SCHEDULING AND ROUTING MODELS FOR AIRLINE SYSTEMS

Robert W. Simpson

MIT

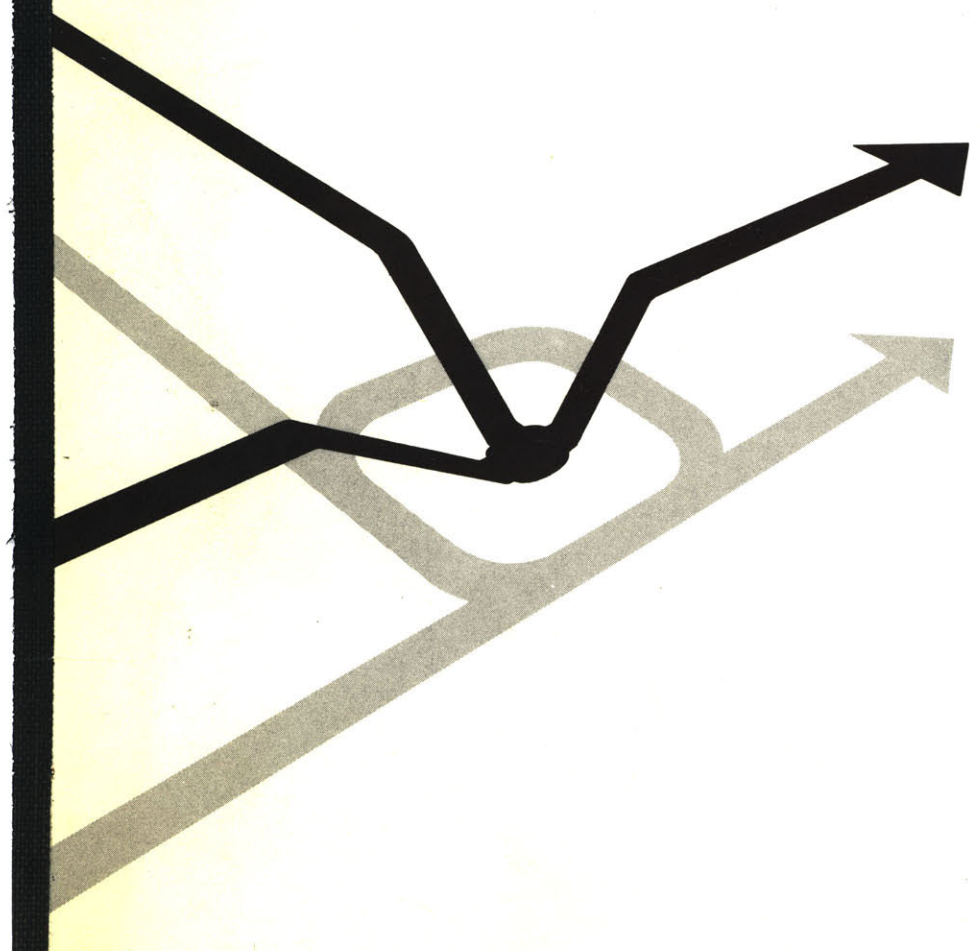**R68-3**

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FOREWORD

This report attempts to put together all of the optimal
computer models concerned with scheduling and routing problems for
passenger transportation systems. By placing them in one place,
classifying them, and using a consistent notation, it is hoped
that the models' relationships to each other can be seen, and that
a clear picture of the state of the art in model building and
solving can be shown. The emphasis of the report is on optimal
models which use well-known optimization techniques from mathe-
matical programming. Work which uses heuristic computer methods
in this area is quite extensive, but is not described here.

The models are oriented towards public transportation
systems operating on a short haul network. Generally a cyclic
or repetitive schedule of services is assumed, and a single
vehicle rather than a train of vehicles is being dispatched.
Within those assumptions, the models can find applicability to
schedule planning for a wide range of public transportation
systems, not necessarily just airline systems. The research is
supported in part by the Office of High Speed Ground Transpor-
tation, Department of Transportation, and is pointed towards
producing schedules for both high speed trains and future
V/STOL aircraft. The models are useful to planners and regula-
tors in studying problems in corporate planning, in transporta-
tion systems planning, and in regulation of transportation
industries.

An extensive bibliography accompanies each class of models
in this report. If it is not complete (with respect to optimal
models), I would appreciate receiving additional references

from interested readers. One of the reasons for writing this report is to give a good bibliography for various groups of present researchers who seem to be unaware of segments of the literature, or of each other's activities.

Much of the content of this report has been taken from lecture notes prepared by the author for an MIT graduate course, "Flight Transportation Operations Analysis", given by the author for the past few years. Students from that course will recognize the examples as being homework problems involving "Tech Airways", and I am indebted to them since some of their computer solutions are used as examples in the report.

As well, the report gives an overview of current research activity in this area in the MIT Flight Transportation Laboratory. A previous report, FTL R68-5 by Professor Amos Levin describes some of the Fleet Routing models and computational methods for solving them. Other reports and theses from the laboratory are referenced where appropriate. I must also recognize the work performed by Dave Benbasset, Norm Clerman, and Thor Paalson in providing computer runs for several of the examples.

Definitions and Symbols

1. Indices

p,q = station origins and destinations for traffic

pq = a city pair, or market

i,j = stations on a route

ij = a link between two stations on a route map

a = aircraft type

m = number of intermediate stops on a route

f = flight

s = service

r = route

T = time period for planning

2. Traffic Symbols

P_T = average traffic, passengers/cycle

P_{pq} = average traffic/cycle for system in period T from p to q

P_{pqra} = the proportion of traffic P_{pq} on route r, and aircraft type a

P_f = average traffic/cycle on flight f

LF = average load factor over planning period

p_{pq}^k = slope of market share curve for segment k, market pq

p_t = number of passengers arriving in interval (t,t+1)

RPM_T = revenue passenger miles for system in period T

$$\therefore \quad RPM_T = \sum_a \sum_{pq} \sum_r d_{pqr} \cdot P_{pqraT}$$

v

3. Aircraft Symbols

A_a = number of active aircraft of type a

A'_a = upper limit on A_a

S_a = seat capacity of type a

u_a = average fleet total block hours/cycle

U_a = average active aircraft utilization, block hours/aircraft-cycle

$$\therefore \quad u_a = U_a \cdot A_a$$

A_{paT} = number of aircraft a bought under purchase arrangement p which are in the fleet in period T.

s_{paT} = number of aircraft a bought under p which are sold at the beginning of period T subsequent to delivery.

$$\therefore \quad A_{paT} = A_{paT-1} - s_{paT}$$

L_{laT} = number of aircraft a leased under lease l covering periods T_l.

BU_{pa}, BL_{pa} = upper, lower bounds on buying aircraft a in period T under plan p.

SU_{paT}, SL_{paT} = upper, lower bounds on selling aircraft a in period T under plan p.

LU_{la}, LL_{la} = upper, lower bounds on leasing aircraft a under lease l

4. Value Symbols

DC_{raT} = marginal direct operating cost for aircraft a on route r in period T (generally without ownership costs)

CHR_{aT} = marginal direct hourly operating cost for aircraft a in period T

$$\therefore \quad DC_{raT} = CHR_{raT} \cdot t_{ra} = c_0 + c_1 \cdot d_r$$

where c_0, c_1 are aircraft cost coefficients

OC_a = aircraft ownership cost/cycle

IC_T = system indirect costs for period T

$$\therefore \quad IC_T = c_2 + c_3 \cdot P_T + c_4 \cdot D + c_5 \cdot (RPM_T)$$

where c_2, c_3, c_4, c_5 are system cost coefficients

Y_f = revenue yield per passenger on flight f

r_f = net revenue for flight f = yield x passengers

I_{fT} = net income for a flight f in period T

PP_{paT} = present value of progress payments for aircraft a in period T under purchase arrangement p which covers periods T_p. T_p has an initial period T_{p1}.

LP_{laT} = present value of lease payments for aircraft a in period T under lease arrangement l which covers period T_l.

MV_{paT} = present value of forecast market value or selling aircraft a purchased under plan p in period T.

BV_{paT} = present value of the depreciated value (book value) in period T for aircraft a purchased under plan p.

DEP_{paT} = present value of depreciation cost in period T for aircraft a purchased under plan p.

$$\therefore \ DEP_{paT} = BV_{paT-1} - BV_{paT}$$

CF_T = cash flow for system in period T

CH_T = cash on hand for system in period T

$$\therefore \ CH_T = CH_{T-1} + CF_T$$

d_i = present value of capital raised, under debt plan i which describes the repayment plan over future periods, T_i. T_i has an initial period T_{i1}.

DV_{iT} = proportion of d_i scheduled to remain as debt in year T

DP_{iT} = proportion of d_i scheduled to be repaid in year T as principal

$$\therefore \ DP_{iT} = DV_{iT-1} - DV_{iT}$$

Definitions and Symbols (continued)

IP_{iT} = proportion of d_i scheduled to be repaid in year T as debt costs

D/A = maximum allowable debt/asset ratio for airline

A^*_T = airline non-aircraft assets scheduled for period T exclusive of cash on hand

CH_{min} = min. amount of cash on hand desired

TD_T = proportion of before tax profit to be paid in taxes and dividends in year T

Definitions and Symbols (continued)

5. Time Symbols

t = time of day, time of cycle

td_{ijs} = departure time from i for s^{th} service from i to j

ta_{ijs} = arrival time at j for s^{th} service from i to j

tr_{ijs} = ready time at j for s^{th} service from i to j

= time at which aircraft becomes available for next service out of j

Tb_{ijsa} = block time or trip time using aircraft a

Tt_{ijsa} = minimum turnaround time for aircraft a

$$\therefore \quad td_{ijs} + Tb_{ijsa} = ta_{ijs}$$

$$ta_{ijs} + Tt_{ijsa} = tr_{ijs}$$

$$Tb_{ra} = \sum_{ij \,\epsilon\, r} Tb_{ij} = \text{sum of segment block times for aircraft } a \text{ on route } r$$

T = maximum cycle time, or index for planning period

W = passenger waiting time for next departure

TT = timetable consisting of set of t_a, t_d for all services

6. Route Map Symbols (See Figure 1a)

$\underline{Q}$ = set of cities p,q,i,j

$\underline{E}$ = set of edges or links ij in a route map

d_{ij} = distance of link ij

$\underline{R}$ = set of routes r representing specific paths on a route map

d_{pqr} = distance from p to q on route r

Routing = a subroute or portion of route r. An m-stop route contains $\frac{(m+2)(m+1)}{2}$ routings, eg. a 2 stop route ABCD has 6 routings: non-stop AB,BC,CD; one-stop ABC,BCD; two-stop ABCD. A single link routing is called a route segment.

$\underline{R}_{pqm}$ = subset of $\underline{R}$ such that every r contains a routing connecting p to q in exactly m stops

$\underline{R}_{pq} = \bigcup\limits_{m} \underline{R}_{pqm}$ = subset of $\underline{R}$ such that every r contains a routing connecting p and q

$\underline{R}_{ij}$ = subset of $\underline{R}$ such that every r uses link ij

$\underline{R}'_{pqm}$ = subset of $\underline{R}$ such that every r originates at p, and ends at q with exactly m intermediate stops

Definitions and Symbols (continued)

7.   Schedule Map Symbols (See Figure 1b)

$\underline{S}$ = set of non-stop services or flights between cities at specific cycle times = service arcs s

$\underline{G}$ = set of ground arcs g in schedule map

$\underline{C}$ = set of cycle arcs c in schedule map, one for each station

$\underline{A}$ = complete set of arcs in schedule map

$|\underline{A}|$ = number of arcs in schedule map

$$= |\underline{S}| + |\underline{G}| + |\underline{C}| \leqslant 3|\underline{S}| + 2|\underline{Q}|$$

$\underline{N}$ = set of nodes in schedule map

$|\underline{N}|$ = number of nodes in schedule map

$$\leqslant 2|\underline{S}| + 2|\underline{Q}|$$

8.   Flights, Services

$\underline{F}$ = set of flights , f, representing specific paths on a schedule map

Service = opportunity to use a portion of a flight f. An m-stop flight provides $\dfrac{(m+2)(m+1)}{2}$ services corresponding to the routings of its route. A non-stop portion of a flight is called a flight segment.

$\underline{F}_{pqma}$ = subset of $\underline{F}$ such that every f provides m-stop service between p and q using aircraft a

$\underline{F}_{pq}$ = subset of $\underline{F}$ such that every f connects p and q

Definitions and Symbols (continued)

8. Flights, Services (continued)

$\underline{F}_{ij}$ = subset of $\underline{F}$ such that f contains flight segment ij

$\underline{F}'_{pqm}$ = subset of $\underline{F}$ such that every f starts in p, ends in q with exactly m intermediate stops

$\underline{F}_{ra}$ = subset of $\underline{F}$ such that every f is along route r and uses aircraft a

9. Frequency of Service/Cycle

$n = |\underline{F}|$ = total number of flights/cycle for the system schedule

$n_{pqma} = |\underline{F}_{pqma}|$ = number of flights/cycle which connect p and q with m-stop service using aircraft a

$n_{pq} = |\underline{F}_{pq}|$ = number of flights/cycle connecting p and q

$n_{ra} = |\underline{F}_{ra}|$ = number of flights/cycle along route r using aircraft a

$n_{ij} = |\underline{F}_{ij}| = \sum_{r \in \underline{R}_{ij}} n_r$ = total number of flights/cycle on link ij

$D = \sum_{ij} n_{ij}$ = total system departures/cycle = flight segments/cycle

$N_{min\ pqm}$ = minimum number of m-stop services/cycle required for market pq

$NM_{min\ pq}$ = minimum number of services of M stops or less required for market pq

$N_{max\ p}$ = maximum number of daily departures allowed at station p

Example of Definitions

Route Map
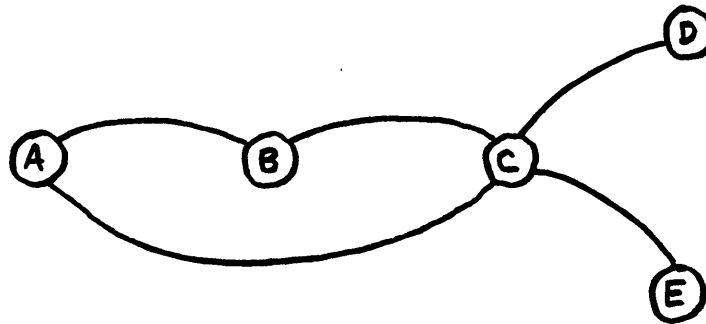
$\underline{R}$, Routes	$\underline{F}$, Flights	n_r	$\underline{R}_{ACO}$	$\underline{R}_{AC1}$	R'_{AC}
r = ABCD	f_r = 1,2	2		ABCD	
ABC	3	1		ABC	ABC
AB	4,5,6	3			
AC	7,8	2	AC		AC
ACD	9,10	2	ACD		
ACE	11,12	2	ACE		
CE	13,14,15	3			

$\underline{F}_{AC}$ = Flights 1, 2, 3, 7, 8, 9, 10

F'_{AC} = 3,7,8

n = $|\underline{F}|$ = 15

n_{AC} = n_{ABCD} + n_{ABC} + n_{AC} + n_{ACD} + n_{ACE} = 2 + 1 + 2 + 2 + 2 = 9

D = $\sum\limits_{ij} n_{ij}$ = n_{AB} + n_{AC} + n_{BC} + n_{CD} + n_{CE} = 6+6+3+4+5 = 24
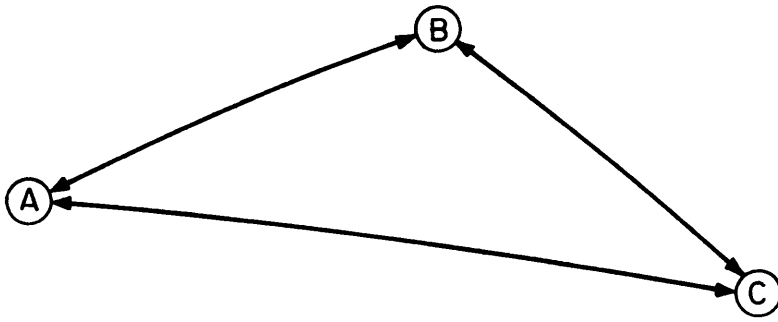
FIG. 1a ROUTE MAP
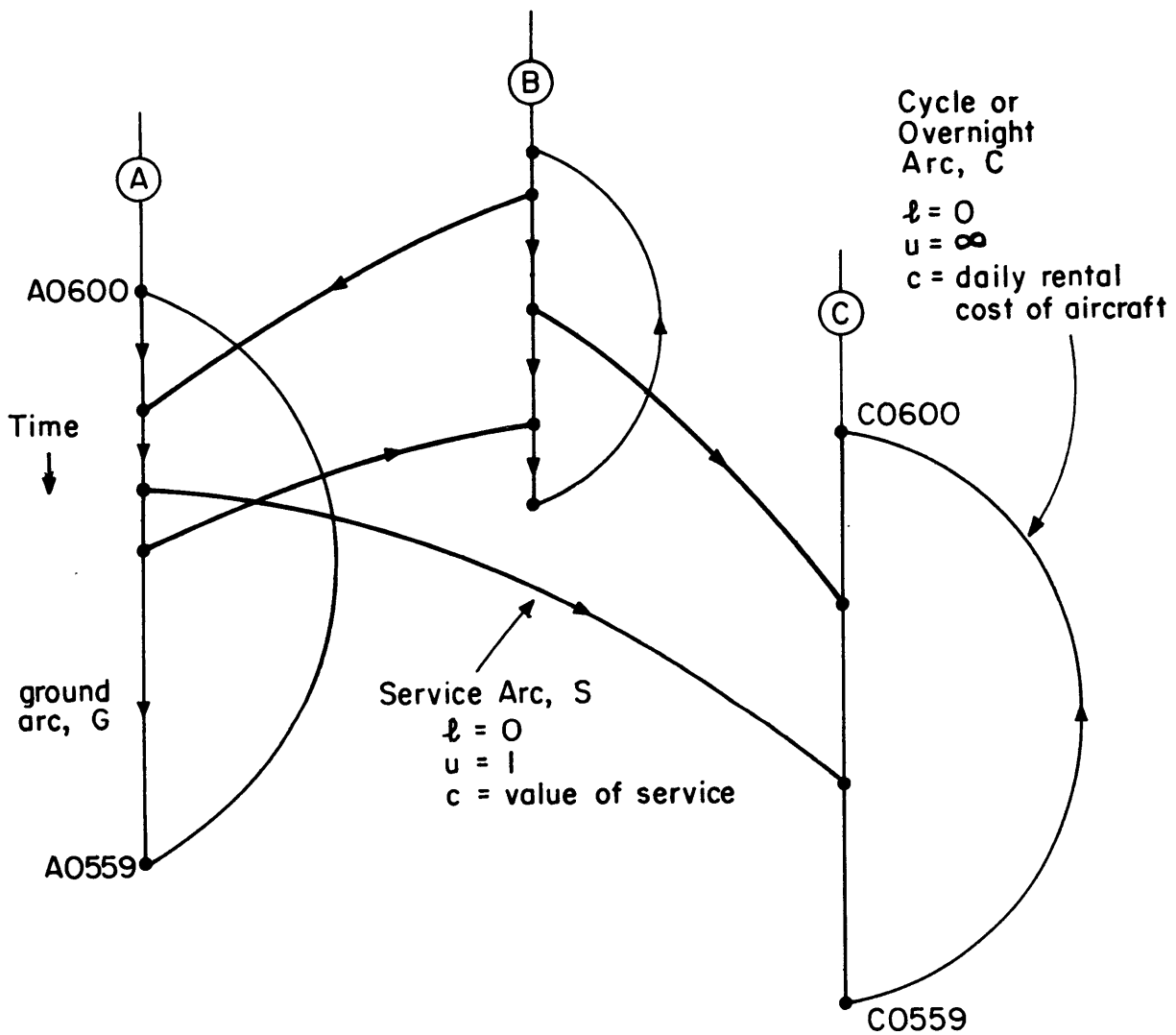
FIG. 1b SCHEDULE MAP

1.0 Introduction

1.1   The Scheduling Process

This report deals with optimization models which might
properly be described as "schedule construction" models.
As such they are part of a much larger scheduling process
called the "schedule planning" process which is followed by
most major airlines. Figure 2 attempts to show the relation-
ships of data and models (or analytic activity) in this
schedule planning process in a diagrammatic fashion. The
rectangular boxes contain deterministic information; the small
circular elements contain forecast, or predicted information
subject to some uncertainties.

The two large circles contain two types of analytic
processes which are amenable to model building. The first is
the traffic forecasting process, which accepts as inputs
information on general economic activity, on levels of adver-
tising, on the levels of service provided by the system compared
to alternative systems, on fares, and which produces as output
a forecast of traffic by route, by flight, etc., as an average
over the period of time considered for the schedule plan.
Computer models of this traffic forecasting process exist, but
are not considered in this report. It is pertinent to note
that for good schedule planning, not only are good averages
for traffic data needed, but also the gradients in traffic
with respect to inputs arising from levels of service offered
by the timetable. It would be nice to be able to predict how
traffic loads will change as daily frequencies, time of departure,
number of stops, etc. are varied. At present, human judgement

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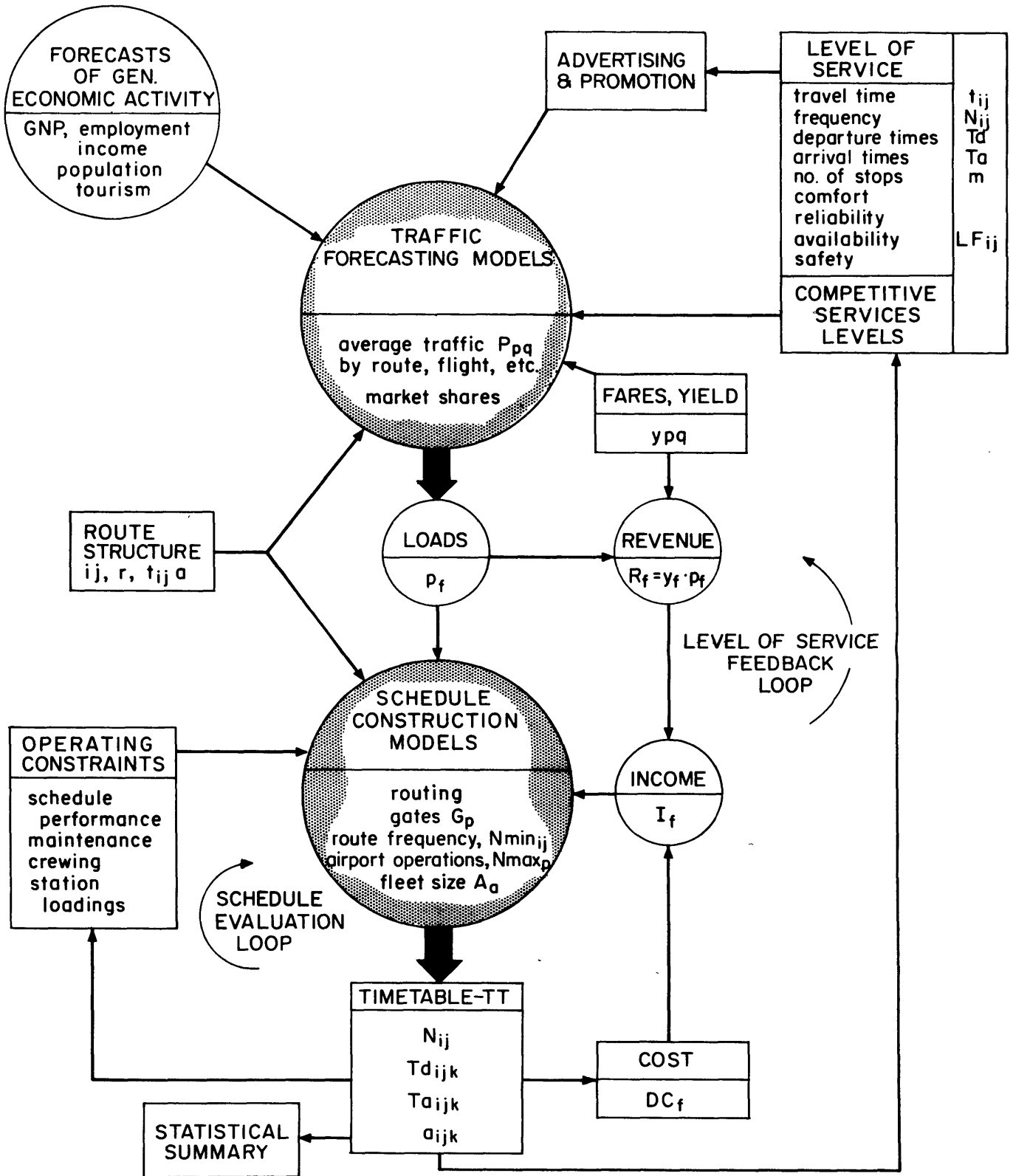
FIG. 2 THE SCHEDULE PLANNING PROCESS

arising from past experience is necessary, and thereby plays a critical part in the schedule planning process.

The second set of models are the schedule construction models described in this report. They accept as input information forecast traffic, profitability by flight, and route structure data, and proceed to construct a timetable for the system subject to various kinds of system constraints. The timetable determines the system operating cost and profitability, and provides a summary of detailed planning data.

The first feedback loop is concerned with considering constraints and factors which are presently outside the schedule construction models such as air crew scheduling, station operation requirements, maintenance requirements of aircraft, and an evaluation of "on time" performance or system schedule reliability. This loop is called the schedule evaluation loop where inputs from the evaluation process cause iterations of the schedule construction process. In most airlines, a wide variety of operating personnel become involved in this process, and the loop is iterated many times in producing a final plan.

The second loop is called the "level of service" feedback loop in this report. It is mainly of concern to academic students of transportation in a rather vague description of how supply and demand levels are determined for transportation systems of all types. Airline thinking recognizes competitive airlines' systems, and talks of "market shares" but very rarely recognizes any overall traffic stimulation through increased service levels. Thus, it is not usual for an airline schedule planning process to determine a timetable, and return

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to the traffic forecasting process to regenerate new traffic data on the basis of the new timetable. It is a real feedback, particularly in competitive airline markets, or short haul markets where the air system competes with automobile traffic, and some of the models in this report will show its inclusion in a very basic way.

For domestic airline systems, the basic schedule plan which is finally produced is generally maintained over an extended period of months. Within that period, a schedule cycle over a day (or a week) is repeated many times without any major changes. Variations in traffic are accommodated by load factors on the vehicles, and no use is made of real time data, or advance reservations information. The schedule plan is "static" in the sense that it is maintained over the period. It is possible to consider "dynamic" scheduling processes, where no fixed schedule plan is maintained, and real time information is used to adapt or augment a schedule plan or to dynamically dispatch vehicles in real time. The Eastern Airlines Shuttle is the only example for present airlines systems. A taxi fleet with radio dispatching or a bank of automatic elevators are examples of a "dynamic" scheduling system. References 1 to 7 describe some recent work in this area.

When a fixed schedule plan has been adopted, the problems which arise in carrying out that plan in the face of weather, aircraft breakdowns, unexpected events, etc. are best described by the phrase "schedule control". Here, some central point in the system is faced with decision making in real time which involves thousands of dollars in revenue and cost as various flights are cancelled, crews are deadheaded, and spare aircraft are ferried to where they are needed.

The Fleet Routing models discussed later in this report are very adept at assisting in this decision making process (see reference 1).

1.2 Classification of Models

This report describes mathematical models of various problems arising in schedule construction. A brief review of available techniques for finding optimal answers to these models is given in the next section. Here we shall attempt to describe the models in general terms, and to classify them into certain groups.

A model is an idealized representation of a problem and its variables in a mathematical form. In general terms, the models of this report can be stated as:

Find a combination of values of $\underline{x}$, (the problem variables) that optimizes an objective function, $R(\underline{x})$ and which satisfies a number of conditions, or constraints, on $\underline{x}$, $g_i(\underline{x}) = 0$.

Typical objective functions and constraints for schedule construction models are shown in Table 1. By choosing one objective, and some combination of the constraints, one can formulate a large number of models. Only models which have been studied by staff and students in the Flight Transportation Laboratory are listed here (partially to enable us to see which models we have or have not yet studied). There are many others, and it is hoped that the scheme of classification will enable us to give a quick, descriptive name to newer models.

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Table 1 - Components of Schedule Construction Models

Objective Functions, $R(\underline{x})$

R1	Minimize Fleet Size
R2	Minimize Operating Cost
R3	Maximize (Revenue-cost) for the system operator
R4	Maximize (Total social benefits - costs) for operator and public

Model Constraints, $g_i(\underline{x})$

i = 1. Demand-capacity relationships

2. Restricted numbers of different types of aircraft

3. Cyclic station balance constraints for aircraft movements

4. Route frequency

5. Restricted number of operations/cycle at a station

6. Fleet continuity constraints

7. Airline Financial constraints

8. Multiple departure times for any service

9. Routing constraints on aircraft services

10. Restrictions on number of gates at a station

These constraint numbers shall be used throughout the report.

Depending on the mathematical characteristics of the model, techniques such as linear, dynamic, or combinatorial programming are used to find an optimal answer for the model. The solution techniques play a strong role in model formulation, since it is easy to conceive of models which are not computationally feasible. The model classification scheme which follows tends to be based on solution techniques, and the particular models discussed are generally computationally feasible for the size of present airline systems.

Fleet Assignment Models

These are generally LP models which assign aircraft types to a set of routes on a route map. They will use objective functions, R_2, R_3 and R_4, and can contain all the constraints except 6, 8, and 9.

Fleet Planning Models

These extend a basic fleet assignment model over a set of planning cycles, and introduce fleet continuity constraints. They are LP models, suitable for decomposition techniques.

Dispatching Models

For a single route, these models will determine an optimal pattern of times for dispatching flights given some information on time of day variation in demand. They generally use dynamic programming. They use objective functions R_3 or R_4, and can use constraints 2, 4, 5, and 8.

Aircraft Routing Models

These models attempt to determine optimal routings for individual aircraft given a schedule map. They may be

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viewed as extensions of the dispatching models onto a network, which add the routing constraints 9.

Fleet Routing Models

Here the optimal set of routings for a fleet is determined without identifying individual vehicles. Network flow methods can be used on single fleet models which use objectives R_1, R_2 or R_3, and constraints 2, 3, and 9. When constraints 4, 5, 8, 10 are added, they impose "bundle" constraints on the network flow and require special combinatorial programming techniques. For different types of aircraft in the fleet, we have to impose "multi-copy" constraints on the network, and this requires similar computational techniques.

This classification scheme is used in this report to organize the work which has been done in this area. In each class, some of the existing models are identified and discussed. Computational experience is given where appropriate.

1.3 Methodology Review

A brief review of the computational techniques used in these models is given here with some comments on present computational capabilities and running times. It is assumed that the reader is generally familiar with mathematical programming. Reference texts are suggested at the end of the report .

1.3.1 Linear Programming - (LP)

The mathematical problem is stated as:

$$\text{Minimize} \quad \left\{ Z = \underline{c}.\underline{x} \right\}$$

$$\text{such that} \quad \underline{A}.\underline{x} = \underline{b}$$

$$\underline{x} \geqslant 0$$

where $\underline{c}$ = cost vector

$\underline{A}$ = constraint matrix

$\underline{b}$ = constraint vector

$\underline{x}$ = problem variables, as real numbers

The model finds an optimal value for a linear objective function subject to a large number of constraints which can be expressed as linear equations in the problem variables. Most computers have standard codings to solve this problem. Their present capacity is approximately 4000 rows (or constraint equations), and virtually unlimited numbers of columns (or variables). Present speeds are very good. For the MPS 360-65 at MIT, the times are roughly expressed by the following formula:

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$$\text{Running time(minutes)} = 4 \times 10^{-6} \times (\text{No. rows} \times \text{No. variables})$$

The range of variation is rather large since simply changing the objective function can cause $\pm 50\%$ variations from the above estimate.

1.3.2 Integer Linear Programming

The problem statement is identical to that above except that $\underline{x}$ must take on integer values. There are now a variety of techniques for finding integer solutions after obtaining the LP solution. The fastest technique for models described in this report appears to be the group theoretic approach developed for solving large scale set covering problems which appear in the crew scheduling process. This technique and its variants are described in reference 7 of the Fleet Routing bibliography.

1.3.3 Combinatorial Programming

The two techniques, branch and bound, and implicit enumeration, are techniques which efficiently search for a best combination of variables as a solution to some programming problem. Both can be used quite successfully on fleet routing models which have the auxiliary or bundle constraints provided the combinatorial size does not become too large.

By using MPS, a Land and Doig routine has been coded for the Flight Transportation Laboratory. It solves the LP problem, chooses a routing variable which should be integer (0,1), sets this variable to either 0 or 1 and computes bounds on the best integer answer by solving the new LP problem. At some point a solution tree (or branching tree) is found where one can show that a best integer answer has been found.

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1.3.4 Network Flow (OKF)

The mathematical problem is a special case of the LP problem called a minimum cost constrained network circulation flow problem:

$$\text{Minimize } Z = \underline{c} \cdot \underline{x}$$

such that 1) Flow is conserved at <u>every</u> node

$$\sum_i x_{ij} - \sum_k x_{jk} = 0 \qquad\qquad \forall j$$

2) Flow in an arc is bounded

$$l_{ij} \leq x_{ij} \leq u_{ij} \qquad\qquad \forall ij$$

where $\underline{x}$ is a circulation flow on a network

$\underline{l}$, $\underline{u}$ are lower and upper bounds on $\underline{x}$

For every node there is a node price π_i . For every arc bound, there is a marginal cost $\bar{c}_{ij}$ representing the marginal change in Z if the arc bound were changed.

Rather than use an LP statement and simplex steps, it is far more efficient to store the network information in lists, to insist that the input flow be conservative so that constraints 1) can be satisfied in an implicit manner, and then use a tree search routine called "labelling" to construct an optimal, feasible network circulation flow. This allows very large problems to be handled and solved very quickly. The FTL version of OKF (Out of Kilter Flow) for the MIT 360-65 can handle up to 15000 arcs and 4500 nodes.

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Computation times depend on the number of arcs which are "out of kilter" for the input circulation.

1.3.5 Dynamic Programming - (DP)

From a graph theory viewpoint, it's possible to classify discrete dynamic programming as a tree construction routine. An optimal tree is built as the routing proceeds rather than searching for an optimal tree on a given network. There is an associated network described by a branching logic which knows the branching or decision process at every node. For complex networks, the branching logic may be inefficiently coded, and may create the necessity of large blocks of storage space to contain information for an expanded network.

The mathematical model is expressed as:

$$\text{Find} \quad \underset{t = 0, N}{\text{Min}} \left\{ Z = R \, (\underline{x}, \, \underline{Y}, \, \underline{\pi}, \, t) \right\}$$

subject to constraints on $\underline{x}$, $\underline{Y}$, or $\underline{\pi}$ variables

$$g_i \, (\underline{x}, \, \underline{Y}, \, \underline{\pi}) = 0$$

and given initial and/or final state space boundary conditions

$$\underline{Y}_{t=0}, \quad \underline{Y}_{t=N}$$

where $\underline{x}$ are called <u>decision</u> or <u>control</u> variables and are represented by arcs of the associated network.

$\underline{Y}$ are called <u>state</u> variables and are represented by the nodes of the associated network at a given stage.

t is a particular state variable used for indexing called the _stage_ variable.

π are the node prices for the tree representing _path_ variables which are determined during the tree construction process and are a function of the tree path to a given node.

If there are d components to the $\underline{Y}$ vector, the state space at any stage consists of d dimensions for the $\underline{Y}$ nodes.

Eg. if d = 2, and we have 100 discrete increments in both dimensions, the state space consists of $100^2 = 10^4$ nodes. If there are also 100 stage increments, we have 10^6 nodes in the associated network, and this determines the size of the low speed memory requirement, N_L, for the computer. The more critical storage is the high speed memory requirement, N_H which is at least twice the number of nodes in the state space, i.e. $N_H = 2 \times 10^4$ in our simple example.

Viewed as a network, there are a variety of tree search routines which may be used. Dynamic programmers usually use a relatively inefficient one involving the following recursive equations;

$$R^*_j = \underset{i_t}{\text{Min}} \left\{ R^*_{i_t} + c_{ij} \right\}$$

where $R^*_{i_t}$ = optimal value of Z from an initial state to the state represented by node i at stage t

c_{ij} = arc cost for going from node i to node j

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1.4 Bibliography - Dynamic Scheduling Models

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Schedule Control

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2.0 Fleet Assignment Models - (LP)

The early applications of linear programming to these problems describe variations of a model originally described in 1954 by Dantzig and Ferguson which assigns aircraft of different types and operating costs to a given set of non-stop routes, ensuring that sufficient seating capacity is supplied, and that aircraft are available. The models assume a fixed time period, T, and that an estimate of the average traffic in that period for each route, P_{pq}, exists. As well, a load factor assumption LF_{pqa} for each aircraft using the route must generally be made in order to relate supply and demand. These models do not route the aircraft, and thus require a second critical assumption regarding aircraft average utilization. The results are generally non-integer assignments which may not be critical depending on the purpose of the model. (One may interpret the fractional assignment as an average over T.)

The first model given here is representative of this type of model. It is posed for an average daily time period where the variables n_{pqa} in the LP represent the frequency of daily service by aircraft on the routes. Thus, the airline "frequency pattern" which is required for initiating time-table construction is the primary output of the model. The model has been posed for other contexts which use equivalent variables.

The second model is an extension to include multi-stop routes as opposed to the direct non-stop city pair route used in the first model. Because of historical reasons or

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legal restrictions, the airline system may have a set of non-stop and multi-stop routes it wishes to operate, and the object of the model as posed here is to determine the frequency pattern n_{ra} for the daily number of trips by aircraft a on multi-stop route r.

The third model extends the first one to include competitive market share of the air traffic on the route pq.

2.1   Model FA-1   Least Cost Frequency Pattern, Non-Stop, (LP)

2.1.1.   Problem Statement

Given a set of routes pq on which there exists an average daily traffic P_{pq} which must be carried by a fleet consisting of limited numbers of different types of aircraft, find the least cost assignment of aircraft to routes such that a minimum frequency of daily service on each route is satisfied, and such that aircraft movements balance at every station while not exceeding a daily allowable maximum imposed by airport congestion.

2.1.2. Model Formulation

Let n_{pqa} be the number of daily non-stop flights on route pq between cities p and q by airplane type a.

Objective Function

Minimize operating costs (for fixed revenue)

$$\text{Minimize} \quad \sum_{pq} \sum_{a} DC_{pqa} \cdot n_{pqa}$$

where DC_{pqa} = an appropriate direct operating cost for an aircraft a on route pq

Constraints

1. Demand on all routes must be carried

$$\sum_{a} LF_{pqa} \cdot S_a \cdot n_{pqa} = P_{pq} \qquad \text{✈}_{pq}$$

where LF_{pqa} = assumed load factor for aircraft a on route pq

S_a = seat capacity for aircraft a

P_{pq} = estimated average for daily traffic from p to q

This requires that passengers on all routes be carried and fixes the revenue. One may wish to relax the equality and allow passengers not to be carried. In this case a slack variable is added, with a cost appropriate for yield per passenger on route pq. The minimization is then over operating cost and lost revenue.

2. Fleet availability must not be exceeded

$$\sum_{pq} Tb_{pqa} \cdot n_{pqa} \leq U_a \cdot A'_a \qquad \rightarrow a$$

where Tb_{pqa} = block hours for aircraft a to fly route pq

U_a = assumed average daily utilization for fleet a

A'_a = number of aircraft in fleet a

This requires that aircraft must be available given a certain fleet size. U_a must be predetermined for the fleet regardless of the set of routes which fleet "a" may eventually be assigned by the optimal solution. Since U_a is strongly determined by average stage length, it may have to be changed after the LP solution is obtained.

3. Aircraft movements must balance over the day at every station

$$\sum_{q} n_{pqa} - \sum_{q} n_{qpa} = 0 \qquad \rightarrow p,a$$

For a cyclic daily schedule, the number of arrivals of a given aircraft must balance the number of departures for every station in order to conserve the flow of aircraft. These constraints ensure that any resultant n vector will be a feasible vector for routing vehicles. If the demand data is symmetric in that $P_{pq} = P_{qp}$, (daily flows are equal both ways) then these constraints may be omitted. If they are omitted, then the LP is simply mapping a set of aircraft

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types onto a set of routes with no network structure. The double index pq can then be replaced by a single index r for a route, or class of routes. If demand is symmetric, the problem size is thereby reduced since the number of routes is halved.

4. Minimum daily frequency must be maintained on each route

$$\sum_a n_{pqa} \geqslant N_{min\ pq} \qquad \mathbf{\mathsf{Y}}\ pq$$

where $N_{min\ pq}$ = a specified minimum level of service

Because of competitive reasons, or traffic generation, or management policy, there may exist a desire to have at least $N_{min\ pq}$ daily services on route pq. This constraint ensures that this will occur, and prevents use of large size, low cost aircraft on routes where the value of n_{pqa} for that route would be very low ($\ll 1$ for example). The value of $N_{min\ pq}$ is related to the estimate for P_{pq} input to the model.

5. Maximum daily departures at a station are limited due to airport capacity

$$\sum_a \sum_q n_{pqa} \leqslant N_{max\ p} \qquad \mathbf{\mathsf{Y}}\ p$$

At airports where capacity quotas have been established, this constraint limits total daily activity.

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2.1.3 Problem Size - (Symmetric Demand)

a) Number of rows $\leq 2|E| + |a| + |Q|$

Constraints 1. $|E|$ = no. of links pq in route map
2. $|a|$ = no. of aircraft types
3. (Omitted)
4. $|E|$ (not necessary for every route)
5. $|Q|$ = no. of cities in route map

b) Number of Variables $\leq |E| \cdot |a|$

Variables n_{pqa} : $|E| \cdot |a|$ = links x aircraft types

e.g. Suppose we have 20 cities, 200 routes, and 5 aircraft types:

Constraints $\leq 400 + 5 + 20 = 425$
Variables $\leq 200(5)$ $= 1000$

For problems of this size, a preprocessor coding accepts a simple description of the input data, and creates the input for the LP coding. A postprocessor coding is also used to give a useful output format.

2.1.4. Comments

Various versions of this model have been reported
in the literature (e.g. references 1, 2, 3) through the years.
Unless the fleet availability constraints, or the minimum
frequency constraints are binding, it produces a rather tri-
vial result of assigning sufficient numbers of the least
cost aircraft to each route. The LP result gives non-integer
answers for n_{pqa}, but since averages are used for traffic,
utilization, and load factor, these values can also be viewed
as averages over T.

Because of its assumptions, this model and its
variants have not received much airline usage for assigning
aircraft to routes. The historical frequency pattern avail-
able for modification as new aircraft are added to the fleet,
and other political or marketing factors have been dominant
in determining airline frequency patterns on a piecemeal
basis.

2.2   Model FA-2   Extension to Multi-stop Services - (LP or ILP)

2.2.1.   Problem Statement

Given a predetermined set of multi-stop routes r, and an average daily traffic P_{pq} between points p,q which must be carried by a fleet consisting of limited numbers of different types of aircraft, find the least cost assignment of aircraft to routes such that various routing and operating constraints are satisfied.

2.2.2 Model Formulation

Let r be any path sequence of edges or arcs in a route map representing a possible multi-stop route connecting city p to city q. Let i and j be consecutive stops on the route where i precedes j, (i < j).

Let $\underline{R}$ be a predetermined set of routes the system would like to consider, and $\underline{R}_{pqm}$ be the subset which connects city p to city q in exactly m stops. Let P_{pqra} be the number of passengers travelling from p to q which uses aircraft a and route r.

Objective Function

Minimize operating cost (for fixed revenue)

$$\text{Min.} \left\{ \sum_{r} \sum_{a} DC_{ra} \cdot n_{ra} \right\}$$

Constraints

1.a) All demand must be served

$$\sum_{r \in \underline{R}_{pq}} \sum_{a} P_{pqra} = P_{pq} \qquad \curlyvee pq$$

where $\underline{R}_{pq} = \bigcup_{m} \underline{R}_{pqm}$ = all routes connecting p & q

1b.) Average daily load on any route segment must not exceed a desirable load factor for an aircraft

Segment load for link ij on route r, aircraft a

$$= \sum_{\substack{p \leq j \\ i < q}} P_{pqra} \qquad \text{for all p, q on route r}$$

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e.g.   Route r, aircraft a

$$1 \qquad i \qquad j \qquad k \qquad m$$
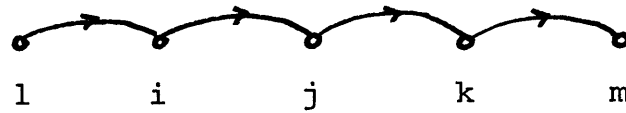
Daily segment load, link $ij = P_{1ja} + P_{1ka} + P_{1ma}$

$$+ P_{ija} + P_{ika} + P_{ima}$$

The route segment constraint may then be written for each aircraft on the route

$$\sum_{\substack{p < j \\ i < q}} P_{pqra} - LF_{ra} \cdot S_a \cdot n_{ra} \leqq 0 \qquad \forall \text{ segments } ij \\ \forall \, r, a$$

or alternatively,

1c.) Average daily load for all aircraft on any route
segment must not exceed a desirable load factor.

$$\sum_{a} \sum_{\substack{p < j \\ i < q}} P_{pqra} - \sum_{a} LF_{ra} \cdot S_a \cdot n_{ra} \leqq 0 \quad \forall \text{ segments } ij \\ \forall \, r$$

Here we reduce the number of constraints 1b) by
cumulating them, and simply insist that for every
segment of a route, there must be enough daily
seats over all aircraft flying the route to cover
the variations in average daily load on the seg-
ment. If any aircraft route becomes overloaded

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on a segment, passengers can be carried on another
aircraft type flying the same route, and offering
through plane service.

or alternatively,

1d.) Average daily load for all aircraft flying a seg-
ment by any route must not exceed a desirable
load factor

$$\sum_{r \in \underline{R}_{pq}} \sum_{a} \sum_{\substack{p < j \\ i < q}} P_{pqra} - \sum_{r \in \underline{R}_{pq}} \sum_{a} LF_{ra} \cdot S_a \cdot n_{ra} \leq 0 \quad \forall \text{segments } ij$$

Here we further reduce the number of constraints
by simply insisting that there must be sufficient
seats on every link of the route map to cover the
variations in daily load expected on the link.
If any route segment becomes overloaded, passen-
gers can be carried on another route traversing
that link by changing planes.

2.) Fleet availability must not be exceeded

$$\sum_{r} Tb_{ra} \cdot n_{ra} \leq U_a \cdot A'_a \quad \forall a$$

where Tb_{ra} = block time on route r

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3.) Aircraft movements must balance at every station

$$\sum_{q} \sum_{r \in \underline{R}'_{pq}} n_{ra} - \sum_{q} \sum_{r \in \underline{R}'_{qp}} n_{ra} = 0 \qquad \forall p, a$$

where $\underline{R}'_{pq}$ is the subset of $\underline{R}$ for routes starting

from city p and ending in city q

4a.) A minimum m-stop service must be supplied for every
city pair market

$$\sum_{r \in \underline{R}_{pqm}} \sum_{a} n_{ra} \geqslant N_{min\ pqm} \qquad \forall pq, m$$

or alternatively,

4b.) A minimum level of service must be supplied for
every city pair market

$$\sum_{m = 1, M} \sum_{r \in \underline{R}_{pqm}} \sum_{a} n_{ra} \geqslant NM_{min\ pq} \qquad \forall pq$$

where $NM_{min\ pq}$ represents a minimum number of
daily services for market pq with M or less stops.

5) Maximum daily departures at a station are limited due
to airport capacity

$$\sum_{a} \sum_{j} \sum_{r \in \underline{R}_{ij}} n_{ra} \leqslant N_{max\ p} \qquad \forall i$$

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2.2.3 Problem Size - (Symmetric Demand)

a) Number of rows $\leq |E| + |a| + |Q| + 2|pq| + \sum_{m} \sum_{pq} |\underline{R}_{pqm}|$

$$+ \sum_{ij} |\underline{R}_{ij}| + \sum_{a} \sum_{ij} |\underline{R}_{ija}|$$

Constraints 1.a) $|pq|$ - number of O&D city pairs

1.b) $\sum_{a} \sum_{ij} |\underline{R}_{ija}| = $ aircraft-route segments

or 1.c) $\sum_{ij} |\underline{R}_{ij}| = $ route segments

or 1.d) $|E| = $ edges, or links in route map

2. $|a| = $ aircraft types

3. (Omitted)

4.a) $\sum_{m} \sum_{pq} |\underline{R}_{pqm}| = $ city pair m stop services

or 4.b) $|pq|$

5. $|Q| - $ no. of cities

b) Number of variables $\leq |a| \cdot (|\underline{R}| + \sum_{pq} |\underline{R}_{pq}|)$

Variables $n_{ra} : |\underline{R}| \cdot |a| = $ aircraft-routes

$P_{pqra} : |a| \cdot (\sum_{pq} |\underline{R}_{pq}|) = $ aircraft-city pair routes

e.g. Suppose we have 20 cities, 200 city pairs, 5 aircraft types, 100 links in the route map, 500 routes, 700 route segments, 600 city pair routes, and 400 city pair m stop services, and assume constraints 1b) are not used.

Constraints $\leq 100+5+20+2(200)+400+700 = 1625$

Variables $\leq 5(500 + 600) = 5500$

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2.2.4 Comments

This problem generally has more constraints than the original problem, and many more variables. If there were sufficient aircraft available, and the minimum frequency constraints were not binding, the LP solution would choose the cheapest aircraft-route for every city pair and assign sufficient daily frequency to that service to carry passengers at exactly the assumed load factor. Because of the cost structure of airline service, this would mean the most direct or least stop service available using the best aircraft. If non-stop routes for every city pair existed in R, they would be used reducing this problem to the original model, FA-1.

However, if a city pair does not have a non-stop route, it must choose the cheapest multistop route, or a routing portion of some other route. If the latter occurs, the whole route is flown with a lower load factor appearing on the remaining segments.

Also, if the minimum frequency requirements for every city pair are not satisfied by the simplistic solution, the model will rearrange the routings to meet the daily required frequencies at lowest incremental cost. This rearrangement also may cause the lower load factor to appear on some flight segments.

While these two possibilities may cause flight segments with available space to appear in the system, the LP solution usually avoids this by rearranging the frequency pattern to fill this available free space. A very small number of flight segments are slack, and one may generalize the model results

by stating that the LP solution tends to find a frequency pattern of multistop routes such that all flight segments are at equal load factors (max. allowable load factor). In airline terms, the LP solution performs the "load building" job in using multistop routes.

To make this model meaningful, it should be solved as an ILP where n_{ra} must be integer. As has been noted in the literature[1], only the ILP model correctly describes the problem of determining a least cost frequency pattern for multistop or transshipment routes. The integrality constraint also causes slack flight segments at lower load factors to appear.

The objective function of this model may be modified to include terms which express the travel costs of passengers for using indirect, multi-stop routings, i.e. the objective function is of class R4 where total social costs are being optimized. An example of this model is used in the work associated with reference 5.

1. The discussion in Chapter 3.4 of reference 3 describes this problem.

2.3. Model FA-3   Maximum Income, Competitive Market Share (LP)

2.3.1. Problem Statement

Given a set of non-stop routes pq for which there exists a market share curve relating traffic carried to daily frequency n_{pq}, an aircraft fleet consisting of limited numbers of different types of aircraft, find the maximum income assignment of aircraft to routes such that a minimum daily frequency of service is maintained, and such that aircraft movements balance at every station while not exceeding an allowable maximum imposed by airport congestion.

2.3.2. Model Formulation

Let P_{pq} be a convex, stepwise, linear function of n_{pq}, the total daily frequency of service on route pq as indicated by Figure 3. Let p_{pq}^k be the slope of the kth segment, n_k^m be the breakpoints in total frequency, and n_{pq}^k the value of total frequency for the kth segment.

The model formulation as an extension of model FA-1 is given by Figure 4.

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P_{pq}
Average Traffic

P^3

P^2

p^k = Slope of Market Share Curve on Segment k

P^1

n_1　　　　n_2　　　　n_3

n^1　　n^2　　n^3

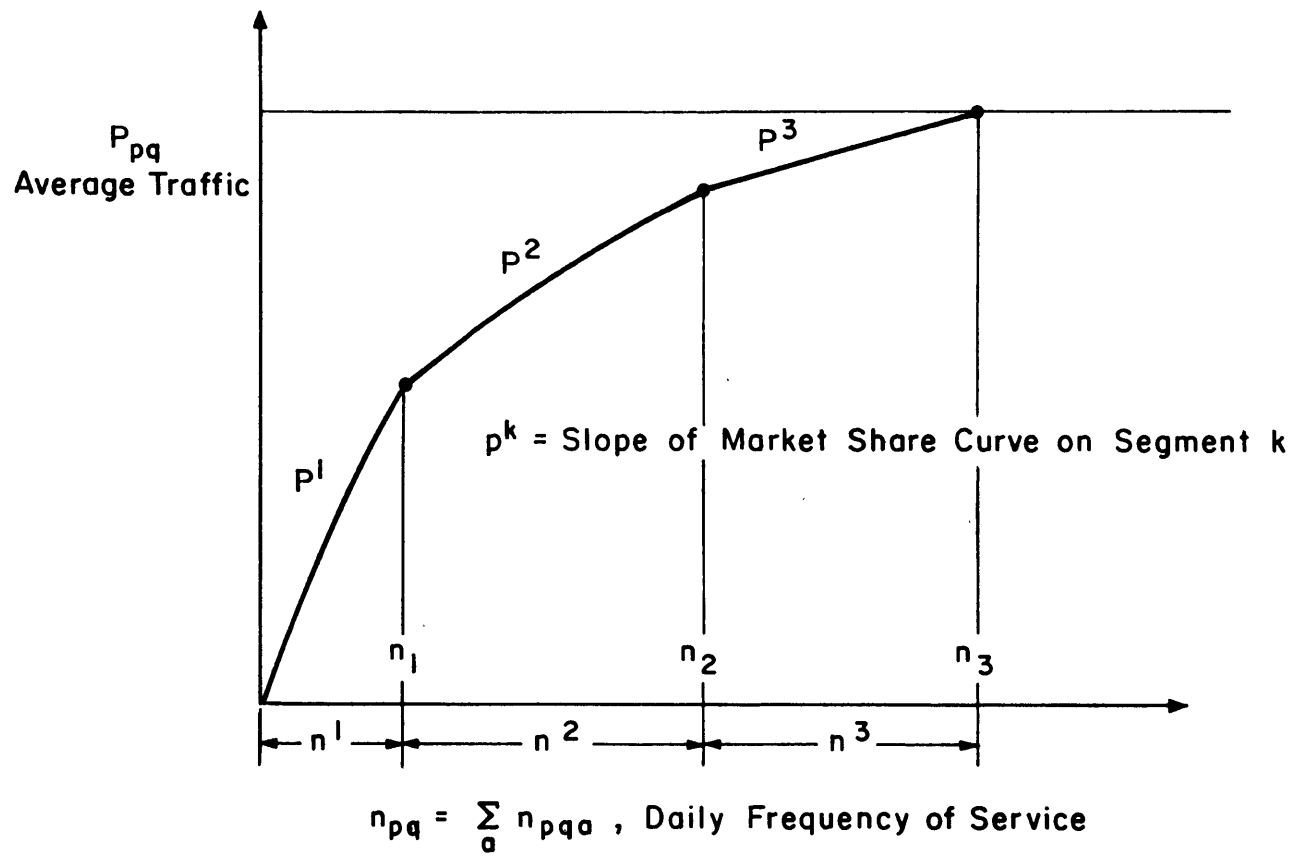$n_{pq} = \sum_a n_{pqa}$, Daily Frequency of Service

FIG. 3 A TYPICAL MARKET SHARE CURVE

OBJECTIVE FUNCTION – Maximize Income for System

$$\text{Max}\left\{ \sum_{pq} y_{pq} \cdot P_{pq} - \sum_{pq} \sum_{a} DC_{pqa} \cdot n_{pqa} \right\}$$

CONSTRAINTS

1.a) Maximum allowable aircraft load factor on all routes

$$\sum_{a} LF\,max_{pqa} \cdot S_a \cdot n_{pqa} - P_{pq} \geq 0 \qquad \forall pq$$

1.b) Traffic carried is a function of total daily frequency

$$\sum_{k} P_{pq}^{k} \cdot n_{pq}^{k} - P_{pq} \geq 0 \qquad \forall pq$$

1.c) Total daily frequency is sum of segment frequencies

$$\sum_{a} n_{pqa} - \sum_{k} n_{pq}^{k} = 0 \qquad \forall pq$$

1.d) Segment frequencies are bounded at breakpoints

$$0 \leq n_{pq}^{k} \leq n_{k}^{pq} - n_{k-1}^{pq} \qquad \forall pq,k$$

2. Fleet availability must not be exceeded

$$\sum_{pq} Tb_{pqa} \cdot n_{pqa} \leq U_a \cdot A'_a \qquad \forall a$$

3. Daily aircraft movements must balance at a station

$$\sum_{q} n_{pqa} - \sum_{q} n_{qpa} = 0 \qquad \forall p,a$$

4. Minimum daily frequency on each route

$$\sum_{a} n_{pqa} \geq N\,min_{pq} \qquad \forall pq$$

5. Maximum daily departures at a station

$$\sum_{a} \sum_{q} n_{pqa} \leq N\,max_p \qquad \forall p$$

FIG. 4. MARKET SHARE MODEL – FA3

2.3.3 Problem Size

a) Number of rows $\leq 3|E| + |a| + |Q|$

Constraints 1.a) $|E|$ = no. of links pq in
route map

b) $|E|$

c) $|E|$

2. $|a|$ = no. of aircraft types

3. (Omitted)

4. (Omitted)

· 5. $|Q|$ = no. of cities in route map

b) Number of variables $\leq |E| \cdot (|a| + \bar{K} + 1)$

Variables n_{pqa} : $|E| \cdot |a|$

n_{pqa}^{k} : $|E| \cdot \bar{K}$ where $\bar{K}$ = average no.
of breakpoints in market share
curve for city pairs

P_{pq} : $|E|$

c) Number of bounds $\leq |E| \cdot \bar{K}$

Constraints 1d) $|E| \cdot \bar{K}$

e.g. For 20 cities, 200 city pair links, 5 aircraft types,
and $\bar{K} = 3$

Constraints	$\leq$	600 + 5 + 20	= 625
Variables	$\leq$	200 (5+3+1)	= 1800
Bounds	$\leq$	600	

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2.3.4. Comments

This model links together a traffic forecasting model for market share, with a schedule construction model. The passengers carried, the system revenue and market load factor are all variables in the optimization. The previous load factor assumption is not used, except as an upper bound constraint upon physically realizable load factors. Without this constraint, the usage of a small size vehicle on a large, busy route could lead to load factors over 100%.

Fare structure now becomes an important parameter for the model since it determines the levels of service to be offered by a profit seeking carrier. If subsidy income can be related to passengers carried and daily frequencies by type of aircraft, it also becomes a parameter which determines service levels. These problems would be of interest to industry planners and regulators.

The traffic-frequency curves, or market share curves have to be known for all competitive routes pq. To calculate them, the rule of thumb "market share = frequency share" could be used, with an estimate of expected competitive frequencies. The market share for the particular airline system may then be directly computed and approximated by the step-wise linear curve. Better market share predictions may, of course, be available, and the model creates an incentive for obtaining good data.

The minimum frequency constraints of previous

models are no longer necessary on a competitive route. (In fact, they were a particular form of traffic-frequency curve as illustrated in Figure 5a.) However, for political or strategic reasons, the carrier may wish to maintain a minimum frequency which might be uneconomic at present. This is shown in Figure 5b. The breakpoints shown in that figure are the usual points in the optimal solution, although any point on the boundaries is possible. One can put a breakpoint for every integer frequency, and thereby obtain integer solutions.

The initial value p^1_{pq} for a given market share curve represents the load which is generated by offering one service frequency. If it is less than the (max. load factor x seating capacity) of the smallest plane on the route, then constraints 1a) are not necessary. Also, to be "economic" any p value must be greater than the lowest breakeven load for any airplane;

i.e. $$p^k_{pq} \geq \frac{DC_{pqa}}{y_{pq}} \quad \text{for some a on pq}$$

If this is not true, then the demand curve may be truncated by dropping segment k and any subsequent segments.

The actual load factor achieved on a given competitive route is now output from the model.

$$LF_{pq} = \frac{P_{pq}}{\sum_a S_a \cdot n_{pqa}}$$

The model usually indicates optimal load factors which are less than the maximum allowable for highly competitive routes.
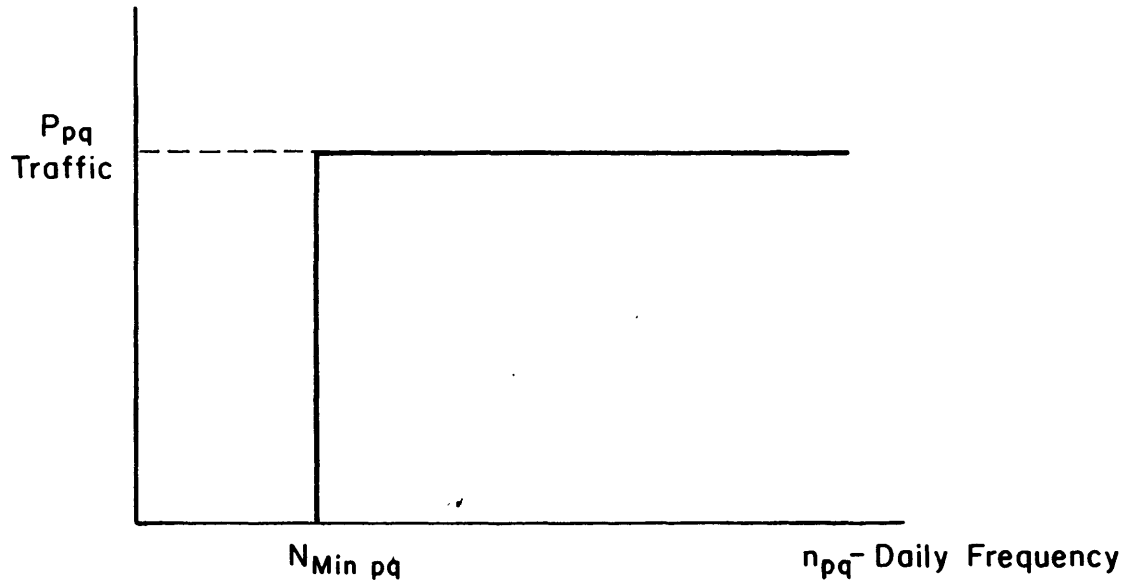
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FIG. 5a MINIMUM FREQUENCY MARKET SHARE CURVES
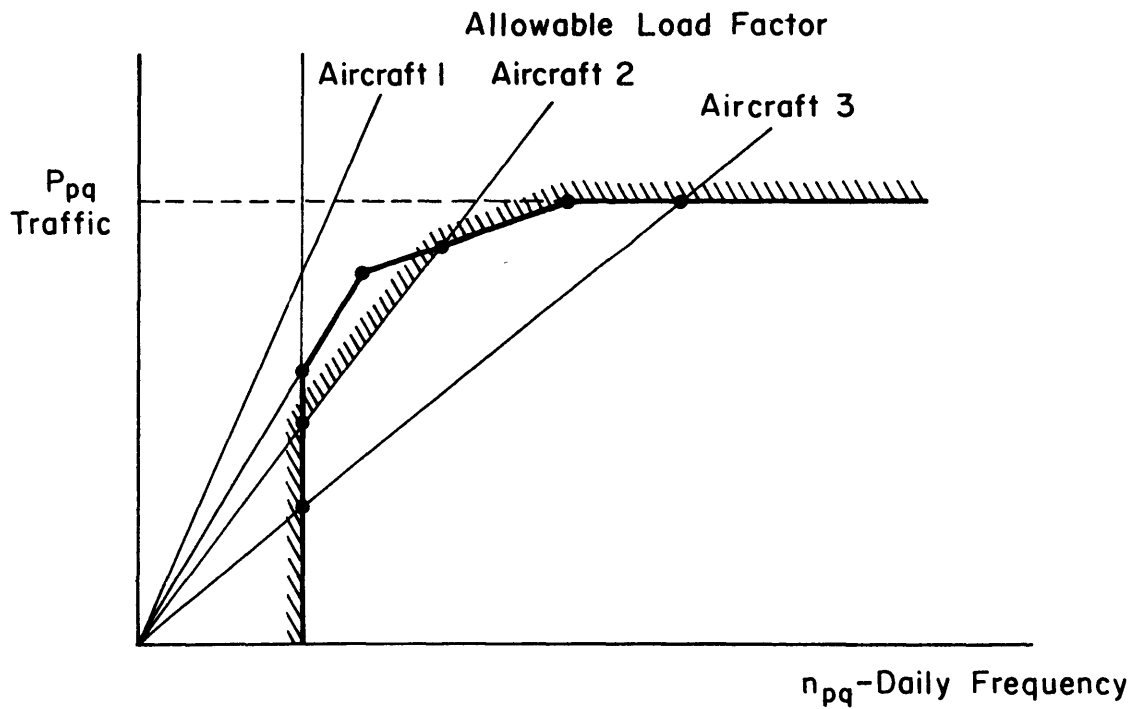
FIG. 5b BREAKPOINTS FOR MARKET SHARE CURVES

2.4 Tech Airways Example - Market Share

An example of model FA-3, the market share model, is given here. It is a homework problem for a Tech Airways system of 1990. The route map is shown in Figure 6b; it has 24 non-stop routes connecting 10 cities. Four types of aircraft are considered, and a description of their economic performance, along with assumed net yield, allowable load factor, etc. is given in Table 2.

Typical market share curves are given by Figure 6a and the market size is given for each route along with the results for the route in Table 4. There were four cases of different fleet sizes as described in Table 3.

The LP problem size was 148 rows and 432 variables. The four cases were solved with MPS on an IBM 360-65 in less than one minute. The results in Table 4 give the passengers carried, the market load factor, and the daily frequency (integerized) for each route. Notice that type of aircraft and frequency of service change quite considerably as the fleet sizes were increased. Also, because of competitive reasons, a number of the routes are flown at less than the maximum allowable load factor, and not all the passengers available to the system are necessarily carried.
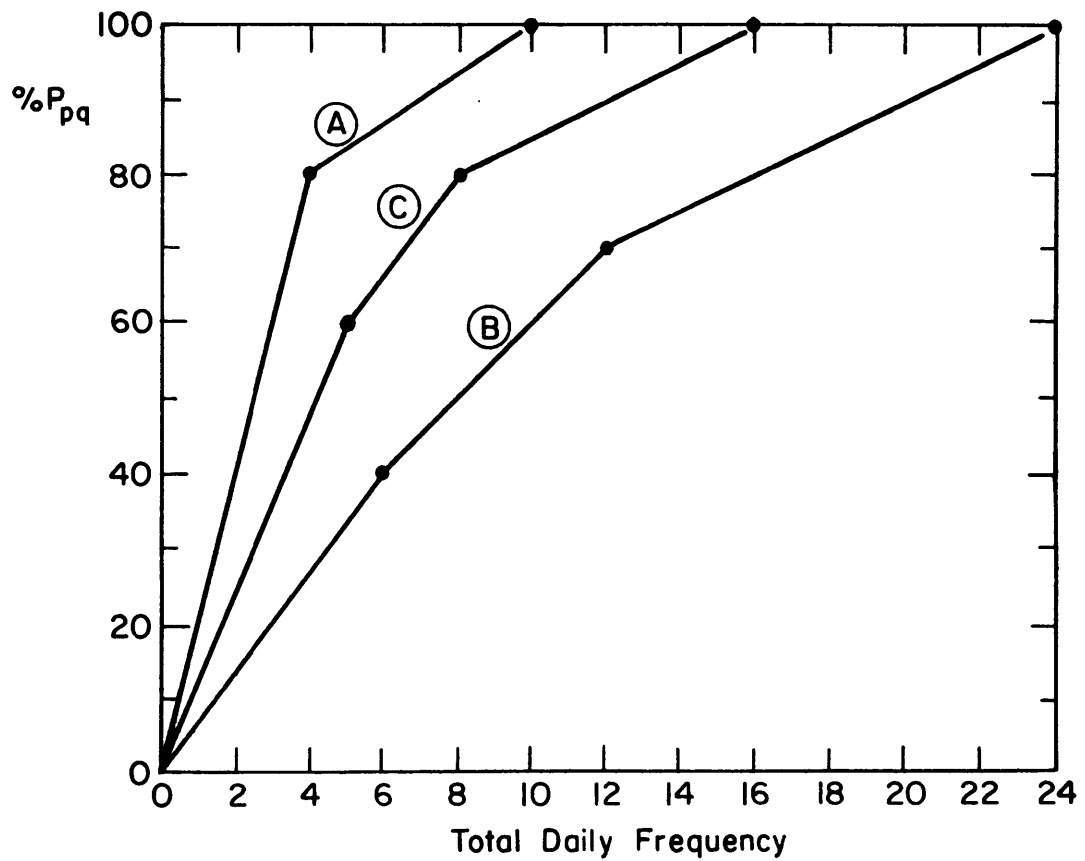
FIG. 6a MARKET COMPETITION CURVES – TECH. AIRWAYS

    Ⓐ LONG HAUL ROUTES – FROM CITIES I AND 2
        TO ANY OTHER CITY

    Ⓑ COMPETITIVE SHORT HAUL – ROUTES 4-9,
        9-10, 8-9

    Ⓒ SHORT HAUL ROUTES – ALL THE REMAINDER

SEATTLE

1

2

PORTLAND

1700 MILES

DETROIT

10

CHICAGO

6

OMAHA

3

9

7

5

CINCINATTI

DESMOINES

4

8 ST. LOUIS

KANSAS CITY

DAILY DEMAND

━━━━━ >400

───── 100 – 399

──── 50 – 99

(blank) < 50

SCALE

250 MILES
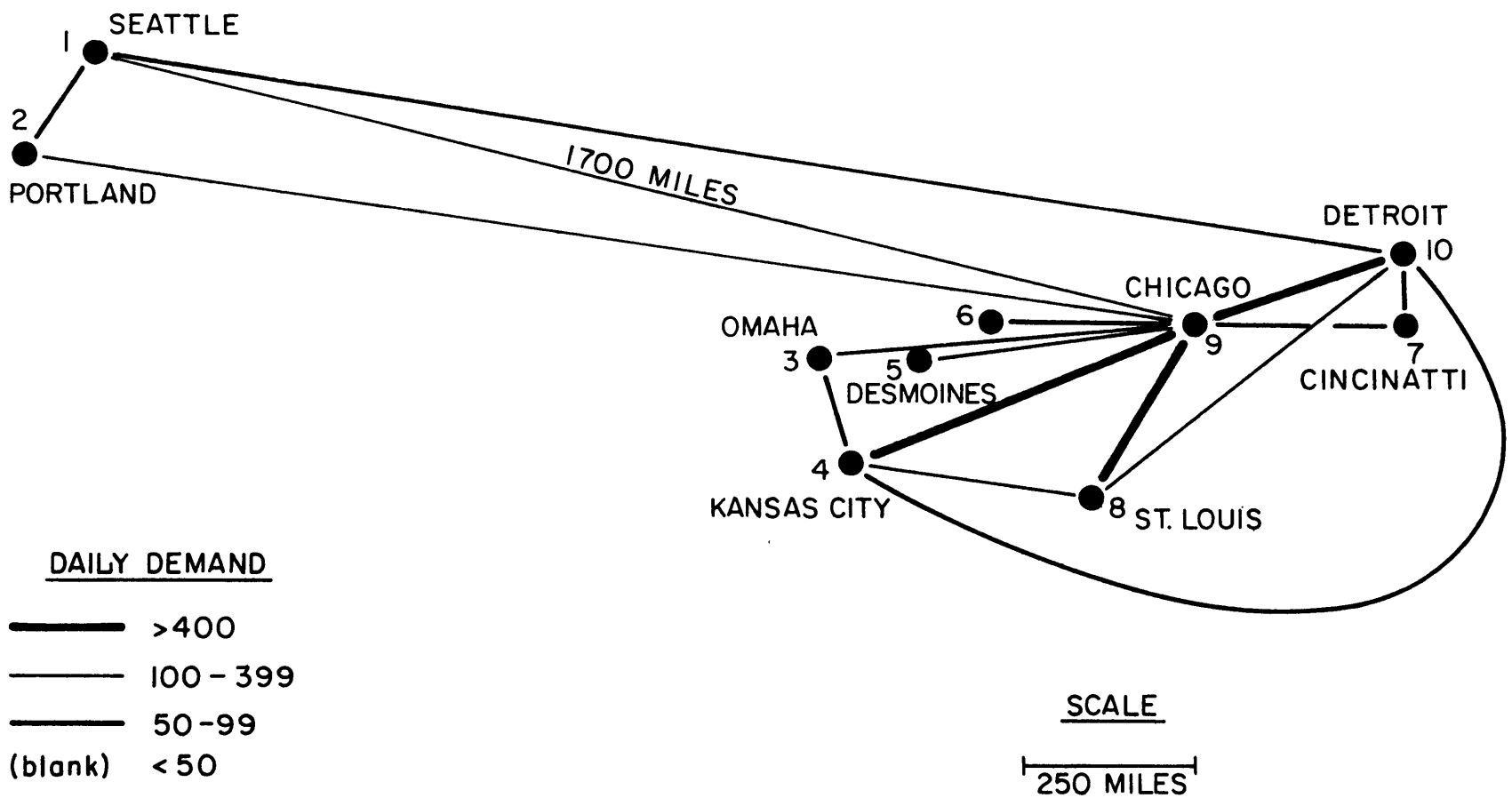
FIG. 6b. ROUTE MAP – TECH AIRWAYS

TABLE 2

Tech Airways Data

1. Aircraft Characteristics

Type	DOC $1/hr.	Capacity seats	Utilization hrs./day	Block time hrs.	Max. Range miles
DC-9	450	90	7	.179+.00207D	1000
B727	550	120	8	.226+.00185D	2000
B707	800	150	9	.300+.00192D	3000
DC10	1050	300	10	.300+.00175D	3000

$$D = Distance$$

2. Net Yield = Fares - less indirect operating costs

$$= \$1.00 + 0.060D$$

3. Max. Allowable Load Factor   all routes = 65%

4. Ownership Costs/Day

DC-9	$600
B-727	$700
B-707	$1000
DC-10	$2000

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TABLE 3

FLEET AVAILABILITY CASES

	DC-9	DC-10	B727	B707	Results
Case #1	25	10	40	20	All aircraft were fully utilized
Case #2	43	12	50	28	Only 16 B707 were used
Case #3	56	14	60	20	Only 9 B707, and 50 DC-9 were used
Case #4	∞ (30)	∞ (28)	∞ (34)	∞ (9)	Unlimited aircraft available. Optimal no. of each aircraft shown ( )

TABLE 4. TECH AIRWAYS RESULTS

CASE	#1	#2	#3	#4
Route 1-3 - 1188 mi.;Mkt.size-119				
Passengers	0	35	95	95
Load Factor %	0	29	20	20
# of Flights:				
DC-10	0	0	0	0
B 727	0	1	4	4
B 707	0	0	0	0
Route 1-10 - 1972 mi.;Mkt.size-450				
Passengers	360	371	371	371
Load Factor %	65	65	65	65
# of Flights:				
DC-10	0	0	0	0
B 727	2	5	5	5
B 707	2	0	0	0
Route 3-8 - 447 mi.;Mkt. size-298				
Passengers	0	184	244	244
Load Factor %	0	41	34	34
# of Flights:				
DC-9	0	5	8	8
DC-10	0	0	0	0
B 727	0	0	0	0
B 707	0	0	0	0
Route 4-8 - 353 mi.;Mkt.size-1080				
Passengers	950	1050	1050	1050
Load Factor %	65	65	65	65
# of Flights:				
DC-9	0	10	10	10
DC-10	0	0	0	0
B 727	12	6	6	6
B 707	0	0	0	0
Route 4-10 - 854 mi.;Mkt.size-748				
Passengers	598	657	748	748
Load Factor %	65	65	65	65
# of Flights:				
DC-9	5	11	16	16
DC-10	0	0	0	0
B 727	0	0	0	0
B 707	3	0	0	0
Route 8-10 - 570 mi.;Mkt.size-1260				
Passengers	1114	1260	1260	1260
Load Factor %	65	65	65	65
# of Flights:				
DC-9	0	0	0	0
DC-10	0	0	0	0
B 727	0	15	16	16
B 707	11	1	0	0

CASE	#1	#2	#3	#4
Route 1-9 - 1697 mi.;Mkt.size-1840				
Passengers	1840	1840	1840	1840
Load Factor %	65	65	65	65
# of Flights:				
DC-10	9	9	9	9
B 727	0	0	1	1
B 707	1	1	0	0
Route 2-9 - 1718 mi.;Mkt.size-1100				
Passengers	1100	1100	1100	1100
Load Factor %	65	65	65	65
# of Flights:				
DC-10	1	1	3	3
B 727	0	0	7	7
B 707	9	9	0	0
Route 3-9 - 552 mi.;Mkt.size-1320				
Passengers	1295	1321	1321	1321
Load Factor %	65	65	65	65
# of Flights:				
DC-9	0	0	0	0
DC-10	0	0	1	1
B 727	10	12	15	15
B 707	5	4	0	0
Route 4-9 - 538 mi.;Mkt.size-3772				
Passengers	3772	3772	3772	3772
Load Factor %	65	65	65	65
# of Flights:				
DC-9	0	0	0	0
DC-10	14	15	16	16
B 727	0	0	8	8
B 707	11	9	0	0
Route 8-9 - 291 mi.;Mkt.size-7750				
Passengers	5319	7633	7633	7633
Load Factor %	65	65	65	65
# of Flights:				
DC-9	90	85	92	0
DC-10	0	0	0	39
B 727	0	34	28	0
B 707	0	0	0	0
Route 9-10 - 316 mi.;Mkt.size-10350				
Passengers	10350	10350	10350	10350
Load Factor %	65	65	65	65
# of Flights:				
DC-9	1	0	0	0
DC-10	0	10	12	53
B 727	132	107	103	0
B 707	0	0	0	0

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2.5 BIBLIOGRAPHY - Fleet Assignment Models

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3.0 Fleet Planning Models - (LP Decomposition)

Fleet Planning models are concerned with determining an
optimal program for buying, selling and leasing aircraft for
the airline system over some future planning horizon. They are
extensions of the Fleet Assignment models in that they repeat
some basic fleet assignment problem over multiple periods, T_i,
which make up the planning horizon. Typically, the periods
may be a year, (or a peak or off-peak season), and the planning
horizon extends perhaps 5-7 years into the future. Because of
a need for expressing fleet continuity between these periods,
the fleet assignment problem for a given time period is not
independent of other time periods. Fleet composition at the
end of one time period becomes the available fleet for the
next time period.

These models tend to be strategic models by nature. The
details of each period may not be fully represented in that
individual routes may be aggregated to form classes of routes
such as short: medium: long haul or domestic: international —
and aircraft may be similarly grouped into 1960; 1970; 1980, or
short: medium: long haul — subsonic jet, supersonic transport,
or V/STOL - 40: 80: 120 passenger, etc.

The models do require good forecasts of future traffic on
the routes, of future competition, future yields, future
availability and price of newer types of aircraft, future
selling prices for present aircraft, etc. Given such data,
an optimal fleet plan program can be determined, and sensiti-
vity of that program to variations in the forecast easily
determined.

The models are useful to transport aircraft manufacturers
in studying industry wide needs for present or newer types
of aircraft, and in investigating the introduction of new air-
craft versus possible new aircraft programs initiated by their
manufacturing competitors. It also becomes an aid in the
sales approach to a prospective airline system in studying
their future needs, and in determining the value of possible
interim leasing arrangements which could be offered.

For the operator, the models are useful for longer range
planning of fleet requirements, and in carrying out the
financial planning associated with buying, selling and
leasing aircraft.

The models are also useful to government planners and
regulators in studying the effects of new route awards,
subsidy programs, future airport loadings, industry profitability,
and introduction of new aircraft such as the jumbo jet or SST.

3.1 Model FP-3 Maximum Income Fleet Planning - (LP)

3.1.1 Problem Statement

Given a forecast of traffic and competitive frequencies for a set of routes covering some future set of time periods and a forecast of prices for buying, selling and pleasing available types of aircraft, determine the programs for owning, operating, and financing the airline fleet which maximize reported profit.

3.1.2 Model Formulation

This model is an extension of the basic Fleet Assign-
ment model FA-3, using r instead of pq to designate non-stop
routes. The basic model is repeated over a sequence of peri-
ods T using forecasts of future data describing an average
cycle of that period. Let n_{raT} be the average number of
frequencies/cycle operated on route r by aircraft a in peri-
od T, and let NCP be the number of cycles per period.

Aircraft are assumed to be bought at the beginning of
a period. Let A_{paT} be the number of aircraft a bought under
purchase plan p. This purchase plan consists of a schedule
of present value progress payments PP_{paT} to the manufacturer
prior to delivery and a schedule of depreciation costs DEP_{paT}
to be used for accounting purposes subsequent to delivery.
There may be a number of such purchase plans corresponding
to a given aircraft purchase.

Aircraft are assumed to be leased at the beginning
of a time period. Let l_{la} be the number of aircraft a
leased under lease arrangement l. This lease arrangement
describes the flow of cash payments to the leasing agency
for aircraft a during the periods T_l associated with the
lease. Let LP_{laT} be the present value of a payment.

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Aircraft are assumed to be sold at the beginning

of a time period. Let s_{paT} be the number of aircraft a

purchased under plan p sold at the beginning of period T.

The market value for selling aircraft a in period T is

forecast to be MV_{aT}, and the depreciated book value BV_{paT}.

The system indirect operating costs for period T, IC_T,

may be expressed as a linear function of system variables

such as revenue passenger miles, RPM_T, passengers boarded,

P_T, and total system departures, D,

$$RPM_T = \sum_r d_r \cdot P_{rT} \qquad \text{where } d_r = \text{route distance}$$

$$P_T = \sum_r P_{rT}$$

$$D = \sum_{r \in R_{-ij}} \sum_a n_{raT}$$

$$IC_T = c_2 + c_3 \cdot P_T + c_4 \cdot D + c_5 \cdot RPM_T$$

The c values are cost coefficients determined for the

particular airline system. This expression gives the present

value of system indirect costs per cycle, and is multiplied

by NCP.

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Interest or debt cost payments per period are scheduled under the repayment plan described by IP_{iT}, the proportion of the debt borrowing, d_i covering periods T_i which is paid as debt cost in period T. Fixed debt plans already obligated can be grouped together under one schedule to initiate the model.

Objective Function

Find the program of buying, selling, leasing, and operating aircraft, and a financial program for raising new capital which maximizes the present value of system profit before taxes over some future planning horizon.

$$\text{Maximize } Z = \sum_T Z_T$$

where Z_T = before tax profit (or loss) in period T

$$= NCP \sum_r y_{rT} \cdot P_{rT} \qquad \text{System revenues/period}$$

$$- NCP \cdot \sum_r \sum_a DC_{ra} \cdot n_{raT} \qquad \text{System DOC (less depreciation)/period}$$

$$- NCP \cdot IC_T \qquad \text{System IOC/period}$$

$$- \sum_l \sum_a LP_{laT} \cdot L_{laT} \qquad \text{Aircraft leasing costs/period}$$

$$- \sum_i IP_{iT} \cdot d_i \qquad \text{Debt costs/period}$$

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$$- \sum_p \sum_a DEP_{paT} \cdot A_{paT} \qquad \text{Aircraft Depreciation Costs/period}$$

$$+ \sum_p \sum_a (MV_{paT} - BV_{paT}) \cdot s_{paT} \qquad \text{Capital gains from Aircraft Sales}$$

Constraints

1a) Maximum allowable load factor for the route

$$\sum_a LF\ max_{raT} \cdot S_a \cdot n_{raT} - P_{rT} \geqslant 0 \qquad \forall\ r,\ T$$

1b) Traffic carried is a function of daily frequency

$$\sum_k p^k_{rT} \cdot n^k_{rT} - P_{rT} \geqslant 0 \qquad \forall\ r,\ T$$

1c) Total daily frequency is sum of segment frequencies

$$\sum_a n_{raT} - \sum_k n^k_{rT} = 0 \qquad \forall\ r,\ T$$

1d) Segment frequencies are bounded at the breakpoints

$$0 \leqslant n^k \leqslant n^r_k - n^r_{k-1} \qquad \forall\ r,\ k,\ T$$

2) Fleet Availability

$$Tb_{raT} \cdot n_{raT} - U_{aT} \cdot (\sum_p A_{paT} + \sum_l L_{laT}) \leqslant 0 \qquad \forall\ a, T$$

where U_{aT} is the forecast of aircraft average block times/ cycle for fleet a in period T.

4) A minimum frequency of service may be required on a route

$$\sum_a n_{raT} \geqslant N \min_{rT} \qquad \forall\, r,\, T$$

where $N \min_{rT}$ = minimum frequency of service on route r, period T

5) A maximum for daily departures at a station exists

$$\sum_a \sum_q \sum_{r \varepsilon R'_{pq}} n_{raT} \leqslant N \max_{pT} \qquad \forall\, p,\, T$$

where $N \max_{pt}$ = maximum daily or cycle operations at station p in year T due to airport congestion

6) Fleet Continuity is assured

a) Purchased Aircraft

$$A_{paT} - (A_{paT-1} - s_{paT}) = 0 \qquad \forall\, p, a, T > T_{ps}$$

For the initial period, the initial fleet is described by its residual payment and depreciation schedules. Initial fleet size and the prospective short term deliveries of aircraft on order are controlled by bounds on A_{paT}.

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b) Leased Aircraft

$$L_{laT} - L_{la} = 0 \qquad \forall \; l, \; a, \; T \in T_l$$

If L_{la} aircraft are leased under lease l, it equals the number available to the system in a year T_l of the lease, L_{laT}.

7) Airline Financial Constraints

There are a variety of financial constraints which may be placed upon a model of the airline financing problems. Here we shall insist that "cash on hand" remain above a given level, and that new debt can be incurred only such that the ratio of total debt to total assets remains below a given value.

The cash income for the system differs from before tax profit because of the variance between declared depreciation schedules and ownership costs through progress payments and debt costs, and between book value and market value. Thus the cash flow in period T, CF_T becomes:

$$CF_T = Z_T + \sum_p \sum_a (DEP_{paT} - PP_{paT}) \cdot A_{paT} \quad \text{depreciation or progress payments}$$

$$+ \sum_p \sum_a BV_{paT} \cdot s_{paT} \quad \text{book value on aircraft sales}$$

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$$- \sum_i DP_{iT} \cdot d_i \qquad\qquad\qquad \text{principal payments on debt}$$

$$- TD_T \cdot Z_T \qquad\qquad\qquad\qquad \text{taxes and dividends}$$

$$+ \sum_i d_i \qquad\qquad\qquad\qquad \text{new capital raised in T}$$

Thus, cash flow is before tax profit with the hypothetical costs of aircraft depreciation and book value removed, and the real costs of progress payments, principal payments on debt, and taxes introduced. Capital raised through new debt is treated as cash income.

We now write a constraint defining the average cash on hand for the system. The cash flow, CF_T, is allowed to be positive or negative for a given period, providing cash on hand does not fall below a required level, expressed as a lower bound on CH_T.

7a) Cash on Hand Constraint

$$CH_T - (CH_{T-1} + CF_{T-1}) = 0 \qquad\qquad \Upsilon\ T$$

There is a limit on the ability of the system to raise new capital. Here this is expressed as a limit on the ratio of debt to total airline assets, D/A, commonly called the debt/asset ratio. It is equivalent to another ratio called the debt/equity ratio.

The assets of an airline can be expressed as the sum of cash on hand (or current assets), non-aircraft assets,

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and aircraft assets expressed as a program of progress payments and book value for each aircraft purchase. Typically the aircraft assets represent about 85% of the total.

The long term debts of an airline for a period can be expressed in terms of the repayment schedules for debts d_i which the airline has incurred in the past, and will incur in its future.

7b) Debt-Asset Constraint

$$D/A \left(\sum_p \sum_a (PP_{paT} + BV_{paT}) \cdot A_{paT} + CH_T + A^*_T \right) - \sum_i DV_{iT} d_i \gtrless 0 \quad \forall T$$

This constraint restricts the debt program from exceeding a desired ratio of the asset program for all periods T. If it becomes tight, the airline cannot raise new capital and must be able to maintain cash on hand levels either through profitability, or selling of aircraft.

Bounds

1) The number of aircraft which are purchased in any plan is subject to upper and lower bounds

$$BL_{pa} \lesssim A_{paT_{pl}} \lesssim BU_{pa} \qquad \forall a, p$$

The system may have incurred an obligation to purchase aircraft a in year T_{pl}. The upper bounds usually arise from availability of new aircraft in the early years of the manufacturing program.

2) The number sold in any year is restricted

$$SL_{paT} \leq s_{paT} \leq SU_{paT} \qquad \forall \; a, \; T, \; p$$

Management policies may restrict the selling program for the present fleet. Constraint 6a) places an upper bound on s_{paT} which prevents selling aircraft not in the fleet. A policy not to sell any of aircraft a before year T can change both the aircraft acquisition program and the operating program.

3) The number of aircraft available under any lease arrangement is restricted

$$LL_{la} \leq L_{la} \leq LU_{la} \qquad \forall \; l$$

4) The amount of debt which can be raised under any debt plan may be bounded

$$DL_i \leq d_i \leq DU_i \qquad \forall \; i$$

5) The cash on hand must not fall below a given desired level for period T.

$$CH_T \geq CH_{MIN_T} \qquad \forall \; T$$

3.1.3 Problem Size

a) Number of rows $\leqslant T \cdot (3 \, |E| + |a| + |Q|) +$
$$T \cdot (2 + |a| \cdot P \cdot \frac{(T-1)}{2}) + \overline{T}_1 \cdot |1|$$

Constraints

1.a)	$T \cdot |E|$	
.b)	$T \cdot |E|$	
.c)	$T \cdot |E|$	where P = number of purchase plans per period
2.	$T \cdot |a|$	
5.	$T \cdot |Q|$	$\overline{T}_1$ = average duration of lease
6.a)	$T \cdot |a| \cdot P \cdot (T-1)/2$	
.b)	$\overline{T}_1 \cdot |1|$	
7.a)	T	
.b)	T	

b) Number of variables $\leqslant T \cdot |E| \cdot (\overline{K} + |a| + 1) + T \cdot (2|a| \cdot P + I + 3)$
$$+ |1| \cdot (1 + \overline{T}_1)$$

Variables

n_{rT}^{k}	:	$T \cdot |E| \cdot \overline{K}$	
n_{raT}	:	$T \cdot |E| \cdot a$	where $\overline{K}$ = average number of segments per route
P_{rT}	:	$T \cdot |E|$	
A_{paT}	:	$T \cdot |a| \cdot P$	I = number of debt plans per period
s_{paT}	:	$T \cdot |a| \cdot P$	
L_{laT}	:	$\overline{T}_1 \cdot |1|$	
L_{la}	:	$|1|$	
d_i	:	$T \cdot I$	
CH_T	:	T	
CF_T	:	T	
z_T	:	T	

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c) Number of bounds $\quad T \cdot |E| \cdot \overline{K} + T \cdot (2|a| \cdot P + 2I + 1)$
$$+ 2|a| \cdot P + 2 \cdot |1|$$

Constraints 1d) $\quad T \cdot |E| \cdot \overline{K}$

Bounds
1. $\quad 2 |a| \cdot P$
2. $\quad 2 |a| \cdot P \cdot T$
3. $\quad 2 |1|$
4. $\quad T \cdot (2I)$
5. $\quad T$

e.g. Suppose we have the same problem as used in the Fleet Assignment chapter with 20 cities, 200 city pairs, 5 aircraft types, $\overline{K} = 3$, 5 lease arrangements averaging 4 periods duration, two types of purchase arrangements and two types of debt repayment for each aircraft purchase, and T = 10 periods.

Then, constraints $\quad 10(625) + 10(2 + 10.9/2) + 4(5) = 6740$

variables $\quad 10 \cdot 200 \cdot (3+5+1) + 10(20+5) + 5(1+4) = 18275$

bounds $\quad 10 \cdot 200 \cdot 3 + 10.2 \cdot (11) + 10 + 10(5) = 6280$

3.1.4 Comments

Various types of leasing arrangements may be
modelled. A lease-option arrangement makes little sense
in a deterministic model since if the option should be
exercised, it will be cheaper to purchase the aircraft
initially. However, often a manufacturer may respond
to a new aircraft from a competitor by offering a favor-
able lease of his present aircraft in the interim period
until his next new aircraft becomes available. By using
the bounds on leasing and buying, or by writing a con-
straint expressing the obligation to buy the new aircraft
as a linear function of the number on interim lease, such
offers may be incorporated in the model. Similarly, any
trade-in arrangement can be modelled where the manufacturer
agrees to purchase present fleet at guaranteed prices in
exchange for new aircraft sold to the airline.

There is an initial cost associated with intro-
ducing a new type of aircraft in the fleet which is not in
the model. As well, there are strong operating cost reasons
for not having two similar types of aircraft from different
manufacturers in the fleet. If problems associated with
these costs arise, they can be handled by parametrically
controlling fleet acquisition variables, A_{paT}.

Associated with a decision to buy aircraft is a
number of ways of raising the money required to make progress
payments and the delivery payment. Independent of a repay-
ment schedule for the loans required, there are progress
payments, and a depreciation schedule for the aircraft

which gives tax benefits, and perhaps an investment tax credit. It is possible to consider a variety of the possible financial planning problems in the model if it seems pertinent. Any financial constraint affects both the buying and selling programs, and also the operating program - the frequency pattern, aircraft used on a route, total traffic and revenue - all can be affected by these constraints.

The model uses a great deal of forecast information as input data, and the results generally are sensitive to the assumed forecasts. Since the data is uncertain, sensitivity testing should be performed.

Because of the sequential, periodic structure of the problem, this model is easily formulated as an LP decomposition problem. The optimal frequency pattern for each period could be determined independently, except that constraints 6, and 7 link the problem for various periods. The next section shows how this decomposition may be carried out since for most airline formulations, the size of this model would be beyond the capacity of present computer LP codings.

3.2 Model FPD-3 Maximum Income Fleet Planning - Decomposition

3.2.1 Problem Statement

This is identical to model FP-3.

3.2.2 Model Formulation

The decomposition may be carried out in a number of ways. Here we shall take constraints 2, as well as constraints 6, and 7 into the master problem. In this way the buying, selling, leasing variables, and the financial planning constraints are all contained in a master LP of reasonably small size. The large subproblems become frequency pattern determinations for each period. A Dantzig-Wolfe iteration technique shall be used since it seems to be preferable to any imbedded iteration technique.

Let λ_{Tj} be the fraction of the solution at iteration j for subproblem T to be used in the master problem formulation. Let Z_{Tj} be the true value of the subproblem objective function when the augmented dual costs are removed. The formulation for the master problem is given by figure 7a. The sets of bounds from model FP-3 are not shown. Constraints 0 ensure a feasible set of subproblem solutions.

Let π_{2aTj-1} be the dual variables associated with the fleet availability constraints 2 of the j-1 master problem solution. The cost of operating a given route is modified as shown in the objective function of figure 7b) such as to express the cost of the fleet constraints from the master in the subproblem solution.

OBJECTIVE FUNCTION – Maximize the present value of future income

$$\text{Maximize } Z = \sum_{T} \sum_{j} \lambda_{Tj} \cdot Z_{Tj} + \sum_{p} \sum_{a} (MV_{paT} - BV_{paT}) \cdot s_{paT}$$

$$- \sum_{p} \sum_{a} DEP_{paT} \cdot A_{paT} - \sum_{\ell} \sum_{a} LP_{\ell aT} L_{\ell at} - \sum_{i} IP_{iT} d_{i}$$

CONSTRAINTS

0. **Combine the sub-problem solutions**

$$\sum_{j} \lambda_{Tj} = 1 \qquad\qquad \forall\, T$$

2. **Fleet availability**

$$\sum_{j} \sum_{r} \lambda_{Tj} \cdot Tb_{raT} \cdot n_{raTj} - U_{aT} \left(\sum_{p} A_{paT} + \sum_{\ell} L_{\ell aT} \right) \le 0 \qquad \forall\, a, T$$

6.a) **Fleet continuity**

$$A_{paT} - (A_{paT-1} - s_{paT}) = 0 \qquad\qquad \forall\, p, a, T > T_{p1}$$

6.b) **Leased Fleet**

$$L_{\ell aT} - L_{\ell a} = 0 \qquad\qquad \forall\, \ell, a, T \in T_{\ell}$$

7.a) **Cash on Hand Constraint**

$$CH_{T+1} - (CH_{T} + CF_{T}) = 0 \qquad\qquad \forall\, T$$

$$\text{where } CF_{T} = \sum_{j} \lambda_{Tj} Z_{Tj} + \sum_{p} \sum_{a} ((DEP_{paT} - PP_{paT}) \cdot A_{paT} - BV_{paT} \cdot s_{paT}) - \sum_{i} DP_{iT} \cdot d_{i}$$

7.b) **Debt-Asset Constraint**

$$D/A \left(\sum_{p} \sum_{a} (PP_{paT} + BV_{paT}) \cdot A_{paT} + CH_{T} + A_{T}^{*} \right) - \sum_{i} DV_{iT} \cdot d_{i} \ge 0 \qquad \forall\, T$$

FIG. 7a THE MASTER PROBLEM, FLEET PLANNING MODEL FPD-3

OBJECTIVE FUNCTION – Maximize "operating income" at iteration j
for system in year T

$$\text{Max } Z^*_{Tj} = NCP\left(\sum_r y_{rT} \cdot P_{rTj} - \sum_r \sum_a (DC_{raT} + \pi_{2aTj-1} \cdot Tb_{raT}) \cdot n_{raTj} - IC_T\right)$$

where π_{2aTj-1} are dual prices from the master problem
for constraints 2.

CONSTRAINTS

l.a) Maximum allowable aircraft load factor for route

$$\sum_a LFmax_{raT} \cdot S_a \cdot n_{raTj} - P_{rTj} \geq 0 \qquad \gamma_r$$

l.b) Traffic carried is a function of total daily frequency

$$\sum_k P^k_{rT} \cdot n^k_{rTj} - P_{rTj} \geq 0 \qquad \gamma_r$$

l.c) Total daily frequency is sum of segment frequency

$$\sum_a n_{raTj} - \sum_k n^k_{rTj} = 0 \qquad \gamma_r$$

l.d) Segment frequencies are bounded at breakpoints

$$0 \leq n^k_r \leq n^r_k - n^r_{k-1} \qquad \gamma_{r,k}$$

4. Minimum daily frequency on each route

$$\sum_a n_{raTj} \geq Nmin_{rT} \qquad \gamma_r$$

5 Maximum daily departures at a station

$$\sum_q \sum_a \sum_{R'pq} n_{raTj} \leq Nmax_{pT} \qquad \gamma_p$$

FIG. 7b. THE SUB-PROBLEM FOR PERIOD T, MODEL FPD-3

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The iterative technique starts by finding a set of solutions to the subproblems which produce a feasible master solution. Then, the dual prices from the master are added to the subproblems, and starting from the old solutions, a new solution to each subproblem is obtained. If the new subproblem solution can be useful in improving the present master solution, a new variable λ_{Tj} of cost z_{Tj} is added to the master problem. The master is then solved starting from its old solution. This iteration cycle is continued until at some point no new subproblem solutions will improve the master problem, or if desired, until the improvements in the master problem are very small. Because of the similarity of the subproblem structures, and the weakness of the linking constraints, this problem is ideally suited for decomposition and seems to converge very fast.

3.2.3 Problem Size

Master Problem

a) Number of rows $\leqslant T \cdot (3 + |a| + |a| \cdot P \cdot \frac{(T-1)}{2}) + |l| \cdot \overline{T}_l$

Constraints   0. T

          2. $T \cdot |a|$

          6a.) $T \cdot |a| \cdot P \cdot \frac{(T-1)}{2}$

          b.) $\overline{T}_l \cdot |l|$

          7a.) T

          b.) T

b) Number of variables $\leqslant T \cdot (2|a| \cdot P + I + 2 + |j|)$

$$+ |l| \cdot (1 + \overline{T}_l)$$

Variables λ_{Tj} : $T \cdot |j|$

          A_{paT} : $T \cdot |a| \cdot P$

          s_{paT} : n

          L_{laT} : $\overline{T}_l \cdot |l|$

          L_{la} : $|l|$

          d_i : $T \cdot I$

          CH_T : T

          CF_T : T

c) Number of bounds $\leq T \cdot (2|a| \cdot P + 2I + 1) + 2|a| \cdot P$

$$+ 2|l|$$

Bounds   1. $2|a| \cdot P$

          2. $2|a| \cdot P \cdot T$

          3. $2|l|$

          4. $T \cdot 2I$

          5. T

Subproblem T_j

    a)   Number of rows $\leqslant 3\ |E| + |Q|$

            Constraints 1a) $|E|$

                        b) $|E|$

                        c) $|E|$

                        5. $|Q|$

    b)   Number of variables $\leqslant |E| \cdot (\overline{K} + |a| + 1)$

            Variables n_{rT}^{k} : $|E| \cdot \overline{K}$

                     n_{raT} : $|E| \cdot |a|$

                     P_{rT} : $|E|$

    c)   Number of bounds $\leqslant |E| \cdot \overline{K}$

            Constraints 1d) $|E| \cdot \overline{K}$

e.g.   for our example used with Model FP-3

Master Problem

Constraints $\leqslant 10\ (3 + 5 + \dfrac{10 \cdot 9}{2}) + 5(4)$ = 550

Variables $\leqslant 10\ (20+2+2+ j\ ) + 5(1+4)$ = $265 + 10|j|$

Bounds $\leqslant 10\ (20+4+1) + 20 + 10$ = 280

Subproblem

Constraints $\leqslant 600 + 20$ = 620

Variables $\leqslant 200\ (3+5+1)$ = 1800

Bounds $\leqslant 200\ (3)$ = 600

3.2.4 Comments

This decomposition is attractive since it creates a large subproblem solely concerned with determining an optimal frequency pattern for the system, and has a small master problem which contains all the essential components of fleet planning, and financial planning for the fleet. Given a solution j for subproblem T, the data needed for the master problem is only the true value of the objective Z_{Tj}, and usage of each aircraft type,

$$u_{aTj} = \sum_{r} Tb_{raT} \cdot n_{raTj}$$

Given the master problem solution, a new "operating" cost for every route is computed using the dual price associated with the availability constraint for each aircraft type.

As seen from the problem sizes for master and subproblem, they become computable within present codings. The master problem grows in number of variables as the iterations progress, but convergence seems fairly rapid. The subproblems are of same size as a basic Fleet Assignment model, and although there are T such problems to be solved at each iteration, the problems are similar. Old solutions can be saved to ensure that only a small number of simplex steps are required to obtain a new solution.

3.3   Example - Tech Airways Decomposition

The previous Tech Airways example is extended over three successive periods (years) assuming an average growth in traffic of 14% on all routes. Table 5 shows an assumed initial fleet composition, the prices for buying, selling, or leasing, and the constraints on buying, selling, or leasing. There is a 3 year lease available for either 8 B727's or 5 B707's and a lease for one DC-10 over the last two years.

The results are shown in Table 6. It shows that B707's were sold off as fast as possible during the period. These results are a function of assumed buying, selling, and leasing prices which are not realistic. One of the routes is shown to indicate how frequency patterns can change throughout the period. The route has roughly 16 flights/day on the average for each year , with larger sized aircraft being introduced as demand grows.

The decomposition technique required 10 iterations for this problem with the optimal value available on iteration 6. The subproblem took 221 simplex steps initially, a few dozen steps over the first two iterations, and no more than 9 simplex steps were needed over the remaining iterations. The master problem required 17 steps on the first two iterations, and less than 6 on the next two, and only one step on the remaining iterations.

Table 5 - Tech Airways Fleet Planning Constraints

	Fleet Size	PP $/day	Buying Constraints, BU Year 1, 2, 3	SP $/day	Selling Constraints, SU Year 1, 2, 3	LP $/day	Leases Year 1, 2, 3
	20	800	∞, ∞, ∞	400	∞, ∞, ∞	600	5 — 5 0
7	40	900	5, 5, 0	500	5, 5, ∞	700	8 — 8 — 8
7	15	1500	5, 5, 0	800	5, 5, ∞	1000	5 — 5 — 5
)	0	2500	5, 5, 5	1800	0, 0, 0	2000	0 1 — 1

Table 6 - Fleet Planning Results

6.1 Aircraft Acquisition Program

	Initial Fleet	Year	1			2			3		
			b	s	l	b	s	l	b	s	l
DC-9	20		7.3								
B-727	40		5			5					
B-707	15			5			5			5	
DC-10	0		5			5		1	5		1

6.2 Typical Route Frequency Assignment

Route 3-9   Average Flights/day

	Year	1	2	3
DC-9		6.7	0	0
B-727		9.3	15.4	13.3
B-707		0	0	0
DC-10		0	.7	2.7

3.4 BIBLIOGRAPHY-Fleet Planning Models

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4.0 Dispatching Models - (DP)

In the Fleet Assignment and Planning models, time of day did not enter as a problem variable. We now introduce it in the simplest way. Dispatching models are concerned with problems of determining an optimal time pattern for departures at a single station and along a single route.

In the schedule construction process, one may have determined an optimal frequency pattern for the system, and the next step is to find a good set of times for dispatching n_{pq} flights on route pq. If this is done route by route without any regard for network considerations, an "initial timetable" is constructed. This step is generally performed manually although a number of initial timetable generators have been coded. The dispatching models pose some optimization problems which can be solved (generally using dynamic programming), and which may be useful in such generators.

As basic input data for all models, a time of day variation in demand $P_{pq}(t)$ is assumed known. This may be viewed as the arrival distribution at the station of passengers for a non-reservation system, or as the latent time distribution of demand in the absence of any fixed timetable. Traffic for any scheduled service can be generated from this distribution using various assumptions, eg. all demand arrives and awaits the next schedule time, all demand is attracted to the nearest scheduled service, etc. A loss in demand may be incorporated as a function of waiting time for service. All of these considerations effectively become a traffic forecasting model for the route as a function of the dispatch schedule and its level of service.

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Competitive services could also be assumed, in which case a "market share" traffic model would be necessary. The weakness of dispatching models lies in the lack of good analytical traffic models for the various airline situations which exist in the real world. If such models do not exist, then human judgement will be applied in determining good service times for each route. The existence of methods to select a good set of service times given various constraints creates a need for research into developing good traffic forecasting and market share models as a function of time of day schedule patterns.

4.1 Model D-1 Minimum Social Cost (DP)

4.1.1 Problem Statement

Given a time of day distribution $P_{pq}(t)$ for non-stop service on route pq, find the number of dispatches and their daily pattern which minimizes a weighted sum of costs for operating a dispatch and passenger waiting time.

4.1.2 Model Formulation

The associated network for the dynamic programming formulation of this model is shown in Figure 8. The stage variable is time, t, and the single state variable, y, is the number of passengers waiting for service at time t. The decisions at almost every state node are "dispatch" or "no-dispatch" and are represented by the arcs of the network.

The "no-dispatch" arcs go from a state y at time t to a state $(y + p_t)$ at time t + 1, where p_t is the number of passengers due to arrive in interval (t, t + 1) determined from $P_{pq}(t)$. Since p_t may be predetermined, the network can be preconstructed, and is shown for constant p_t in Figure 8. The cost of a "no dispatch" arc can also be precalculated. If it is to be a linear function of passenger waiting time, it takes the form

$$c_{ij} = K(y + \tfrac{1}{2}p_t) \cdot \Delta t \qquad \text{where } K = \$/\text{minute}$$

The 'dispatch' arcs go from state y, to the greater of y = 0, or $y = y-S_a$ for a dispatch at time t. The arc cost is given as DC_{pq}, an operating cost for a dispatch.

If the network is preconstructed, the problem is simply a least cost path problem from state (0,0) to state (0,T).
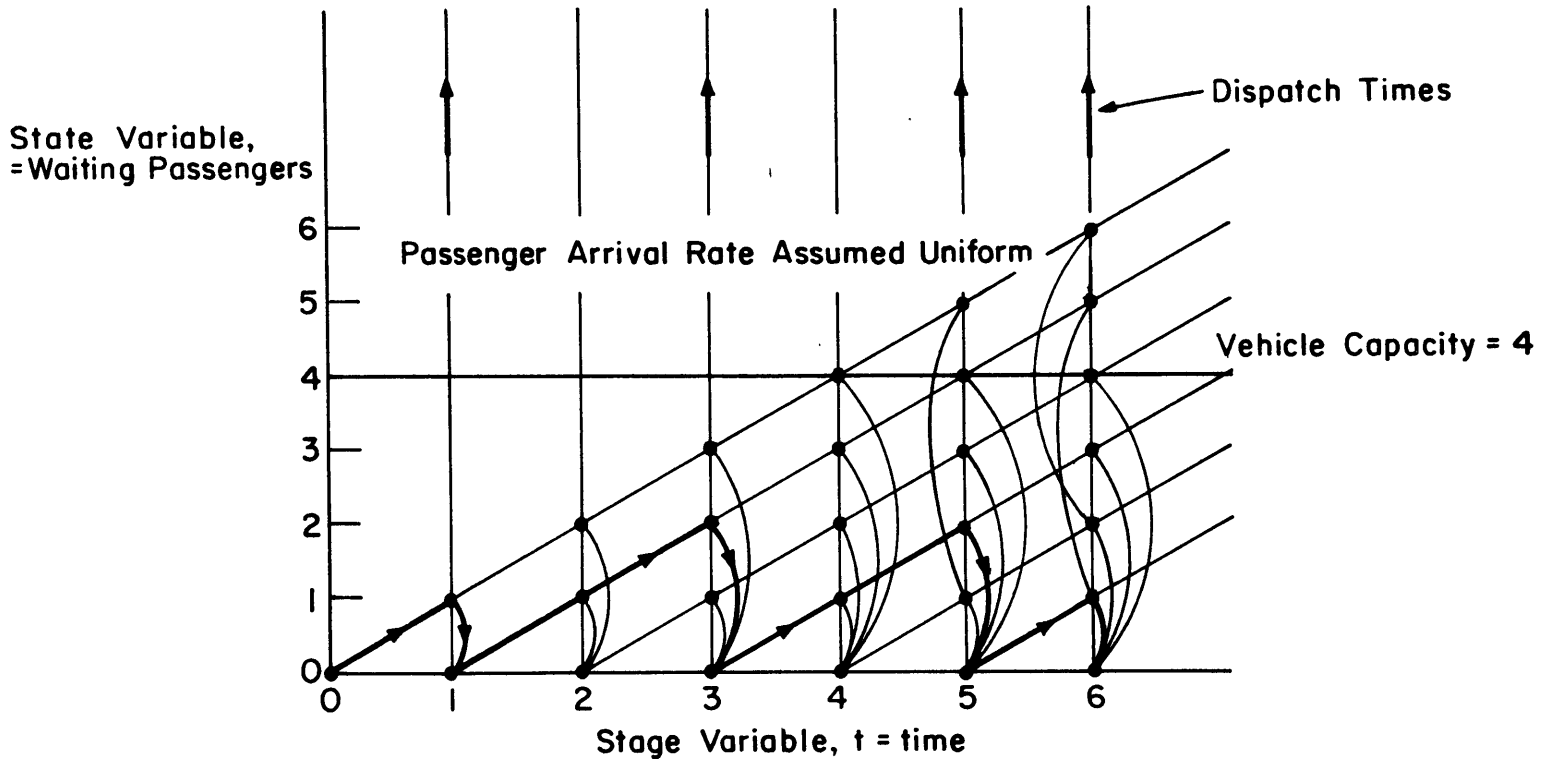
FIG. 8 NETWORK FOR DISPATCHING PROBLEM

4.1.3 Problem Size

State Space - If dynamic programming is used, the dominant storage space requirement is a high speed memory space for twice the number of nodes at a given stage. This is $N_H = 2P_{max}$ where P_{max} is the maximum number of passengers allowed to be waiting. There will be $N_h = T \cdot P_{max}$ nodes in the associated network where T is the maximum stage value, and secondary storage of at least this size must be provided.

Arcs - There are generally two arcs for every node (except for the nodes $y = 0$, $y = P_{max}$). A computation time can be associated with each arc.

eg. If we use 5 minute intervals between 6 am. and 12 pm. T = 216, and if P_{max} = 100 passengers, there are 21,600 nodes and 43,200 arcs in the preconstructed network of figure 8. Dynamic programming would require at least 21,600 storage spaces to determine the optimal path. The high speed memory requirement is only 200.

4.1.4 Comments

There are several extensions to this model where the delay cost becomes non-linear, more than one vehicle is dispatched at a given time, different kinds of vehicles and dispatching costs exist, a line of stations is collapsed into a collective $P_{pq_1 q_2}$ .. for dispatching along the line, etc. See reference 1 of the dispatching model references.

The model effectively assumes an infinite supply of different types of vehicles at p. It ignores the cost of owning the vehicle, or the arrival of vehicles at p during the day. It assumes a fixed demand independent of the number or pattern of dispatches, and that passengers will wait for the next available seat.

4.2 Model D-2 Least Passenger Delay for n Dispatches

4.2.1 Problem Statement

Given a time of day demand distribution $P_{pq}(t)$ for a non-stop route pq find the daily pattern of n dispatch times which minimize passenger waiting time.

4.2.2 Model Formulation

The model can be formulated exactly as D-1, and a parametric or Lagrange variable added to the dispatch cost. By parametrically controlling this variable, the number of dispatches in D-1 can be controlled, and a few iterations will normally give exactly n dispatches.

However, it is shown in reference 3 that the optimum minimum waiting time distribution is a unique function of the first two dispatch times. An optimum dispatch pattern for n dispatches given an initial dispatch time can quickly be found by varying the second dispatch time until the n+1 dispatch occurs at the initial time of the next day's cycle. Then the initial dispatch time can be varied to find the optimal dispatching pattern. This computational technique is faster than dynamic programming but is less flexible in considering more than one vehicle, vehicle capacities, etc.

4.2.3 Comments

This model does create a good pattern of dispatching if exactly n_{pq} dispatches are desired, and has been used in generating initial timetables given the frequency pattern for a network. At high frequencies of daily dispatch, it is not significantly different from "equal load" dispatching patterns.
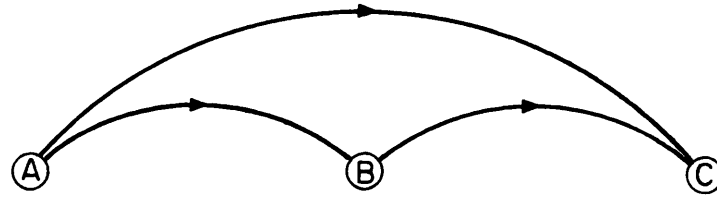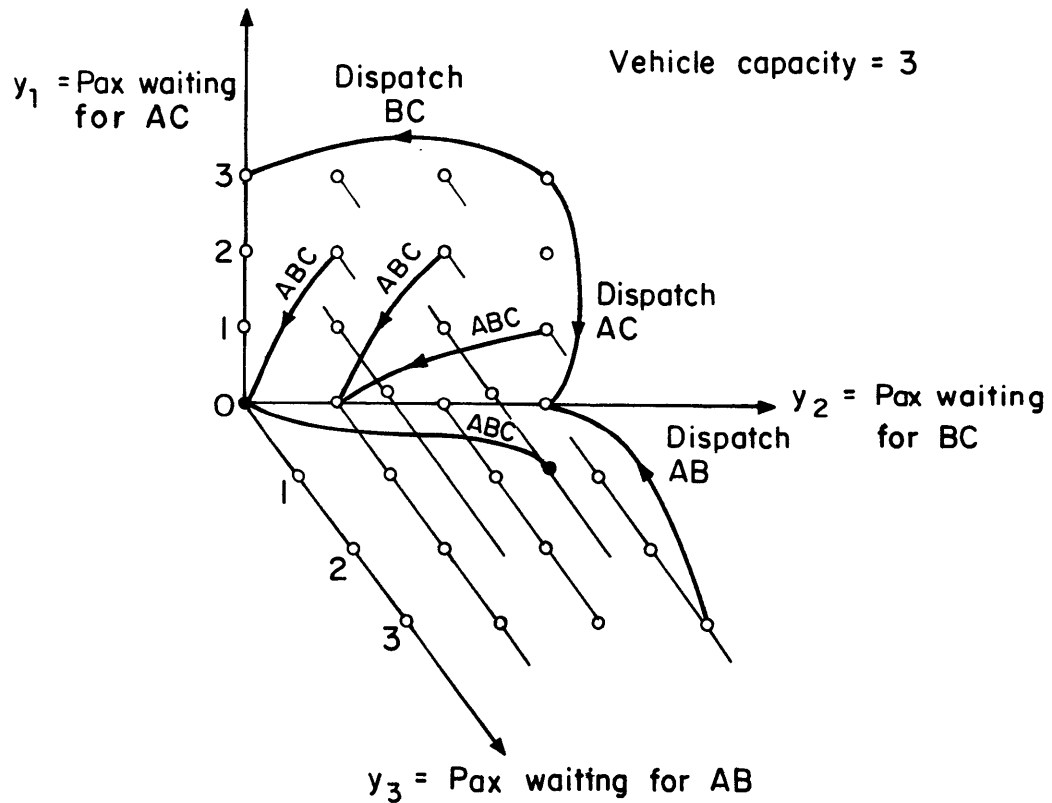
Demands AB, AC, BC
Dispatch Routings AB, AC, BC, ABC

FIG. 9 STATE SPACE DIAGRAM FOR MULTI-STOP DISPATCHING

4.3   Model D-3   Maximum Income (DP)

4.3.1   Problem Statement

Given a time of day distribution $P_{pq}(t,w)$ where the traffic carried is a function of time of day and passenger wait for service, w, find the number and daily pattern of dispatches which maximizes operator income.

4.3.2   Model Formulation

The passenger wait for service can be defined as a function of πd, the time from the previous dispatch on the tree path. This is a "path" variable in dynamic programming and a πd can be associated with every node in the tree. If $P_{pq}(t,w)$ is known, and πd is given for nodes in the tree, the number of people waiting to be served at t+1 can be computed. This determines the j node for a "no-dispatch" arc. Here the network cannot be preconstructed since the "no-dispatch" arcs are a function of a path variable, πd.

The costs of "no-dispatch" arcs are all zero. For "dispatch" arcs, the cost is r_d = passenger revenue - dispatch cost = system income for the dispatch.

4.3.3   Problem Size

Here, πd must be known for every node at a given stage, so there must be at least $4P_{max}$ storage spaces of high speed memory. For our example, this is still only 400 spaces.

4.3.4   Comments

For the operator, the difficult task is correctly defining $P_{pq}(t,w)$ for a market pq where alternative nodes or competitive services may be offered. This is a micro traffic forecasting (or market share) model which should be

able to predict for all daily patterns, competitive services, etc. what the traffic load will be on every dispatch. Here it is assumed that factors other than passenger waiting time canbe ignored in such a model, and for airline scheduling this is not realistic. For example, on some routes arrival times rather than departure times or wait for service may dominate the determination of P_{pq}. However, if the model for P_{pq} is known, it should be possible to incorporate it in a dynamic programming model such as D-3.

4.4 Model D-4   Minimum Social Cost on a Multi-Stop Route (DP)

4.4.1 Problem Statement

Given time of day demand distributions between points i and j on a route r from p to q, and a set of possible dispatch routings, determine the number and pattern for every dispatch routing which minimizes a weighted sum of dispatching and passenger waiting costs.

4.4.2 Model Formulation

For every station p, we have a set of demands $P_{pq}(t)$ for stations q downstream of p. For every such demand, a dimension is added to the state space, which causes the state space to increase very rapidly. The stage variable can be taken as time for the first station, and all other $P_{pq}(t)$ translated to the equivalent time using interstation travel times.

For every dimension, there is a dispatch set of arcs similar to previous dispatch arcs. However, now one may use a multi-dimensional dispatch routing satisfying more than one demand. These "dispatch" arcs, go from a node in the state space back towards $y_i=0$ at time t such that on-board loads do not exceed capacity. Figure 10 shows a 3 dimensional state space at a given time t, and shows a few of the possible dispatch arcs. Simple priority rules for loading a dispatch may reduce the number of dispatch arcs which are possible; eg. all possible passengers at a given station are picked up giving them priority over downstream passengers.

4.4.3 Problem Size

For an m-stop route, the number of demands which can be considered is

$$n = \frac{(m+2)\ (m+1)}{2}$$

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and the number of dispatch routings which can be considered is

$$r = \sum_{p=1}^{m} \frac{(p+2)(p+1)}{2}$$

State Space - The dominant requirement for high speed memory is twice the number of nodes at a given stage which now becomes $N_H = 2(P_{max})^n$. Secondary storage of size $N_n = T \cdot (P_{max})^n$ must also be provided.

Arcs - There are at least r branches for every node, so the number of arcs for the problem is $r \cdot T \cdot (P_{max})^n$.

eg. for the simple route of figure 9 and the example for model D-1, high speed storage requires 2×10^6 spaces, secondary storage 2.16×10^8 spaces and there are at least 8.64×10^8 computations to be performed on the arcs. This high speed memory requirement is beyond present computers for this very minimal extension to a multistop route.

4.4.4 Comments

Since the state space size increases exponentially with the number of stops on a multi-stop route, and even for a one-stop route becomes computationally infeasible, this approach is not practical. The model is included to demonstrate this fact. If one attempts to go to a network where a number of multi-stop routes are available between city pairs, the model is even more impractical.

The combinatorial complexity of this model which arises when one attempts to go to a network is due to every time of day being considered feasible for dispatching

a vehicle on every possible route through the station. Both the optimal number and the pattern of dispatches are determined. If the number of dispatches desired on a given route is known (say, by results from a Fleet Assignment model), then a range of feasible departure times for every service can be specified. Rather than attempt to construct a complete timetable from zero specifications, such an approach allows a partial specification of the timetable as input, and the best timetable is then selected from this much smaller combinatorial set. This approach is described later by the "multi-departure" fleet routing models.

4.5 Example - Least Delay for n Dispatches, D-2

An example of least passenger delay dispatching is shown
using a $P_{pq}(t)$ corresponding to a weekly average of daily
demand on the Eastern Airlines Shuttle. The distribution is
given by a histogram of average hourly traffic, and flights
can be dispatched at 6 minute intervals. Passengers are
assumed to arrive and wait for the next service.

There are three dispatching patterns shown in figure 10.
The first is the full load pattern, where a dispatch occurs
as soon as the vehicle is full. The average wait for this
case is 0.74 hours, and 12 dispatches are made.

The second uses 18 dispatches at a 66% load factor and
reduces the average wait to 0.49 hours. The dispatching
pattern is shown, with the height of the bars representing
the on board load.

The third pattern uses 28 dispatches at a 44% load
factor and reduces the average wait to 0.34 hours. Again the
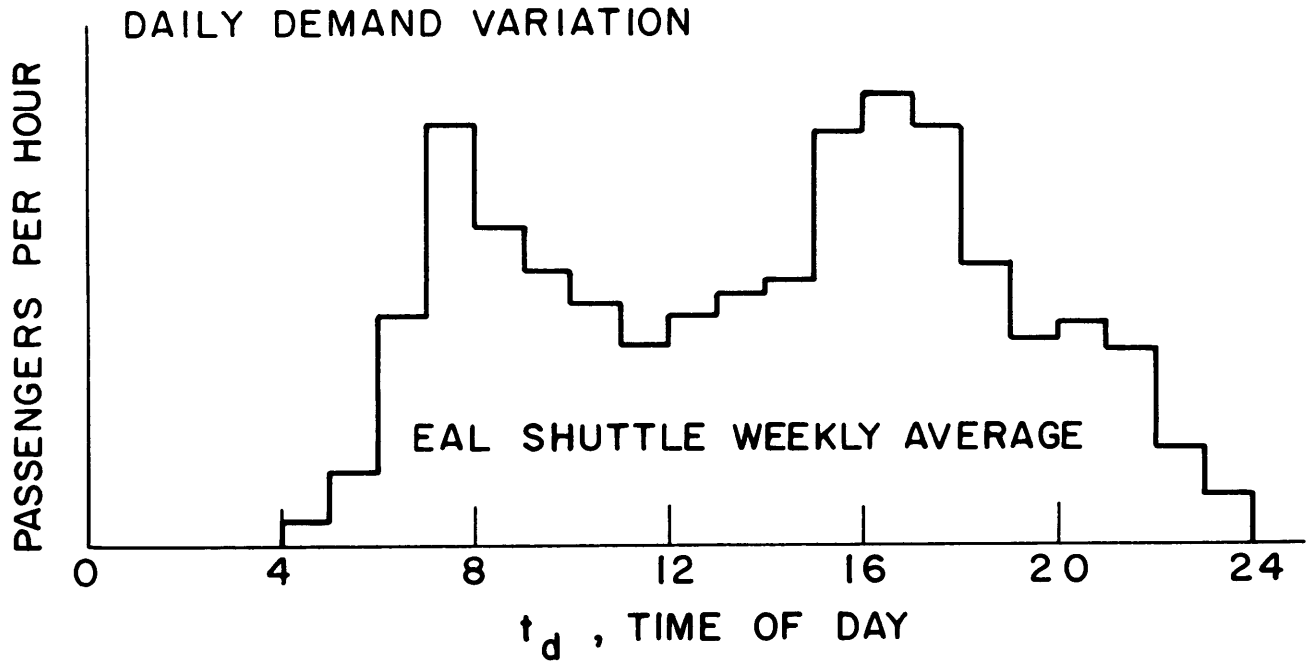time pattern for the 28 dispatches is shown with the on board
loads.
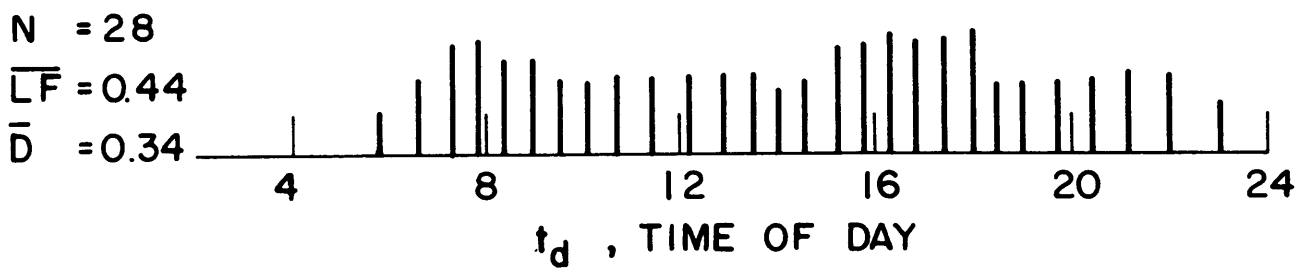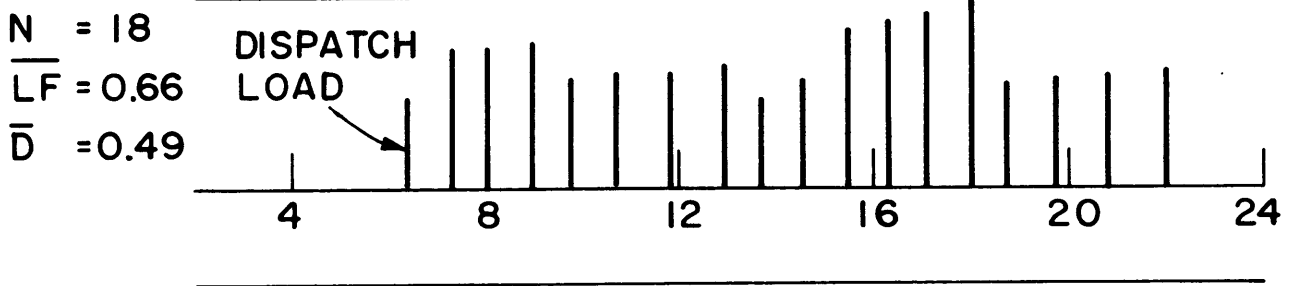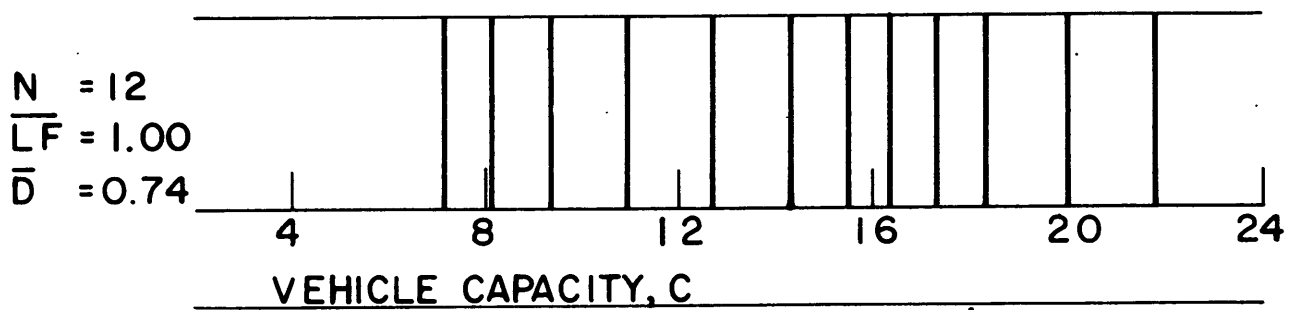
FIG. 10 OPTIMAL DISPATCHING PATTERNS

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5.0 Vehicle Routing Models - (DP)

In the previous models, routing constraints for vehicles have not been applied. We introduce them in this section which considers the problems of routing individual vehicles operating non-stop services on a given network structure. Generally, it is assumed that a description of scheduled services $\underline{S}$, or the demand distribution $P_{pq}(t)$ is known. The models propose to find an optimal set of routings for every individual vehicle in the fleet.

Since routing constraints are applied, the number of aircraft in the system becomes an explicit variable. In the Fleet Assignment and Planning models, the critical assumption of daily utilization was made in order to relate number of aircraft to usage in terms of block hours per cycle. Here, the number of aircraft is known, and the actual utilization of each aircraft, and fleet average utilization are given as output.

Although these models preceded the Dispatching models, they may be regarded as extensions of them onto the network. The solution technique is again dynamic programming and once again, computational problems arise as the size of the state space increases as a power of the number of vehicles in the fleet. As a result, a sequential optimization routine has been sub- stituted in practice which leads to a set of "good" but not necessarily optimal set of routings for the fleet.

Only two models of the vehicle routing class are described here. There are many variations possible some of which have been described in the literature. Here it is assumed that $\underline{S}$ is given rather than $P_{pq(t)}$ in order to relate these models to the Fleet Routing models described later.

5.1 Model AR-1   Maximum Income, Single Aircraft

5.1.1 Problem Statement

Given a schedule of possible non-stop services, S with an income for each service, find the routing for a single aircraft which maximizes its income.

5.1.2 Model Formulation

The state space diagram for the dynamic programming formulation of this problem is shown in Figure 11. A node now represents a space-time position for the aircraft. The diagram is essentially a route map with added "route" nodes at stage increment, Δ t, intervals along each non-stop route. For every route node, the aircraft is automatically transferred to the next node as the stage variable is incremented. There is no decision at these nodes.

At station nodes, there may be a decision to "dispatch" a flight if S indicates there is a service from the station at that time. Such station nodes are called "event" nodes (station arrival nodes are also event nodes), and the arc representing the decision to dispatch is called the "dispatch" arc. The cost or value of dispatch arcs is the given income for the service. All other arcs have zero cost.

By replicating the state space diagram for every time t, we form the network associated with the dynamic program. The existence of a given schedule, S allows this replication to be simplified by omitting routes from non-departure nodes at a station, and the result is essentially a schedule map with a great many added route nodes. This is shown in Figure 12, and has been used in some dynamic programming formulations.
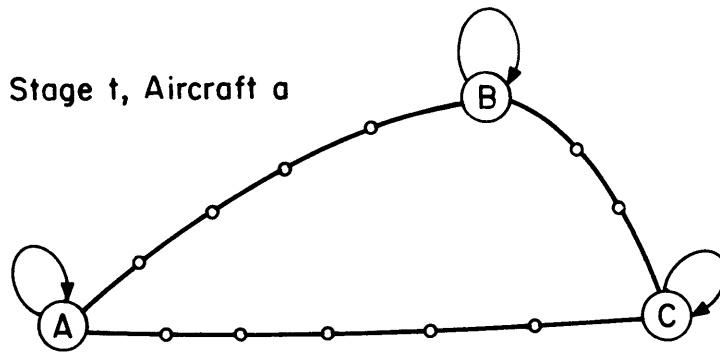
Stage t, Aircraft a

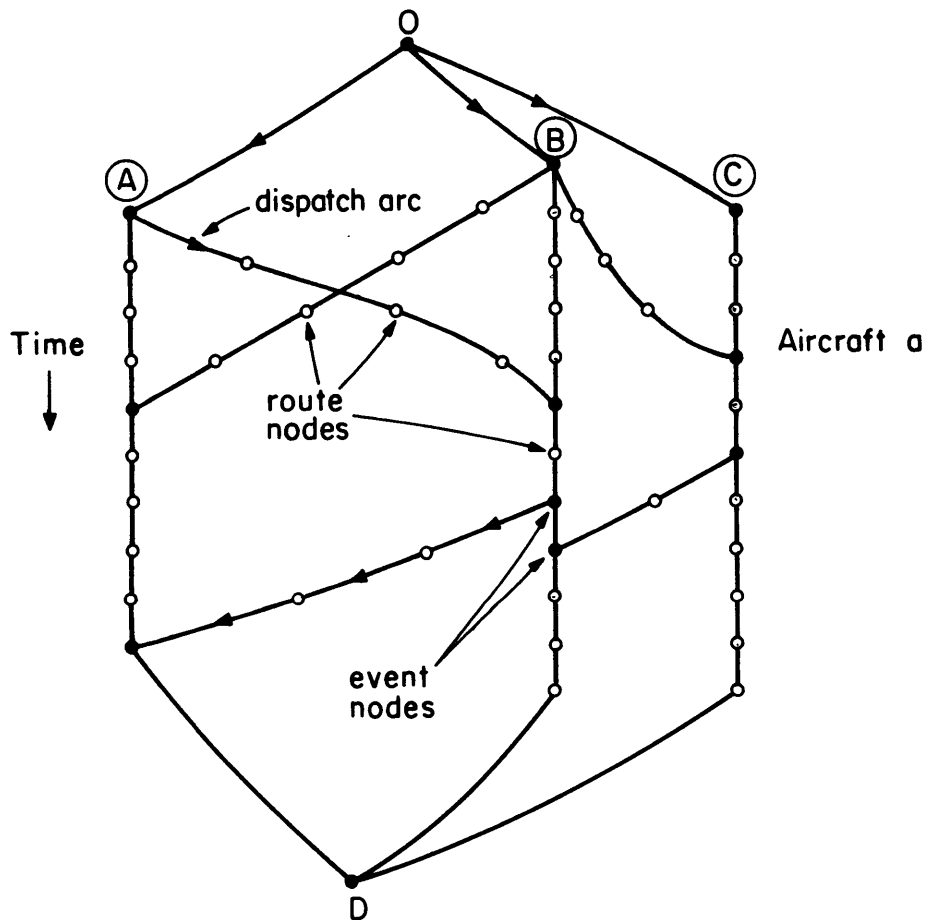FIG. II STATE SPACE DIAGRAM FOR AIRCRAFT ROUTING

FIG. 12 DYNAMIC PROGRAMMING NETWORK FOR AIRCRAFT ROUTING

However, it is possible to redefine the state space such that an aircraft is k units of stage away from a given station. Thus, two aircraft arriving at a station at the same time but along different routes are lumped together in the same state. Figure 13 shows the state space diagram for this definition. A single inbound path is established for each station and dispatch arcs are joined to this path at appropriate times. This diagram is again replicated for every stage.

The problem is now a shortest path problem on the associated networks. Figure 12 shows the origin, O, of this path connected to all stations at the earliest event node, and the destination, D, connected to all latest event nodes.

5.1.3 Problem Size

State Space - The number of nodes in the state space diagrams depends on the number of stations, and the total length of the route map measured in stage increments. The reduced state diagram has a similar dependency upon problem structure. If L is taken as the node length of the longest route, an upper bound on the number of nodes is ($|\underline{Q}|$. L) where $|\underline{Q}|$ is the number of stations. The high-speed memory requirement is therefore $N_H \leq 2 |\underline{Q}| . L$.

The total number of nodes in the associated network is $N_L \leq T . |\underline{Q}| . L$ where T is the number of stage increments. This is low-speed memory requirement.

Arcs - There is one arc for every node in the network except for the event nodes which also have a dispatch arc. For the reduced state space, there would be less than $N_L + |\underline{S}|$ arcs.
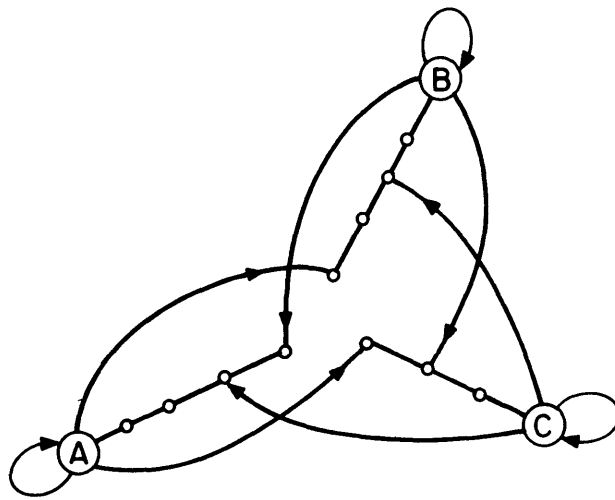
FIG. 13 REDUCED STATE SPACE DIAGRAM FOR VEHICLE ROUTING

eg. for 20 cities, and 500 services in $\underline{S}$ of longest length 50, there would be a high speed memory requirement for 2(20 x 50) = 2000. If the problem were continued over 300 stages, the secondary memory requirement would be 3×10^5 spaces. Computation would be done on $(3 \times 10^5 + 500)$ arcs.

5.1.4 Comments

The dynamic programming formulation is an extremely inefficient formulation for this problem. It is much more efficiently solved as a least cost path problem on a schedule map which eliminates all non-event nodes and their arcs.

Most formulations have ignored station balance constraints. The time dimension should be continued over several cycles of the schedule until the aircraft overnights back at its origin station. Then there will be several aircraft on this routing cycle with a sequence of individual aircraft overnighting at each station on the cycle.

Path constraints such as flying time between overnights at maintenance bases, interconnections between flight segments can be incorporated into this formulation. These constraints are the only reason for this formulation of the problem, since otherwise the least cost path methods may be used.

5.2 Model AR-2 Maximum Income, Multi-Aircraft

5.2.1 Problem Statement

Given a schedule of possible services, $\underline{S}$, with an income for each service, find the set of individual aircraft routings which maximizes system income.

5.2.2 Model Formulation

Extending the previous formulation to include more than one aircraft adds a new dimension to the state space diagram for each additional aircraft. Thus, for every node in the first aircraft's state diagram, a complete state space diagram exists for describing the position of the second aircraft.

Now, each aircraft can be a different type of vehicle with different capacity costs and speeds, and more than one vehicle can be considered for dispatch on a given service. A fixed number of aircraft can be specified, or by using a given cycle ownership cost, the most profitable number of aircraft can be determined. Station balance constraints should be applied for a cyclic schedule.

5.2.3 Problem Size

State Space - The high speed memory requirement now becomes that required for a single aircraft raised to the power A' , where A' is the maximum number of aircraft considered. If we use the upper bound estimate for the reduced state space from model AR-1, the high speed memory requirement is roughly

$$N_H \leq 2 \, (\, |\underline{Q}| \, . \, L)^{A'}$$

The low speed memory requirement for a stage length T becomes

$$N_L \leqslant T \ (|\underline{Q}| \ . \ L)^{A'}$$

The number of arcs on which computations are done becomes

$$N_C \leqslant N_L + |\underline{S}| \ . \ A'$$

eg. for the previous example of 20 cities, 500 services, longest length of trip = 50, 300 stages, and now considering 10 vehicles

$$N_H \leqslant 2 \ (1000)^{10} = 2 \ x \ 10^{30}$$

$$N_L \leqslant 300 \ (1000)^{10} = 3 \ x \ 10^{32}$$

$$N_C \leqslant N_L + 5000 = 3 \ x \ 10^{32}$$

This is clearly computationally infeasible even if just two aircraft were used instead of ten.

5.2.4 Comments

The model can be extended such that $P_{pq}(t)$, etc. exists for every route as in the dispatching models. It is then possible to extend any of the dispatching models on to a given route map, and open up every time of day for a possible dispatch on a route.

However, as with the attempt to extend the dispatching models to multi-stop services, the attempt to extend them to more than one vehicle is more conceptual than practical because of the exponential growth of the size of the state space. The key

requirement here is the identification of the best routing for individual vehicles. If that is dropped, and we consider routing a fleet of similar vehicles, this same problem may be posed as a "fleet routing" problem of the next section.

In the absence of computational feasibility, a number of approximate computation methods have been used. The aircraft manufacturers and airlines have adopted a sequential method of solving model AR-1, then removing the single aircraft's optimal services or demand from the problem, and repeating for every aircraft until either aircraft or profitable routings are exhausted. Computation times seem to be measured in hours for present airline systems. This sequential process is not optimal, but a theoretical iterative improvement process does exist (reference 3). As well, reference 5 suggests an iterative approximate procedure. Unless there is a real reason for identifying the routing of each individual vehicle, such optimal seeking procedures are unprofitable in view of the models of the next section.

5.3 Aircraft Routing Example

A simple example of an aircraft routing problem is described in reference 3. It consists of 47 individual routings on a network given by Figure 14. Data on aircraft, flight services, their values, ferry flights, etc. is described in reference 3. Computation of these 47 routings apparently required 3 minutes on a Burroughs B-5500. As each routing was found, its services were removed from the problem before the best routing for the next aircraft was computed. The process is therefore sequential, and can be sub-optimal.

Figure 15 shows the variation from the optimal results for this example. Since the units of income are arbitrary, the best measure of optimality is the difference in numbers of aircraft required for a given income. This was as much as three aircraft, and at the end, when all of the profitable flights are flown, and the system income only differs by the cost of ferry flights, a higher income can be gained with 45 aircraft than this sequential routine achieves with 47 aircraft.
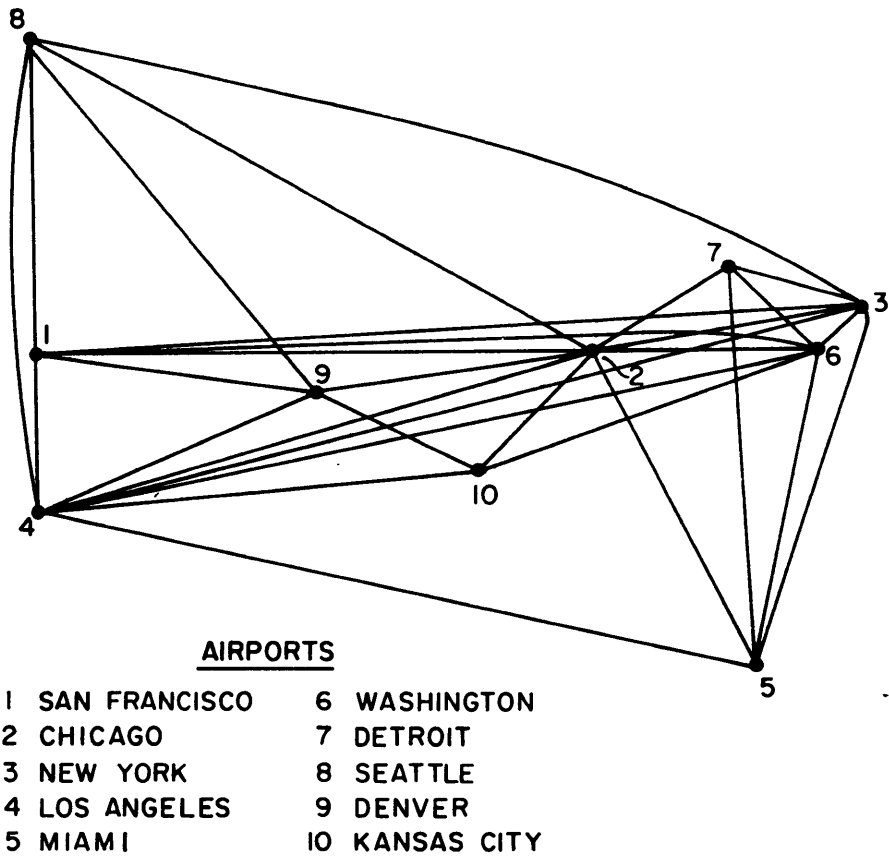
AIRPORTS

1	SAN FRANCISCO	6	WASHINGTON
2	CHICAGO	7	DETROIT
3	NEW YORK	8	SEATTLE
4	LOS ANGELES	9	DENVER
5	MIAMI	10	KANSAS CITY

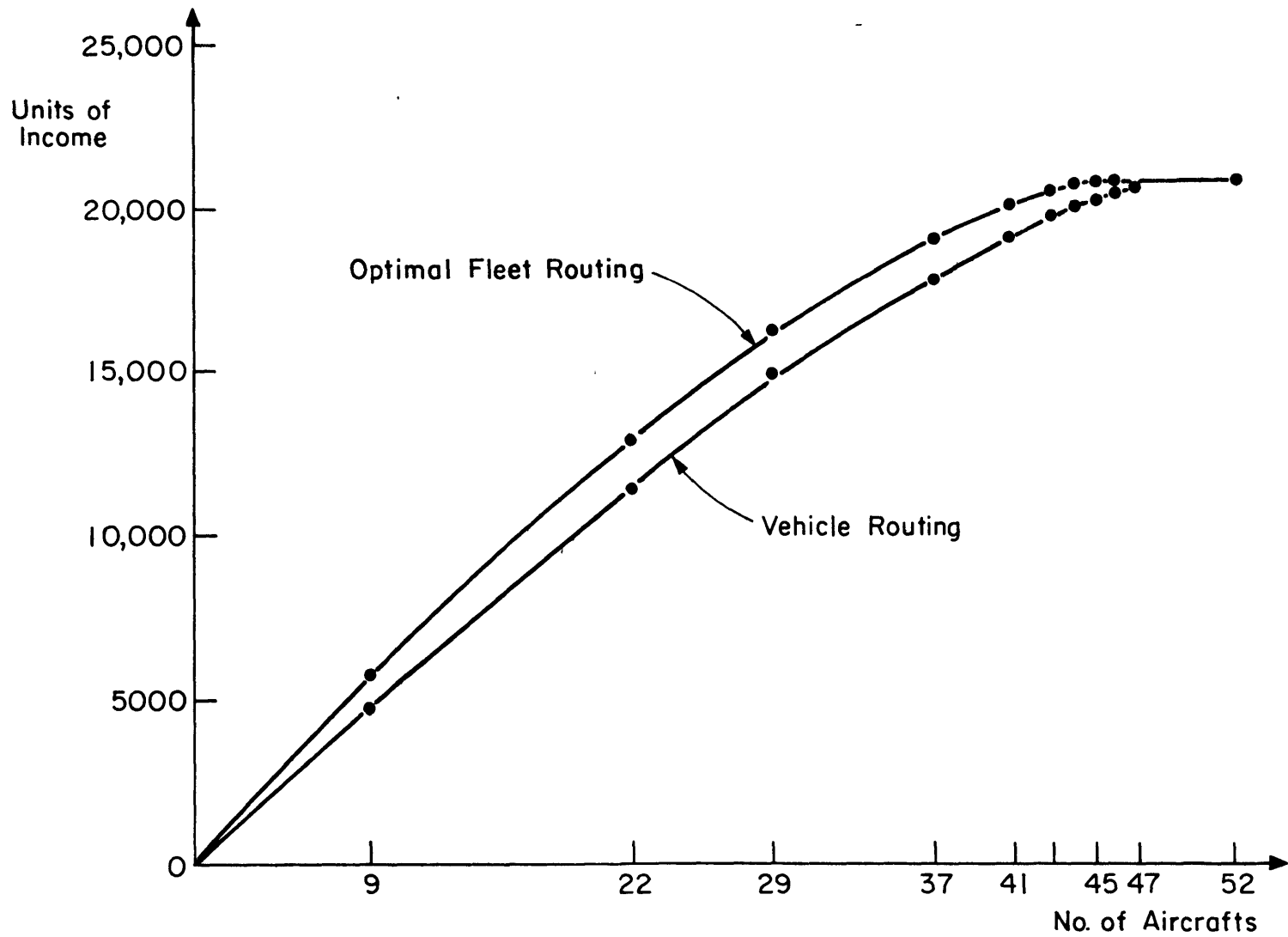FIG. 14 ROUTE MAP, VEHICLE ROUTING EXAMPLE

FIG. 15 COMPARISON OF VEHICLE ROUTING AND OPTIMAL RESULTS

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6.0 Fleet Routing Models

Fleet routing models consider every vehicle in the fleet to be essentially similar, and find an optimal routing pattern for the entire fleet without identifying individual aircraft. When two or more aircraft are available at a station to take a given service, the aircraft used is not explicitly identified by the solution. Given an optimal fleet routing, individual aircraft routings may later be constructed, and there exist a very large number of such decompositions of the optimal fleet routing. This large number of decomposed sets of routings caused the combinatorial complexity of model AR-2. It also allows any constraints or individual aircraft routings arising from turnaround times, maintenance, etc. to be satisfied <u>after</u> the optimal fleet routing has been found.

The models of the next few sections all assume a cyclic timetable TT consisting of a set of possible flight services <u>S</u> between stations. This data is used to construct a <u>"schedule map"</u> (see Figure 2). All of the fleet routing models can be posed as network flow problems on such a network where the circulation flows must be integer.

The schedule map consists of vertical time lines for each station consisting of <u>"ground"</u> arcs which join event nodes at the station. An <u>"overnight"</u> or <u>"cycle"</u> arc joins the last event node of the cycle back to the earliest event node at that station. Flight services are represented by <u>"flight"</u> or <u>"service"</u>, or <u>"dispatch"</u> arcs leaving a station i at time td_{ij}, and arriving at a time ta_{ij} or arriving at

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a "ready" time tr_{ij} which is the first time at which an aircraft for this service could be ready to depart.

For all routing models, the network flow must be integer valued. It is not meaningful to have 1/3 of an aircraft follow one branch while the remainder continues on, and rounding such values does not lead to optimal or even feasible routings. The routing problems are combinatorial programming problems, and require special computational techniques, which tend to be problem dependent, and even dependent upon the size of a given problem. The fleet routing models have been grouped according to computational techniques.

The first group are single fleet, fixed timetable models for which efficient network flow algorithms like the Out of Kilter method are applicable. The second and third groups add auxiliary constraints called "bundle" constraints to the network flow problem, and require a variety of appropriate computational techniques from present day combinatorial programming. The fourth group of models are the multi-fleet models where copies of the schedule map are repeated for each fleet of a different type of aircraft, with auxiliary constraints on network flows in the individual copies.

While most of the airline fleet routing models are several years old, they have not received much attention in the literature. Apart from the first group for which large scale problems can be easily solved, there has not been any successful computational techniques until the last few years. Now these models have a variety of techniques to choose from, and the question of which technique is best seems to be dependent on the particular problem and its size.

6.1 Single Fleet, Fixed Timetable Models - (OKF)

This section deals with models which can be solved using the Out of Kilter algorithm of network flow theory. This algorithm finds a least cost network circulation flow on a network subject to bounds on arc flow. It is briefly described in Section 1.

These models assume that a fleet of a single type of vehicle is to be routed on a schedule map. The flow on a given arc represents the number of aircraft using the arc and must be integer valued. A fixed timetable of services S is given, and the value of each service in terms of net income may be known. A known minimum turnaround time is added to each flight time to calculate a tr_{ij} "ready to depart" time as the arrival event node in the schedule map. Thus, a network flow into a station can immediately depart, and this case would represent an aircraft arriving and making a minimum time connection to a departure service. Network flow in ground arcs therefore represents spare ground time in making connections.

The first model gives a minimum fleet size and its fleet routing to cover the schedule. As indicated in reference 6, this is a trivial counting problem, but is given here as a means of introducing fleet routing models. The second model determines maximum income, the associated services flown and fleet routings, and the number of aircraft which can be profitably flown. If this fleet size exceeds the available fleet, the third model shows how the OKF routine may be parametrically controlled to produce the most profitable set of routings given A' aircraft. The

last model shows a network structure which can be used to include one-stop flight itineraries in the problem and still use the OKF algorithm.

6.1.1 Model FR-1 Minimum Fleet Size (OKF)

6.1.1.1 Problem Statement

Given a fixed schedule of non-stop services which must be flown by a fleet of aircraft of similar type, what is the minimal fleet size required and the fleet routing?

6.1.1.2 Model Formulation

A schedule map network is constructed using the set of services $\underline{S}$, their departure times at every station, td_{ij} and calculating the "ready" times, tr_{ij} for given minimum turnaround times. Let the flow in the network, $\underline{x}$, represent the number of aircraft on any arc of the network.

Flight Service Arcs, $\underline{S}$ - for every service in $\underline{S}$ construct a flight service arc leaving station i at time td_{ij}, and becoming ready to depart at station j at time tr_{ij}.

$$\text{Put} \quad \left. \begin{array}{l} u_{ij} = 1, \\ l_{ij} = 1, \end{array} \right\} \text{so only 1 aircraft flies the service}$$

$$c_{ij} = 0$$

Ground Arcs, $\underline{G}$ - for every service two ground arcs are usually constructed in the schedule map. The number of ground arcs can be greatly reduced by collapsing the string of successive event nodes at a station onto all arrival nodes which follow a departure node.

$$\text{Put} \quad u_{ij} = \infty$$
$$l_{ij} = 0$$
$$c_{ij} = 0$$

Cycle Arcs, $\underline{C}$ - from the last event node at every station
a cycle arc returns the overnight aircraft to the first
event node at that station

$$\text{Put } u_{ij} = \infty$$
$$l_{ij} = 0$$
$$c_{ij} = 1, \text{ so that overnight aircraft are counted.}$$

Objective Function,$\underline{Z}$ - since the only arcs which have a
cost are the cycle arcs, Z will take on the value of the
fleet size. We solve for a minimum

$$Z_{opt} = \min \left\{ \sum_{\underline{A}} c_{ij} \cdot x_{ij} = \sum_{\underline{C}} c_{ij} \cdot x_{ij} \right\}$$

This assumes that there is some period of each night
when the entire fleet is scheduled to be on the ground. This
is generally true for domestic short haul systems. Otherwise,
a particular "counting" arc must be constructed corresponding
to some given time like 0001Z (Greenwich time) and inserted
into every flight still airborne at that time.

6.1.1.3 Problem Size

As indicated in the brief description of a schedule
map in the section giving definitions and symbols,

$|\underline{A}|$ = no. of arcs $\leq 3|\underline{S}| + 2|\underline{C}|$

$|\underline{N}|$ = no. of nodes $\leq 2|\underline{S}| + 2|\underline{C}|$

This assumes the trick of collapsing ground arcs has not
been used.

e.g. Suppose we have 20 cities, 200 city pairs with an average
of 5 flights/day, or 1000 services.

$$|\underline{A}| \leqslant 3000 + 40 = 3040$$

$$|\underline{N}| \leqslant 2000 + 40 = 2040$$

This is well within the present coding capabilities given in section 1.

6.1.1.4 Comments

Since every flight must be flown, one can insert flow into the service arcs "a priori", and compute the minimal flow in the loop of ground arcs and cycle arc. This process is the "counting" process of reference 6, and it provides a method for finding a good initial, feasible network flow for any of the routing models. For this model, it gives the optimal answer, and the corresponding fleet routing without using OKF. For other models, it greatly reduces the running time for OKF, but, of course, increases the running time for the preprocessor coding.

This model clearly places time of day routing constraints on aircraft in the fleet. The station balance constraints which ensured that the number of services flown into and out of a station were in balance for the Fleet Assignment models, are implicitly handled by the schedule map construction. For this model where every service is flown, the number of service arcs into a station must equal the number outbound, and this forms a useful check on network construction. In later models, the node conservation constraints ensure that the number of arrivals equals the number of departures for any fleet routing.

6.1.2 Model FR-2 Maximum Income (OKF)

6.1.2.1 Problem Statement

Given a schedule of non-stop services, an associated income for each service, and a cycle ownership cost for an aircraft, what set of services should be flown for maximum system income?

6.1.2.2 Model Formulation

A schedule map is constructed as in model FR-1, with $\underline{x}$ representing the fleet routing network flow.

Flight Service Arcs, $\underline{S}$ - every flight arc now represents a possible service for a city pair, time of day market (e.g., a morning service A to B, or a 9:00 a.m. departure from A to B). A forecast is made for the number of passengers in that market given the expected total daily frequency of service from the system, and the level of competitive services. Effectively, it is assumed that the traffic for a given service is independent of the selection of other services in that city pair market since the expected daily frequency is not guaranteed by this model (See model FR-2 DC5). From the traffic forecast, net revenue, r_{ij}, for the flight (e.g., traffic x fares, less indirect costs for boarding, ticketing, reservations) may be estimated in dollars. A marginal direct operating cost, DC_{ij}, for this type of vehicle (e.g. excluding ownership, or other fixed costs) can be assigned for every service.

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Then, for all arcs in $\underline{S}$

Put $\quad u_{ij} = 1$

$$c_{ij} = DC_{ij} - r_{ij}$$

$$l_{ij} = 0$$

Here, the arc cost c_{ij} for the OKF formulation is the negative of net income, I_{ij}. Effectively, the negative sign changes the OKF minimization to a maximization of total system income.

Ground arcs, $\underline{G}$

Put $\quad u_{ij} = \infty$

$$c_{ij} = 0$$

$$l_{ij} = 0$$

Cycle Arcs, $\underline{C}$

Let OC be the daily or cycle ownership costs for this type of vehicle. As each aircraft traverses the cycle arc, it pays its daily cost.

Put $\quad u_{ij} = \infty$

$$c_{ij} = OC$$

$$l_{ij} = \text{overnight maintenance requirements for each station.}$$

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Objective Function

$$\text{Min} \left\{ -Z = \sum_{\underline{S}} (DC_{ij} - r_{ij}) \cdot x_{ij} + OC \cdot \sum_{\underline{C}} x_{ij} \right\}$$

$$= \text{Max} \left\{ Z = \sum_{\underline{S}} (r_{ij} - DC_{ij}) \cdot x_{ij} - OC \cdot A \right\}$$

$$\text{where } A = \sum_{\underline{C}} x_{ij} = \text{fleet size}$$

$$Z = \text{maximum income for system - dollars/cycle}$$

$$\underline{x}_{ij} = \text{optimal fleet routing}$$

$$\begin{cases} x_{ij} = 1, & \text{service has been selected} \\ x_{ij} = 0, & \text{service is not flown} \end{cases}$$

6.1.2.3 Problem Size

The size is identical to model FR-1.

6.1.2.4 Comments

The optimal fleet routing solution is given by the network circulation, $\underline{x}_{ij}$, and is such that no additional aircraft can be profitably routed. The solution may be decomposed into individual aircraft routings in many ways, but no cycle can be found which is not profitable. The solution represents a true optimal routing given A vehicles, and is equivalent to the

dynamic programming formulation, model AR-2.

Additional information about the optimal solution is available from the $\bar{c}_{ij}$ values. They indicate how Z will change as x_{ij} is changed. For example, if service arc ij is in the solution, with $x_{ij} = 1 = u_{ij}$, $\bar{c}_{ij} < 0$, then $\bar{c}_{ij}$ represents the marginal value of the service to the system given the present solution. It is a lower bound on the income loss if x_{ij} were forced to zero by putting $u_{ij} = 0$, and the next best solution were found. Similarly, for services not in solution, the $\bar{c}_{ij} > 0$ values represent a lower bound on the income loss if we insisted that this service be flown by putting $l_{ij} = 1$.

Because of the schedule map structure, and the daily ownership costs, services which have a positive cost, $(DC_{ij} - r_{ij}) > 0$ or in other words seem to be "income loss" flights should be included in $\underline{S}$ if this loss is less than OC. It is possible because of their position in the schedule map, that such a flight may act as a "ferry" flight, saving an additional aircraft for the fleet. This "income loss" service can then be routed as part of a profitable daily cycle for the fleet, with a $\bar{c}_{ij}$ value for the arc which becomes negative indicating the true marginal value of the service to the system.

If we insist on l_{ij} aircraft overnight at given maintenance stations, a positive $\bar{c}_{ij}$ indicates the marginal daily cost of this requirement for the last vehicle required. This daily cost can be weighed against the costs of performing maintenance elsewhere in the system.

The model is similar to FA-3 in that income is being maximized, and traffic or revenue is a variable in the problem. Its formulation thus gives a representation of constraints 1) from the Fleet Assignment models. The station balance constraints 3), and routing constraints 9) are obviously part of this routing model. Most of the remaining constraints are incorporated in later models.

For large systems, a preprocessor program should be used to construct a reduced schedule map input for OKF. It is generally advantageous to put flow in every profitable service arc, and compute the resulting station flows in the preprocessor. This initial circulation flow greatly reduces the number of arcs which are "out of kilter", and thereby reduces OKF computation times.

Normally, a set of possible services $\underline{S}$ corresponding to an average weekday cycle are used as input. Weekend cycles may be sufficiently different that different $\underline{S}$ sets may be desirable. The cycle period can represent a week or a sequence of dissimilar days; e.g., an average weekday $\underline{S}$ can be followed by a Saturday and Sunday S before the cycle arcs return the fleet to the weekday services. The value of the services, and rental costs have to be appropriately chosen. If the number of aircraft overnighting at each station remained identical, each $\underline{S}$ could be solved separately. If it changes then Friday and Monday transition schedules may be different from the normal weekday schedule. In this case, one may only be able to collapse Tuesday, Wednesday, Thursday into one common daily schedule, and publish varying schedules for Friday, Saturday, Sunday and Monday services. The weekly cycle variation will determine how this problem is approached.

6.1.3 Model FR-3 Maximum Income for a Given Fleet Size

6.1.3.1 Problem Statement

Given a schedule of possible flight services $\underline{S}$, an associated income for each service, and a cycle ownership cost for an aircraft, what set of services should be flown to maximize income such that exactly A' vehicles are used?

6.1.3.2 Model Formulation

This is model FR-2 with a single additional constraint,

$$\sum_{\underline{C}} x_{ij} = A', \text{ or } A - A' = 0$$

This problem may be solved in a number of ways. The simplest seems to be an iterative OKF solution of model FR-2 using a Lagrange multiplier technique. We form a new objective function by adjoining the new constraint:

$$z' = z - \pi \cdot (A-A')$$

At an optimum for z',
$$z' = z$$
$$A = A'$$
$$\pi = \pi'$$

The model is now formulated with the arcs $\underline{S}$, $\underline{G}$, $\underline{C}$ as in model FR-2, except that the cost on the cycle arcs, $\underline{C}$, becomes:

$$c_{ij} = OC + \pi$$

For a given π value, the new OKF problem is solved to obtain

$$z'' = z - \pi \cdot A = z - \pi \cdot \sum_{\underline{C}} x_{ij}$$

where A is the number of aircraft used in the new solution. A solution which uses exactly A' aircraft can be found by iteratively solving the OKF problem using judicious π values.

The following formula for choosing the π values is suggested.

Define: $\pi_1 = 0$ and solve FR-2 for z_1, A_1.

$$\pi_0 = \frac{z_1}{A_1} \text{ , and } A_o = 0$$

Then

$$\pi_i = \pi_{i-1} + \frac{(\pi_{i-1} - \pi_{i-2})}{(A_{i-2} - A_{i-1})} \cdot (A_{i-1} - A')$$

For example,

$$\pi_2 = 0 + \frac{(0 - \frac{z_1}{A_1})}{0 - A_1} \cdot (A_1 - A') = \frac{(A_1 - A')}{A_1} \cdot \frac{z_1}{A_1}$$

Using π_2 produces a solution z_2, A_2.

Then,

$$\pi_3 = \pi_2 + \frac{(\pi_2 - \pi_1)}{(A_1 - A_2)} \cdot (A_2 - A')$$

$$= \frac{A_2 - A'}{A_1 - A_2} \cdot \pi_2$$

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This process should converge fairly rapidly to a solution Z_j', A', π'. The iterative OKF solutions are very fast since only $|C|$ arcs are being changed, and therefore could possibly be out of kilter for the next solution.

6.1.3.3   Problem Size

The size is identical to Model FR-1.

6.1.3.4   Comments

The new constraint is the fleet availability constraint 2) from previous models since unless the number of aircraft available is less than the optimal number, this model would not be useful and Model FR-2 would be used.

This model only differs from FR-2 in the addition of a toll or fee, π to the cycle arcs. As π is increased, the apparent daily ownership cost for each aircraft increases, and every aircraft in the fleet must earn more than $OC + \pi$ to remain profitably routed. As π increases, aircraft and their routings drop out of the solution until π reaches a value where no aircraft can be profitably used. This value of π would be the net income of the single aircraft optimally routed as in Model AR-1.

The relationship of A & Z to the parameter π is illustrated by figure 16. As π is increased, both A and Z decrease. Actually since A is discrete, the A-Z curve is stepwise discontinuous at various points where 1 or more aircraft may drop out of the optimal fleet routing. At these discontinuous points, the value of π represents the marginal income to the system for operating the aircraft which are being dropped.

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i.e. $\pi = \dfrac{\Delta Z}{\Delta A}\bigg|_A$ at points where A changes.

The capability of this model changes the strategy of listing the services for the schedule map. Now one should include every possible service for which an independent estimate of the net income can be estimated, and should not be guided by selecting the best n_{pq} service times for the city pair. All reasonable services should be included in every market since the model will be selecting the best set of services for an optimal routing of a fleet of A' vehicles. Given this routing, the value of π indicates the additional income for adding one more aircraft to the system given the remaining unflown set of market opportunities. This application has been successfully used in airline schedule planning since 1962.

The model also has a very direct application to the problem of real time schedule control. Here the schedule map is constructed from the actual timetable, and real time data from the reservations system is used to update the traffic and income values for each service. As described in reference 1 of section 1, various modifications of the network can be applied to represent aircraft breakdowns, airport closures, flight cancellations, etc. In a matter of seconds, the model gives the schedule controller the most economic decisions for keeping the system operating, and returning it to normal operation.

6.1.3.5 Model FR-3 as a Decomposition Model

It is instructive in view of later models to consider solving model FR-3 by using the Dantzig-Wolfe decomposition. In this case the single additional constraint is made part of the master problem. Let λ_k be the proportion of subproblem solution k used in the master solutions. Then the model is formulated as:

Master Problem - find the optimal mix of subproblem solutions such that exactly A' aircraft are used.

Objective Function

$$\text{Maximize} \left\{ Z = \sum_k \lambda_k \cdot Z_k \right\}$$

where Z_k = value of objective function for kth subproblem

Constraints	Dual Variable
0) $\sum_k \lambda_k = 1$	S
2) $\sum_k \lambda_k \cdot A_k = A'$	π_{k+1}

where $A_k = \sum_C x_{ij}^k$ = aircraft used by kth subproblem solution.

The master problem has only 2 rows: the first assumes that the subproblem solutions form a complete solution; the second ensures that the solution uses exactly A' aircraft.

<u>Sub-Problem k</u>   - maximize system income for the modified costs of iteration k.

Objective Function

$$\text{Maximize} \left\{ Z_k' = Z_k - \pi_{k-1} \cdot A_k \right\}$$

The subproblem may be solved on a network using OKF since it is essentially Model FR-2. The same schedule map is used, and similar to Model FR-3, the costs on all cycle arcs become,

$$c_{ij} = OC + \pi_{k-1}$$

For a given value of π_{k-1} from the master problem, we obtain a solution Z_k', $\underline{x}^k$, A_k for this subproblem where $\underline{x}^k$ is an integer network flow. If this solution is useful to the master problem, a new variable λ_k is created and added to the master. The master is then solved to produce a new π value for the next subproblem solution. This process is repeated until the optimal answer is obtained.

Now the master problem has exactly two equations or constraints which means that only two variables can be non-zero in any solution. They may or may not be the last two variables $(\lambda_{k-1}, \lambda_k)$ generated. If they are, another iterative formula for λ is generated, and we do not have to solve the master problem as an LP.

Choose π_0 such that $Z_0 = 0$, $A_0 = 0$

$\pi_1 = 0$ such that Z_1, A_1 are answers for Model FR-2

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From the master problem, we can solve for the dual variables, S_2 and π_2.

$$S_2 + \pi_2 \cdot A_o = Z_o$$

$$S_2 + \pi_2 \cdot A_1 = Z_1$$

Subtracting $\pi_2 (A_o - A_1) = Z_o - Z_1$

$$\text{or} \quad \pi_2 = \frac{Z_o - Z_1}{A_o - A_1} = \frac{Z_1}{A_1}$$

In general, $\pi_k = \dfrac{Z_{k-1} - Z_{k-2}}{A_{k-1} - A_{k-2}}$

which is the slope of the secant between points k-1 and k-2 on the Z-A curve of figure 16. The shape of this curve may be such that the best solution does not fall on the other side of A' from the present solution. In this case, the two variables in the LP solution will not be the last two generated, but a simple test can produce the proper interpolation for computing π.

This iterative formula is different from the one suggested for Model FR-3. The previous one seems to be preferable because of a faster convergence.

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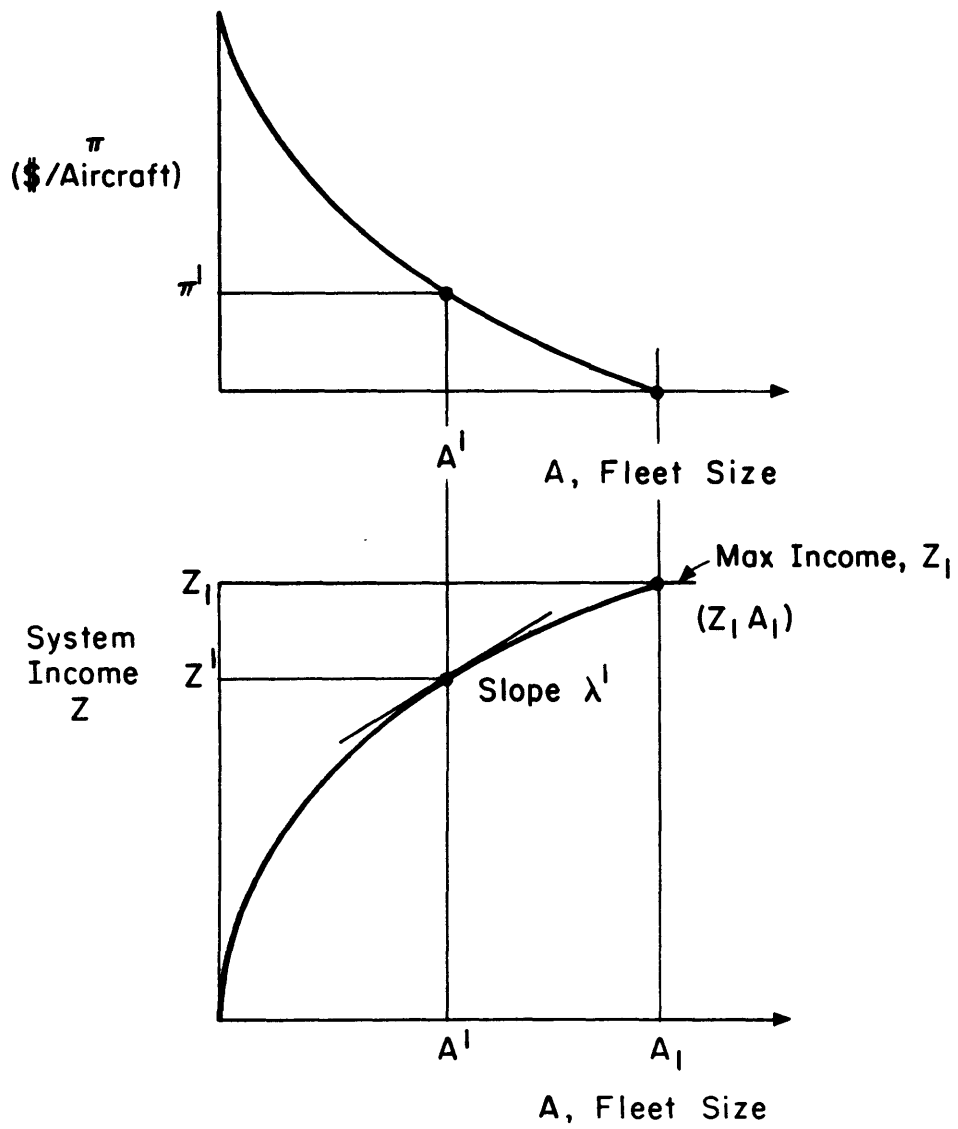
FIG. 16 RELATIONSHIP OF π, A, AND Z, MODEL FR3

6.1.4 Model FR-4 Maximum Income including Multi-Stop Flights

6.1.4.1 Problem Statement

In models FR-1,2,3 the set of arcs $\underline{S}$ represented non-stop flight services with an estimate of the traffic on board the flight segment ij. By appropriately adding a set of "multi stop" arcs $\underline{M}$ to the network, it is possible to consider connecting non-stop flight segments together to form a multi-stop flight. The problem statement "what set of services should be flown, etc." is expanded to read "what set of non-stop services should be flown, and how should they be combined into multi-stop flights" etc.

6.1.4.2 Model Formulation

The arcs $\underline{S}$, $\underline{G}$, and $\underline{C}$ are given values as in model FR-2. We add appropriate sets of "multi-stop" arcs $\underline{M1}$, $\underline{M2}$ as follows.

The "One-Stop" Arc $\underline{M1}$

We now add arcs M1 to the network to allow consideration of connecting certain $\underline{S}$ arcs together into a one-stop flight. Figure 17a shows how two non-stop segments are bridged by a one-stop arc. A node is placed on flight ij and flight jk and a "one-stop" arc joins these two nodes.

This construction allows either ij or jk segments to be operated as non-stop flights independently. Any previous arrivals at j cannot use the attractive "one-stop" arc joining the two flights. There are two zero cost arcs created for use if the flights operate independently. If the flights are
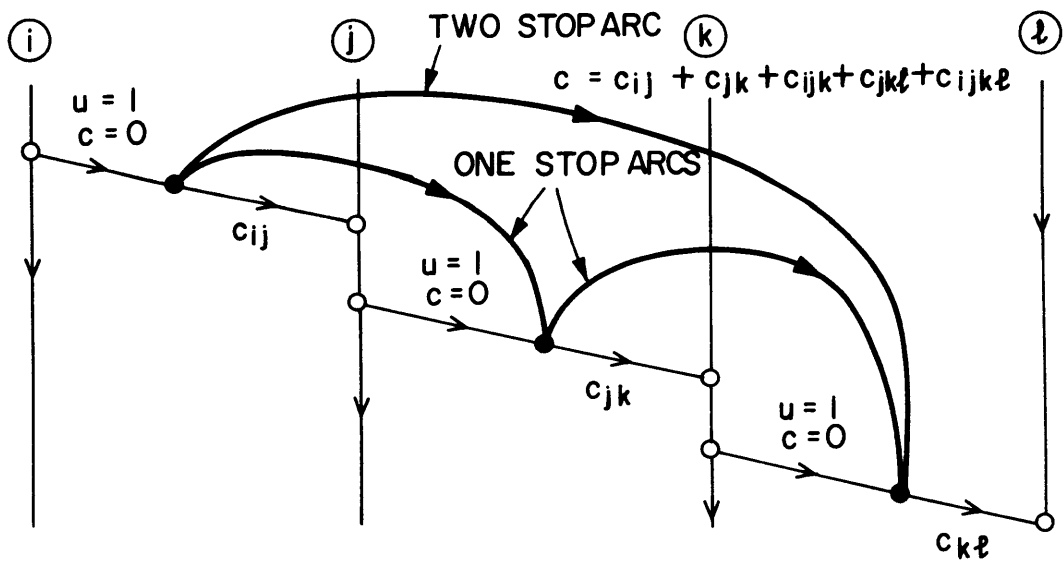
120
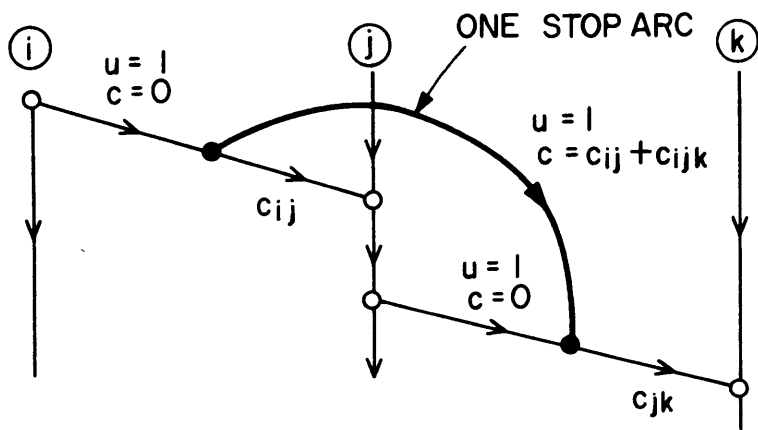
$$c_{ij} = r_{ij} - DC_{ij}$$

$$c_{ijk} = r_{ik}$$

$$c_{jk} = r_{jk} - DC_{jk}$$

FIG. 17a ADDING "ONE STOP" ARCS, M_1

$$c_{jk\ell} = r_{j\ell}$$

$$c_{ijk\ell} = r_{i\ell}$$

FIG. 17b ADDING "TWO STOP" ARCS, M_2

connected, the x_{ij} flow directly transfers to the x_{jk} flow and receives an additional benefit of r_{ik} for doing so.

The "Two-Stop Arcs, <u>M2</u>

Figure 17b shows the network construction for a two-stop flight combination. Here the two "one stop" arcs for ij-jk, and jk-kl are bridged by an il "two stop" arc. Here we can have 3 separate non-stop flights (ij, jk,kl) or two one stops (ijk, jkl), or the two stop flight (ijkl). Unfortunately, it is possible for the two stop flight (ijkl) and non-stop flight (jk) to co-exist, so that an auxiliary or "bundle" condition is necessary:

$$\text{i.e.} \qquad x_{ijkl} + x_{jk} \leq 1$$

The existence of this type of auxiliary constraint places this model with the next set of models which use other solution techniques.

However, if segment jk is not to be considered as part of a service originating in j, the dispatch arc from j to the multi-stop node can be omitted, and this auxiliary condition dropped. In this case, the multi-stop service options are (ijk), (ijk,kl), (kl), (ijkl), (ij), (ij,kl), or none of them.

6.1.4.3 Problem Size

An additional node is required for every service arc which may be considered for potential use as part of a multi-stop flight. One or two stop arcs can be flown out of this

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node connecting to other similar nodes.

If we assume one half the flight services might be considered for multi-stop flights, and they average three possible connections, then for our previous example

No. of arcs $\leqslant$ 3040 + 3(500) = 4540

No. of nodes $\leqslant$ 2040 + 500 = 2540

6.1.4.4 Comments

The extension of model FR-2 to include multi-stop services differs from the extension of FA-1 to model FA-2 in that here the multi-stop flight itineraries are not predetermined. This model allows the optimization process to decide about putting multi-stop flight segments together to form a multi-stop flight itinerary.

If a multi-stop service is predetermined, then the network representation should simply be an arc from the initial departure to the final arrival time, and information should be retained about intermediate station times.

6.1.5 Fleet Routing Example - Tech Airways

An example of a fleet routing problem was constructed using the B 727 frequency pattern for case 4 of the previous Fleet Assignment example for Tech Airways. These 34 aircraft were used on 144 daily non-stop services connecting 9 of the 10 cities. Using n_{pq} results, and an assumed daily variation in demand, optimal dispatching times were computed for each route, thereby constructing an initial timetable of services with a given load and revenue for each service. An appropriate schedule map was then constructed by a preprocessor program which clears the map of unnecessary ground arcs and nodes, and which loads an initial circulation flow. Only 252 arcs and 107 nodes were used, and the OKF coding solved 8 fleet size cases in less than 30 seconds.

The overall set of services flown for each case is too lengthy to be completely shown. Figure 18 shows the variation in system income as fleet size was reduced. The optimal fleet size was 27 aircraft with 9.2 hrs/day average utilization and 8 services were not flown even though they were marginally profitable. As fleet size is reduced, the best set of services to be flown with the restricted fleet was determined. A given service may be dropped, only to reenter a later solution because of its position relative to the flights being flown in the later solution. This is indicated by Table 7 which shows the services operated on route 3-9, and 9-3 as the fleet size was reduced. Notice that the routes dropped may be more "profitable" than some of those retained. Also the service

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from 3-9 at 1851 is not flown on the first two solutions which correspond to 27 and 24 aircraft, and yet it is part of all subsequent solutions for lesser fleet sizes. Because of the time of day routing constraints, the value of a service to the system varies depending upon its position relative to the complete set of services which are going to be flown. The simple net income for the flight segment is not a good indicator of the flight's value, e.g. notice that the 1441 service on 3-9 earns $3410 compared to $1464 for the 1851 service, and yet it is dropped from the optimal solutions for less than 20 aircraft while the 1851 service is retained. Overall, the system can earn more money with 20 aircraft by a set of routings which does not fly the 1441 service and does fly the 1851 service.
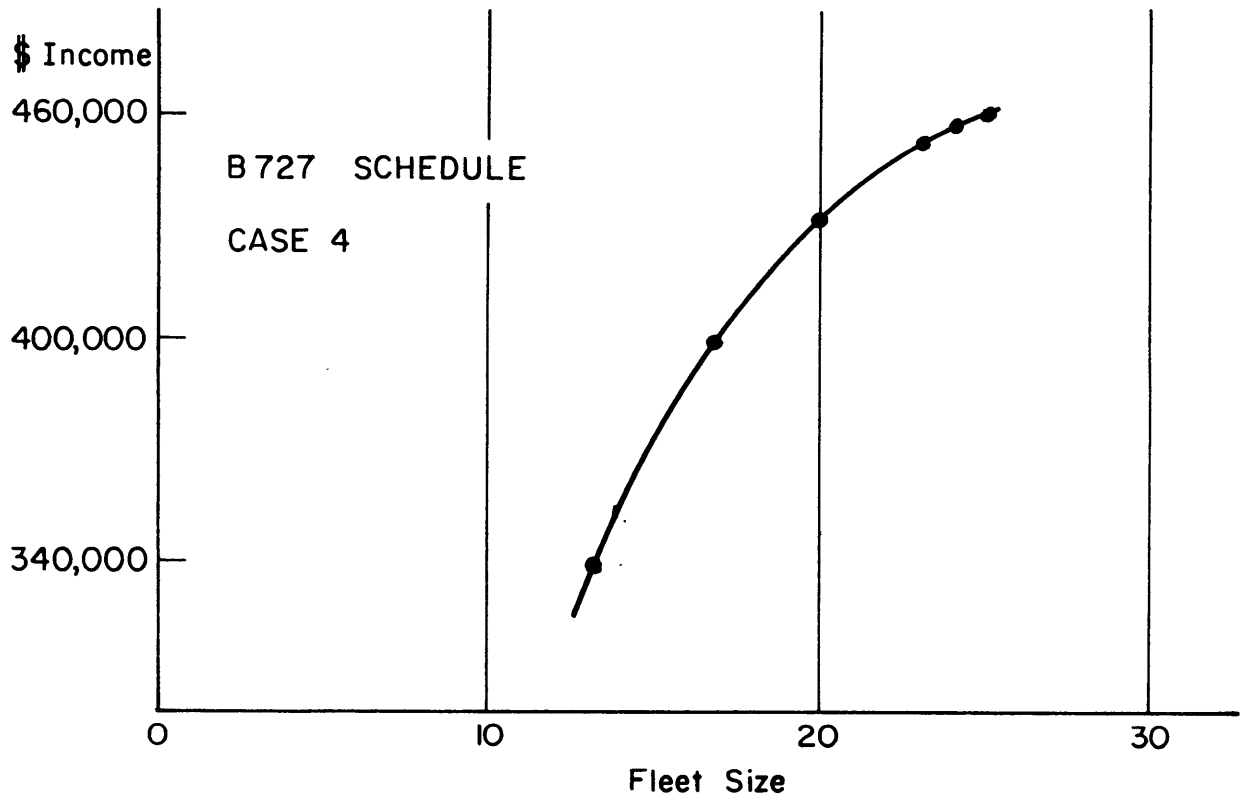
FIG. 18 FLEET ROUTING EXAMPLE, TECH AIRWAYS

TABLE 7. Routes 9-3,3-9, Tech Airways, B727 Schedule

Departure Time	Income	Fleet Size (X = service not flown)						
		27	24	23	20	16	13	
Route 9-3								
645	1771							
759	2386							
832	3410					X	X	
955	1157					X	X	
1037	2386					X	X	
1321	1464				X	X		
1518	3410					X	X	
1716	3410						X	
1941	1464					X		
2027	1464							
Route 3-9								
645	1771							
759	3410					X	X	
913	3410							
955	1157					X	X	
1037	2386					X	X	
1441	3410				X	X	X	
1720	2727							
1811	2727					X	X	
1851	1464	X	X					
2103	1464							

6.2 Single Fleet, Multi-Departure Models

The models of this section avoid the use of a fixed time-table by allowing some variation in the actual departure times for a given service. Instead of opening up every time of day like the dispatching models, it is assumed that a range of departure times exist for every service. Every service is considered independent of other services as assumed in models FR-2, 3, and there may be some variation in load and revenue for different departures within the range. The model now determines not only the best set of services to be flown, but also the associated departure times within each range.

The use of a bundle of arcs to represent a given service is important because of the routing constraints. The variable times for a service allow different connections to be made between services, and can greatly improve aircraft utilization.

Since only one departure time can be used from a bundle of services, a "bundle" constraint must be added to the models. The addition of these constraints destroys the guarantee of integer optimality for an LP statement of the problem, although it seems as though a high percentage of problems would still have integer optimal answers. As a result special computational techniques such as parametrically investigating network flow solutions, implicit enumeration, branch and bound, or the group theoretic ILP approach are used, and all seem to have reasonable success.

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6.2.1 FR-1D Minimum Fleet, Multi-Departure Models

6.2.1.1 Problem Statement

Given a schedule of services to be flown by a fleet
of aircraft of a single type, where a bundle of discrete
departure times are given for each service, what is the
minimal fleet size?

6.2.1.2 Model Formulation

There are a variety of formulations which are possible.
If we follow the network flow formulation, the sets of arcs
$\underline{G}$ and $\underline{C}$ are identical to those in FR-1. The set of flight
services, $\underline{S}$,now consists of bundles of arcs for each service,
where we identify the bundle as set of arcs $\underline{B}_s$ for the sth
service. Previously, the bundle consisted of one member,
and many of the services in this model may still consist of
only one member. A multi-departure bundle is shown graphically
in figure 19.

We then adjoin to the OKF formulation for FR-1 a
set of bundle equality constraints

$$\sum_{ij \in \underline{B}_s} x_{ij} = 1 \qquad\qquad \curlyvee s \in \underline{S}$$

Since x_{ij} can only take the values 0 or 1, we are
effectively selecting the best arc out of the bundle, and
the problem becomes combinatorial in nature. We want to find
the best integer network flow which satisfies these bundle
constraints.

Branch and bound, or implicit enumeration techniques
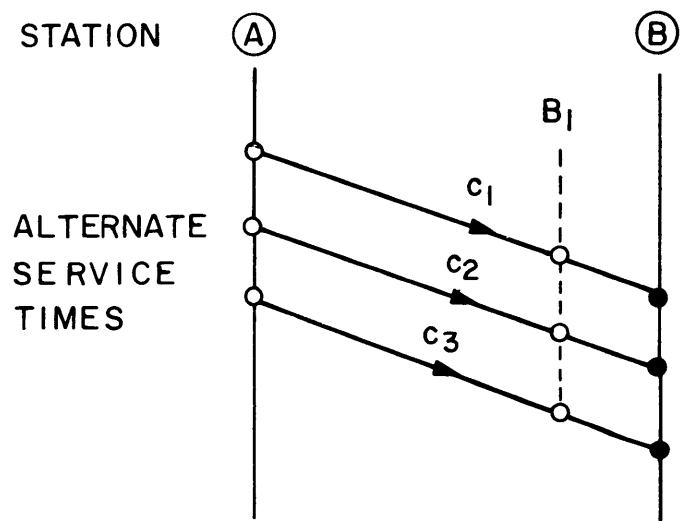
FIGURE 19 MULTI-DEPARTURE BUNDLE

can be used (see reference 5) in an attempt to search the network flow solution space, but since the probability of an integer LP solution seems very high, the preferable technique seems to be to use an LP formulation of the model. Reference 3 uses a Land & Doig algorithm in the event that the LP solution is not integer, with a branching strategy of picking the "earliest" fractional flow arc in the schedule map. The group theoretic approach used on crew scheduling in reference 7 has also been successfully used since experience has shown the size of the determinant associated with the basis of the LP solution is usually very small.

6.2.1.3 Problem Size

For the LP formulation, the number of rows in the LP statement as given in reference 3 is:

$$\text{No. of rows} = |\underline{s}| + \sum_{s=1}^{|\underline{s}|} |B_s|$$

eg. for 500 services averaging 3 departure times per service, the number of rows would be 2000. The LP computation time at this size is currently measured in hours.

6.2.1.4 Comments

A related heuristic routine (reference 6) has been used in a variety of transportation planning studies. Various codings of it exist which can rapidly produce an answer for problems which have up to 10,000 services. Experience on a limited number of comparisons is given in Table 8. For the optimal models (FR-1D) there are three arcs in each bundle: one at each end of the range and one in the middle. For the

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heuristic routine, departures could leave at 6 minute intervals through the range.

Various problems of different size and route maps were solved. For example, problem 3 is the B727, Tech Airways problem used as an example in the previous section. Where the LP model assumed 34 aircraft at 8 hours/day utilization, and the optimal answer for the timetable of the previous section required 27 aircraft, we now see that all services in this timetable can be flown with only 23 aircraft at an average of 11.8 hours/day for an active aircraft. The ranges assumed for departure times are indicated in reference 10. Table 8 shows that the initial timetable required 30 aircraft, and the heuristic routine required 1 pass to reduce this to 25 aircraft. The optimal model required 1.71 minutes where the heuristic took 10 seconds on the same computer.

The purpose of the experiments described in Table 8 was to evaluate the heuristic Reducta. It requires only a small fraction of the LP solution time and can handle extremely large schedules. The results for an improved heuristic called REDUCTA 2 show that it can produce the optimal answer for small fleet problems, but tends to diverge as much as 10% as problem size increases. A complete description of these tests is given in References 10 and 11.

Problem Number	Number of Stations	Number of Flights	Number of Routes	Number of Aircraft in Original Schedule	Reducta Solution	MPS Solution	MPS Integer Solution	REDUCTA 2 (Improved) Solution	Computer Time Reducta	Computer Time MPS	Rows	Columns
1	6	54	10	16	11	11	11	11	.7sec	.55min	153	259
2	4	36	12	15	12	10.5	11	11	7 "	.22 "	98	108
3	8	144	22	30	25	23·	23	24	10 "	1.71 "	406	692
4	8	188	20	28	19	18.0	18	17	10 "	2.63 "	498	872
5	7	56	12	18	11	10.33	11	11	10 "	.44 "	159	269
6	10	296	60	67	52	42.0	NA	45	17 "	16.96"	683	1273
7	26	240	91	72	43	33.1	NA	37	13 "	33.0 "	600	1078

TABLE 8.   COMPARISON OF REDUCTA WITH OPTIMAL RESULTS

6.2.2 Model FR-2D – Maximum Income for Multi-Departure Times

6.2.2.1 Problem Statement

Given a schedule of possible services, where a bundle of discrete departure times are given for each service, and an associated income is known for each departure time, which departure services should be flown, and what is the fleet size required for maximum systems income?

6.2.2.2 Model Formulation

If we follow the network formulation of FR-2, the sets of arcs $\underline{G}$ and $\underline{C}$ are similar. The set of flight services $\underline{S}$ now consists of bundles of arcs, $\underline{B}_s$ for the sth service. Each bundle arc is defined similar to the service arcs where c_{ij} now represents an estimated income (negative value) for the service if operated at this departure time.

We then adjoin to the problem a set of bundle inequality constraints which allow the service not to be operated if that is desirable.

$$\sum_{ij \in \underline{B}_s} x_{ij} \leq 1 \qquad\qquad \forall s \in \underline{S}$$

Since we are seeking an integer network flow, where x_{ij} can be either 0 or 1, the constraint effectively states that, at most, only one arc out of the bundle can be used. The problem thus becomes combinatorial similar to FR-1D

There are a variety of techniques which have been used to solve this problem. One approach is to use a searching technique which solves an OKF network flow problem while controlling the upper bounds on members of each bundle. This

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approach is described in references 8 and 9, and other unpub-
lished FTL work. The model has been solved using a Land & Doig
technique as formulated in reference 3 (ILP 3.4), and using
the group theoretic techniques.

A decomposition approach has also been attempted.
Here the bundle constraints are kept in the master problem
so that the subproblem (model FR-2) can be quickly solved
using OKF. A price is added to each member of a bundle
until the adjoined constraints of the master problem are
satisfied. If the optimal answer to the LP is integer, it is
possible to obtain a single subproblem network solution which
is optimal in this manner. In general, decomposition will give
an optimal mix of the integer subproblem solutions which will
not be integer itself.

6.2.2.3 Problem Size

In the LP formulation, the number of rows is identical
to model FR-1D. If the network flow formulations are used,

$$\text{No. of arcs} \leq 3 \left| \sum_{s=1}^{|\underline{S}|} \underline{B}_s \right| + 2 \left| \underline{c} \right|$$

$$\text{No. of nodes} \leq 2 \left| \sum_{s=1}^{|\underline{S}|} \underline{B}_s \right| + 2 \left| \underline{c} \right|$$

eg. for 500 services averaging 3 departure times/service over
20 cities,

No. of arcs $\leq$ 4540

No. of nodes $\leq$ 3040

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6.2.2.4 Comments

If there were no ownership cost for vehicles, the most profitable arc in each bundle would be flown. As ownership (or cycle arc costs) are increased, fewer aircraft are used since they must earn at least the cycle arc cost during the rest of the day. The multiple departure times for a given service allow many more connections between services and provide opportunities for more services to be flown by a given number of aircraft. It is a valuable extension of the model for the schedule planner.

At the present time, there is no clear answer to the question of a preferable solution technique. This will vary with the problem and its size, and there probably will be further development of improved solution techniques for this problem, and the other "bundle" network problems. Experience with the LP formulation has indicated so far a very high proportion of integer optimal answers. This fortunate occurrence has directed attention towards formulating the model as an LP, and using the Land and Doig or group theoretic techniques to find the ILP answer only when the LP solution turns out to be fractional. As problem sizes increase, and LP solution times increase, the percentage of integer LP solutions may decrease, and a combinatorial search using network methods may become preferable.

6.2.3 Model FR-3D Maximum Income, given Fleet Size

6.2.3.1 Problem Statement

Given a schedule of possible services $\underline{S}$, where a bundle of discrete departure times is given for each service, and an associated income is known for each departure time, which departure services should be flown to maximize system income given the number of aircraft in the available fleet?

6.2.3.2 Model Formulation

This model is a simple extension of FR-3 to handle multi-departure bundles of arcs. The network formulations used on FR-2D are not attractive for this model. It is easier to use the LP formulations of the problem, adding the single additional bundle constraint,

$$\sum_{\underline{C}} x_{ij} = A'$$

Since A' is an integer number, this constraint tends to aid the LP solution in finding an integer solution. One can add a similar constraint to FR-1D and cause the LP solution to give an integer optimum.

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6.3 Single Fleet, Bundle Constraint Models

Since the ILP or combinatorial programming methods allow
optimal solutions to be found, any of the constraints from
section 1 which were applied to the Fleet Assignment models
can be considered for application to the Fleet Routing models.
Here we shall define three such models: the first applies the
constraints ensuring that certain city pair routes are flown
at least a minimum number of times per cycle; the second
applies constraints to the maximum number of cycle operations
at a given station; and the third introduces a new constraint
on the number of aircraft which can be served simultaneously
at a given station.

They are posed here as extensions of model FR-2D the maxi-
mum income, multi-departure times model, and their model
name has been framed so as to indicate this fact. The behaviour
of the models will depend upon the number and type of constraints
added to the basic fleet routing problem. They are just
formulated here for definitional purposes.

6.3.1 Model FR-2DC4 Maximum Income, Route Frequency Constraints

6.3.1.1 Problem Statement

Given a schedule of possible services, $\underline{S}$, and a bundle of departure times for each service with an associated income for each departure, what set of departures should be flown to maximize system income such that the number of services flown on a given route exceeds a minimum required frequency, $N \min_{pq}$?

6.3.1.2 Model Formulation

The problem is similar to model FR-2D, with the additional "service" constraints applied. For a given city pair, pq, we define a bundle $\underline{B}_{pq}$ consisting of all services (or bundles of services) possibly flown for that city pair.

The additional constraint is number 4 in Table 1.

ie.
$$\sum_{ij \in \underline{B}_{pq}} x_{ij} \geqslant N \min pq$$

The constraint is shown in graphical form in figure 20, where B_{AB} represents a constraint over all bundle members for AB services. B_{AB} contains the departure bundles as subsets.
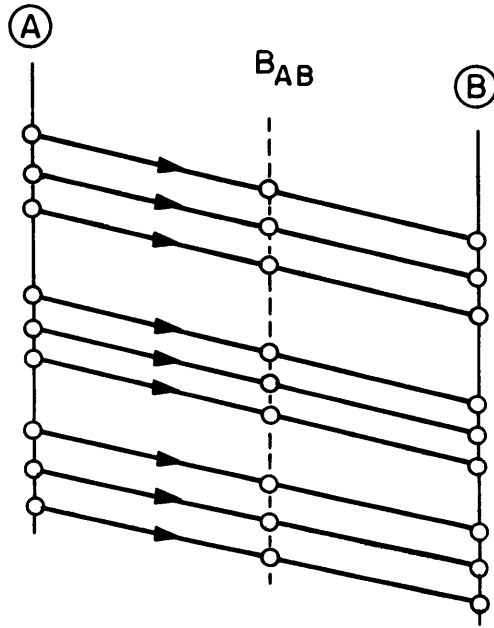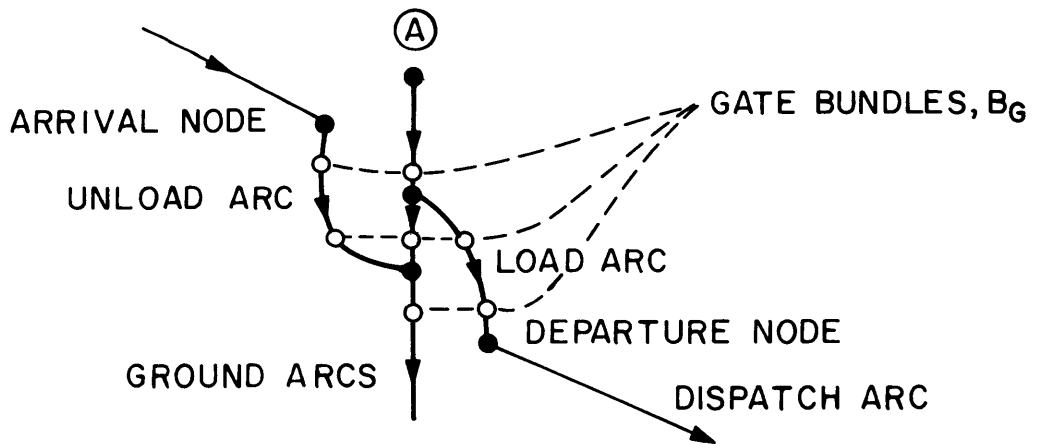
FIGURE 2O FREQUENCY BUNDLES

6.3.2 Model FR-2DC5 - Maximum Income, Airport Constraints

6.3.2.1 Problem Statement

Given a schedule of possible services, $\underline{S}$, and a bundle of departure times for each service with an associated income for each service, what set of departures should be flown to maximize system income such that the number of services flown into a given airport does not exceed a maximum limit imposed by airport congestion, $N\ max_p$?

6.3.2.2 Model Formulation

The problem is similar to model FR-2D, with additional "airport" constraints applied where necessary. For a given station p, we define a bundle $\underline{B}_p$ consisting of all services (or bundles of services) into (out of) the station.

Then, the additional constraint is number 5 in Table 1.

$$\sum_{ij \in \underline{B}_p} x_{ij} \leq N\ max\ p$$

The constraint is shown in graphical form by figure 21 where B_{pa} represents a constraint over all services into city a. The bundle B_{pa} contains other departure bundles as subsets.

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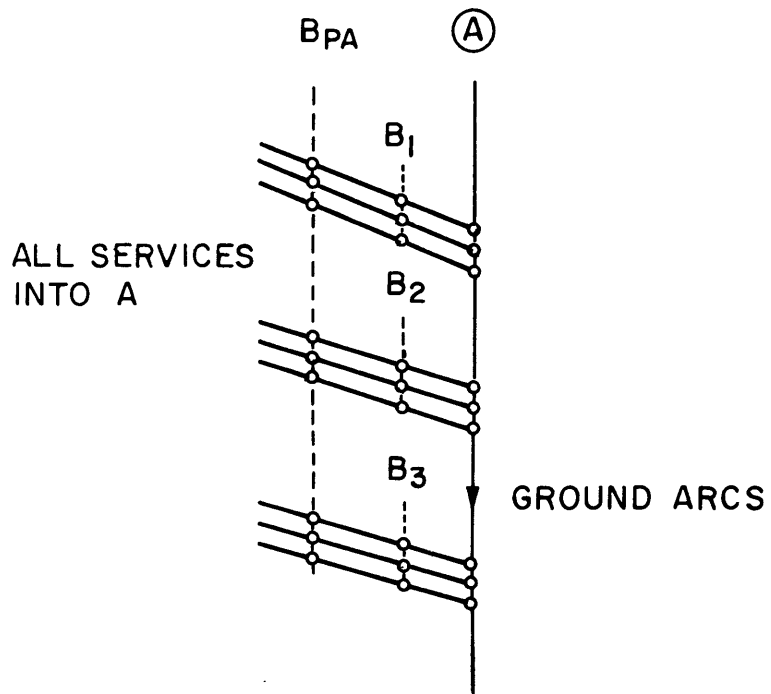
FIGURE 21   AIRPORT BUNDLES

FIGURE 22 GATE BUNDLES

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6.3.3 Model FR-2DC10 - Maximum Income, Gate Constraints

6.3.3.1 Problem Statement

Given a schedule of possible services, $\underline{S}$, and a bundle of departure times for each service with an associated income for each service, what set of departures should be flown to maximize system income such that the number of active aircraft on the ground does not exceed the number of available gates?

6.3.3.2 Model Formulation

The problem is similar to model FR-2D, with additional "gate" constraints which are probably added to the formulation after a tight situation occurs at some station. Although these constraints can be expressed without adding new nodes or arcs, the model is most easily explained by defining two sets of arcs:

1) "unload" arcs, set $\underline{U}$

2) "load" arcs, set $\underline{L}$

For every ground arc in $\underline{G}$, there is an associated bundle $\underline{B}_g$ consisting of the arcs in $\underline{U}$ and $\underline{L}$ which coexist within the time of the ground arc. This is illustrated in figure 22 where the dotted lines indicate the bundles for each ground arc.

We now define a variable N_{gp} = number of aircraft on the ground either loading, unloading, or idle at station p

$$N_{gp} = \sum_{ij \in \underline{B}_g} x_{ij} \quad \text{for every ground arc g.}$$

The gate constraint is a bound upon N_g

$$N_g \leqslant G_p$$

It is possible, of course, to place the constraint upon the unload-load arcs only, assuming that idle aircraft can be towed away from a gate.

6.3.3.3 Comments

The "gate" bundles as formulated here are independent of the departure bundles of service arcs, $\underline{B}_s$, and do not contain any other bundles as subsets.

6.4 Multi-Fleet Routing Models

We now extend the fleet routing models to cover multiple
fleets of different types of aircraft. The different types
of aircraft will have different operating costs, different
capacity, and varying speed and range capabilities which
will make them preferable for use on certain services. For
some services, certain aircraft may not be eligible because
of stage length or traffic load size.

As a result we can associate a "copy" of the schedule
map with each type of aircraft. The copies will not be iden-
tical, but will overlap since certain services will be pos-
sible with two or more types of aircraft. Bundle constraints
are then applied over the arcs representing the same service
appearing in different copies of the schedule map.

While the flow in each copy may be integer, the bundle
constraints may cause the LP statement of the model to give
a fractional flow at the optimum. The special techniques for
integerizing such answers, or for searching for the optimal
combination of integer network copy flows can again be suc-
cesfully applied. Experience again seems to indicate a high
percentage of integer LP solutions.

Here two multi-fleet models are formulated simply for
definitional purposes. All of the previous fleet routing
models may be extended as multi-fleet models, but computa-
tional experience has not been achieved with such extensions.

6.4.1   Model MFR-1   Minimize Total Fleet Size

6.4.1.1   Problem Statement

Given a set of non-stop services $\underline{S}$ to be flown by a fleet consisting of different types of aircraft, and a copy of the schedule map $\underline{S}_a$ constructed for each aircraft type using its speeds and eligibility for each service, find the minimal number of aircraft required, and the routing pattern for each aircraft type such that every service is flown.

6.4.1.2   Model Formulation

This is an extension of model FR-1 to cover the multi-fleet problem. Let the copy of the schedule map for aircraft a be represented by $\underline{S}_a$. Let the set of bundles of arcs representing a given service S in two or more copies be $\underline{M}_s$. Then we adjoin a set of constraints for each service

$$\sum_{ij \in \underline{M}_s} x_{ij} = 1 \qquad\qquad s \in \underline{S}$$

to the multicopy network formulation. Since the values which x_{ij} can take are 0 or 1, we are finding the best combination of integer flows on schedule map copies which satisfies these constraints.

The model can be posed as an LP problem with the multi-copy constraints added. Then the Land & Doig, or group theoretic methods can be used to find the best integer flows. If the number of multi-copy constraints is not too large, and the individual copies are network flow problems solvable by OKF as postulated here, a branch and bound technique similar to that used in reference 8 can be used to search for the best combination of integer network flows.

6.4.1.3 Problem Size

If there are $|a|$ types of aircraft, and MC multi-copy constraints, the number of rows in the LP formulation becomes,

$$\text{No. of rows} \leq 2 \sum_a |\underline{S}_a| + MC$$

where there are $|a|$ copies of the basic LP formulation of FR-1, and an additional MC rows of multicopy constraints.

For the branch and bound search, as many as $2|a|$ network flow problems must be solved at each stage of the solution tree, and there are MC stages of $|a|$ branches at each stage.

6.4.1.4 Comments

The Land & Doig algorithms and the group theoretic ILP methods seem to work successfully on these problems. However, large scale, airline size problems have not yet been tried. The mathematical problem is well known as a multi-copy or multi-commodity network flow problem for which several proposed techniques now exist. The best technique for schedule map or routing networks is yet to be determined since the solution efficiency varies with the size and type of problem.

6.4.2 Model MFR-2 Maximum Income

6.4.2.1 Problem Statement

Given a set of non-stop services $\underline{S}$ to be flown by a fleet consisting of different types of aircraft, and a copy of the schedule map $\underline{S}_a$ constructed for each aircraft type using its speeds, eligibility for service, its income value if it flies the service, and its daily ownership costs, find the combination of routing patterns for each aircraft type which maximizes system income such that every service is flown at most by one aircraft type.

6.4.2.2 Model Formulation

This is an extension of model FR-2 to cover the multi-fleet problem. If the bundle of arcs representing a given service, s, common to two or more copies be $\underline{M}_s$, then we adjoin a constraint

$$\sum_{ij \in \underline{M}_s} x_{ij} \leq 1 \qquad\qquad s \in \underline{S}$$

to the multi-copy formulation. This ensures that the service can be flown by at most one type of aircraft. The model can be formulated as an LP problem which has to be integerized, or as a branch and bound search using network flow solutions.

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7.0 Summary

This report has described the state of the art for computer
models concerned with scheduling problems for public transpor-
tation systems for which optimal solutions can be obtained with
present computer technology. As computer hardware and software
improves, and as new methodology is added to mathematical pro-
gramming techniques, larger and more sophisticated models can
easily be envisaged. There is still much work to be done in
generating good software and gaining computational experience
with the models described in this report.

It is difficult to comment generally on the usefulness
of the models in view of the spectrum of users and their goals.
In some cases, these models have seen many years of successful
application in airline schedule planning. Others have been used
once or twice in various planning studies with apparent satis-
faction to those involved in the study. It must be remembered
that they are models of the real world problems, and as such
are idealized representations. This report has emphasized the
description of the models, and has placed them together in order
to make some analysts realize the shortcomings of their models
in certain aspects. For various purposes, the models can be
extremely useful to an analyst providing he has a broader under-
standing of scheduling problems which permits him to apply the
appropriate model to his problem. Model building is an excel-
lent way to gain this broader understanding since it forces one
to be quite specific about describing the way in which various
factors enter these problems. This process also clearly deline-
ates the need for other studies that should be done , e.g.,

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there are a number of econometric studies of traffic forecasting and market share models suggested by model FA-3 and extensions of it.

There is a clear division of the categories of models of this report as to their usage. Both Fleet Assignment and Fleet Planning models are strategic models in the sense of giving an incomplete answer to schedule construction. Their output is a frequency pattern for the system under the assumed aircraft utilization. While this result can be useful for many long range planning purposes in that some idea of the levels of service and operating costs for the system are obtained, the assumed utilization means the exact fleet size is not known, and the time of day schedule and aircraft routings are unknown. The Fleet Routing models allow this next improvement, and are useful in performing shorter range schedule planning and playing a role in the actual schedule planning process. The purpose of the study, the required accuracy and confidence of its answer will determine whether it is necessary to consider a much larger routing model rather than the more approximate "usage" or "assignment" models.

There are so many directions of future development of computer models in this area, it would be difficult to describe them all: problems of traffic flowing over a network in various ways, the generation of optimal multi-stop itineraries, problems which arise in scheduling cargo or freight delivery systems, the prospect of using real time traffic information for dynamic scheduling or partially dynamic scheduling of transportation systems, a number of detailed problems in schedule control are all areas for a great deal of interesting future work. As well, it seems clear that econometric traffic forecasting models in

which we have some confidence will be required before useful
computer applications can be envisaged. Trend forecasting
will not be useful, and a great deal of study seems to be in-
dicated to relate past traffic statistics to levels of service
in the various markets.

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