :

No entries below this line.