15.093J/2.098J Optimization Methods
Midterm Solutions

Problem 1

1. False
   The problem of minimizing a convex, piecewise-linear function over a polyhedron can be formulated as an LP problem, but not the problem of maximizing.
2. True
   The dual problem is $\min 0p_1$ subject to $p_1 \leq 1$.
3. False
   Multiple optimal solution case.
4. False
   There are more than one basis associated with a degenerate extreme point.
5. False
   Degenerate optimal basic feasible solution.
6. False
   Only problem infeasibility.
7. False
   If we consider the standard simplex algorithm (without multiple optimal solution case).
8. False
   An optimal solution that is not a basic solution.
9. True
   Consider the point $(0, 0, 10, 10)$
10. True
   Hypercube in $\mathbf{R}^n$

Problem 2

(a) Optimal solution $(1, 1, 0)$, optimal profit $3 + 4 = 7$. The solution is not degenerate, only three tight constraints at the optimal solution.

(b) Yes, the dual price is $1.4 > 1$.

(c) Yes, the lower bound for profit is $1.4 \times 1.667 > 1.5$.

(d) The profit must increase more than \$0.6.

(e) 3 is in the allowable decrease range, the current plan is still optimal, thus $x_3 = 0$, $2x_1 + 3x_2 = 2$, and $x_1 - x_2 = 0$ or $x_1 = x_2 = 0.4$ and $x_3 = 0$.

(f) \$0.75 is the limit of the allowable decrease range. Thus the current solution is still optimal. However, the reduced cost of some non-basic variables will become zero. The current solution is nondegenerate; therefore, there will be alternative optimal solution.

(g) The current optimal solution is unique. You can calculate the reduced cost for all nonbasic variables or just need to look at the allowable increase and decrease range of all cost coefficients (no zero limit).

(h) Upper bound $\$1.4 \times 3$, lower bound $\$1.4 \times 1.667$.

(i) If the difference between x_1 and x_2 is increased by 1 then the profit will increase by \$0.2.

Problem 3

(a) $\gamma \geq 0$, we can also argue that in order to have a valid tableau, $\alpha \neq 0$ (basis is invertible)

(b) $\gamma \geq 0$ or $\gamma < 0$ and $\alpha > 0$

(c) $\alpha > 0$, otherwise, we will have an unbounded direction.

(d) $\gamma = 0$, no restriction on α (or $\alpha \neq 0$ as argued above)

(e) $\gamma \geq 0$, $\alpha > 0$: problem is dual unbounded.

(f) Pivot element is α ($\alpha < 0$).

(g) Optimal cost is 10. The solution is degenerat if $\beta = 0$. Optimal basis $[A_1, A_2]$, basis inverse is $\begin{pmatrix} -2 & -3 \\ 0 & \alpha \end{pmatrix}$

(h) $[2 \quad \gamma] = [0 \quad 0] - p'$, thus $p = [-2 \quad -\gamma]'$

(i) $\epsilon \leq \frac{3}{2}$

(j) $2 + 2\epsilon \geq 0$ and $\gamma + 3\epsilon \geq 0$