Recitation Notes 8

Konrad Menzel

November 3, 2006

Disclaimer: This material is only meant to spell out some theoretical footnotes to what we discussed in the lecture. This isn't relevant for the exam and probably not going to be of great help for the current problem set.

1 Motivation

Result from Acemoglu and Pischke model as presented in the lecture yesterday: firms expected profit is

$$\pi(\tau) = (1-p)(1-q)\beta[\tau - v(\tau)] - c(\tau)$$

where

$$v(\tau) = \frac{q(1-p)}{p+(1-p)q}\tau$$

Now assume the worker can't make the investment or finance it through a wage cut in the first period, so that the investment in skills is only what the firm is willing to pay. I.e. the firm will choose the profit-maximizing τ, which satisfies the first-order condition

$$c'(\tau) = (1-p)(1-q)\beta\frac{p+(1-p)q-(1-p)q}{p+(1-p)q} = (1-p)(1-q)\frac{p\beta}{p+(1-p)q} < (1-p)(1-q)$$

whereas the Pareto-optimal level would satisfy $c'(\tau) = (1-p)$. Since the cost function is convex, this means that there would be *underinvestment* in skills relative to first best (again with the caveat that we might imagine alternative settings in which the worker would also contribute to the investment).
So how does that go with the first Welfare theorem? Where exactly is the market imperfection which prevents firms and workers to achieve first-best? There is clearly no information problem in the classical sense (if we forget for a second about the adverse selection problem which outside firms are facing in the Acemoglu and Pischke QJE paper, and which is not really relevant for this question).
If we stare for a while at the formula, we see that the problem is that the firm isn't the full residual claimant for the social returns to the investment in skills for two reasons:

1. The firm can only retain a fraction $1-q$ of the productive workers which had received training. Therefore, there is a positive externality on outside firms

2. Wages are set only after the investment in skills is sunk and workers are in a stronger bargaining position because of their improved outside option

There are several reasons why firms and workers do not write a binding contract ex ante which conditional on the upfront investment

- Any contract has to be enforced by a third party (usually the courts) which may only imperfectly observe the actual investment done by either party even if investment is common knowledge of firm and worker - this is the difference between observability and verifiability
- The original contract may turn out to be suboptimal once the investment is sunk, so that both parties would be willing to renegotiate the terms once the investments have been made
- In order to support the optimal investment level, the contract may impose constraints on future behavior or have a payment structure which are considered to be unfair, illegal, or immoral (e.g. indentured service).
- There are some interim decisions which will depend on a state of the world which is only realized after the contract has been made, and which is sufficiently complex to describe in order to make it prohibitively costly to lay out a complete contingency plan in the original contract.

Particular problem with human capital: inalienable, firm can't claim their investment in skills back if the worker reneges on his promise to continue to work for the firm. Therefore, a firm's investment in a particular worker's skill is always relationship-specific from its point of view.

2 Bargaining

This follows loosely the presentations in Mas-Colell, Whinston & Greene, chapter 22.E., and Cahuc & Zylberberg ch.7.2.

how to split up a fixed joint surplus (e.g. decisions inside the household, bargaining between employers and unions, or a cartel trying to negotiate each firm's share of total profits).

There are 2 players who start off from a (suboptimal) status quo (or *threat point*), which is typically a non-cooperative Nash equilibrium players can return to if the bargaining process breaks down, and the payoffs under the status quo are $\underline{u}_1$ and $\underline{u}_2$ for player 1 and 2, respectively, and players could potentially improve on the status quo by moving to some other payoff vector (u_1, u_2) in the utility possibility set U.

Two major approaches -

2.1 The Strategic Approach

There is the "strategic" approach of Rubinstein-Stahl bargaining, which sets up the problem as a sequence of offers and counter-offers when there is a time cost of haggling over the outcome. The unique subgame-perfect equilibrium of this dynamic game under certainty yields an immediate agreement in which each party gets a fixed share of the surplus (where players who are more patient - i.e. have a lower discount rate - can credibly threaten that they would be the more persistent hagglers, and therefore get a larger share than their impatient opponents). If two players bargaining over transferable utilities (i.e. money), in equilibrium the first player to make a proposal gets a share

$$x_1 = \frac{1-\delta_2}{1-\delta_1\delta_2}\bar{x}$$

of the total "pie" $\bar{x}$.

2.2 The Axiomatic Approach

The "axiomatic" approach doesn't take a stand on the institutional set-up under which the bargaining takes place, but focus on the outcome which is formalized as the *bargaining solution* $f(\underline{u}, U) := \left[{}^{[} u_1^*(\underline{u}, U), u_2^*(\underline{u}, U)\right]$, which can be interpreted as a choice rule implemented by some impartial arbitrator, but agreed upon ex ante by the players.

There are three minimal consistency requirements we'd like to impose on any reasonable bargaining solution:

1. *Individual Rationality (IR)*: Each participant has the right to opt out, and must prefer the bargaining solution over the threat point, i.e.
$$f(\underline{u}, U) \geq \underline{u}$$

2. *Pareto Optimality (P)*: players exploit all mutual benefits, i.e.
$$\nexists u \in U \text{ such that } u > f(\underline{u}, U)$$

3. *Independence of Irrelevant Alternatives (IIA)*: the bargaining solution doesn't change if we remove suboptimal (i.e. "irrelevant") choices from U, i.e.
$$U' \subset U \text{ such that } f(U) \in U' \Longrightarrow f(U') = f(U)$$

These assumptions aren't too restrictive and still allow for many plausible bargaining solutions, e.g.

- *"Utilitarian"* $f(\underline{u}, U)$ maximizes $\sum_{i=1}^2 \gamma_i [u_i^* - \underline{u}_i]$. This looks a bit like the optimum of a welfare maximization problem with a linear social welfare functional.

- *"Egalitarian"* $f(\underline{u}, U)$ maximizes $\min\{\gamma_1(u_1^* - \underline{u}_1), \gamma_2(u_2^* - \underline{u}_2)\}$, which resembles a Leontief-type social welfare function.

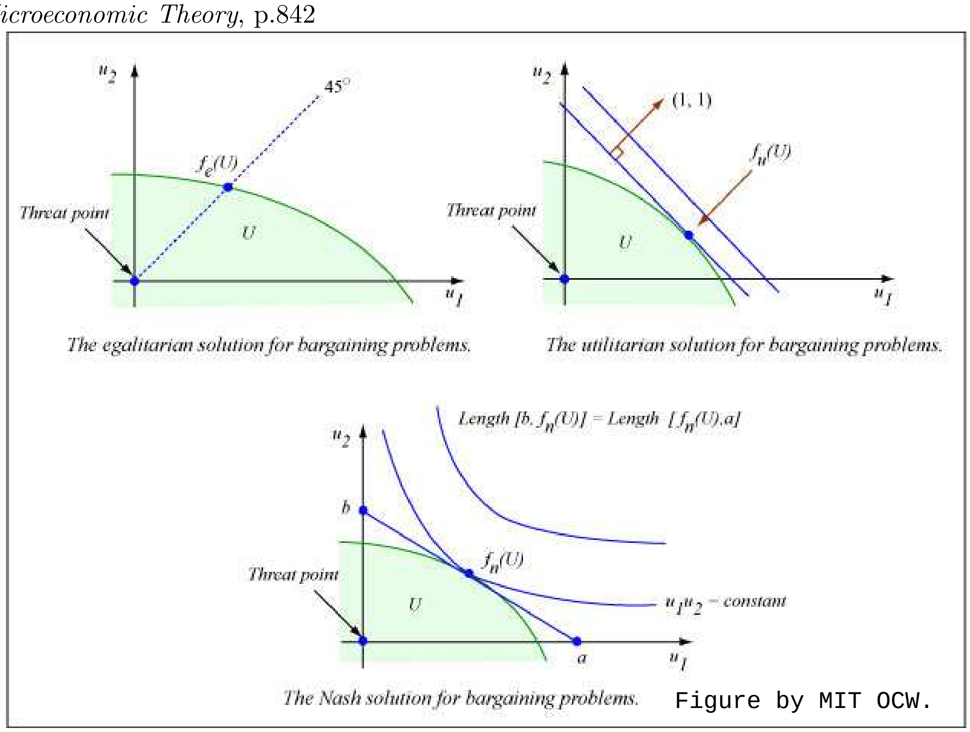
- *"Nash Solution"* $f(\underline{u}, U)$ maximizes $\sum_{i=1}^2 \gamma_i \log(u_i^* - \underline{u}_i)$, which is a Cobb-Douglas welfare functional on the share of the surplus

If utility is transferable (i.e. the utility possibility set is linear) as e.g. in the hold-up models and Rubinstein-Stahl bargaining, the utilitarian solution is either at the corner or includes the whole efficient frontier of the utility possibility set (and therefore doesn't give any useful answer for our bargaining problem).
There is however another caveat: all these "welfare-based" criteria are based on interpersonal utility comparisons, which as we know, is empirically meaningless. It is not surprising that no bargaining solution is robust with respect to an arbitrary monotone transformation of utilities, but there is a weaker requirement we can impose without killing any possible choice rule:

4. Invariance to Positive Linear Utility Transformations (INV): define $a + bU := \{[a_1 + b_1 u_1, a_2 + b_2 u_2] | [u_1, u_2] \in U\}$, where $a, b \in \mathbb{R}_+^2$. Then the choice rule satisfies
$$f([a_1 + b_1 \underline{u}_1, a_2 + b_2 \underline{u}_2], a + bU) = a + bf([\underline{u}_1, \underline{u}_2], U)$$

It turns out that the only bargaining rule satisfying this requirement is the Nash solution, which depends only on the cardinal characteristics of *each* individual utility function, but does not rely on interpersonal comparisons.

Figure 1: Illustration of the three bargaining rules described above from Mas-Colell, Whinston and Greene (1995): *Microeconomic Theory*, p.842



Proposition 1 (MWG Theorem 22.E.1): *If U is convex, any bargaining solution satisfying (IR), (P), (IIA), and (INV) is a Nash solution with utility weights $\gamma_i \in [0,1]$.*

Proof: Assume that a choice rule $f(\cdot)$ satifies the four assumptions, and our goal will be to show that $g(\cdot)$ has to be identical to the Nash solution $f_N(\cdot)$. Can normalize $\underline{u} = 0$, and let $[\hat{u}_1(\gamma), \hat{u}_2(\gamma)]$ be the Nash solution $f_N(0, U)$ for some arbitrary U which corresponds to the maximization problem $\max \sum_{i=1}^2 \gamma_i \log(u_i)$. Define some "artificial" utility possibility set which we'll need for the proof:

$$U'(\gamma) := \left\{ u \in \mathbb{R}^2 \middle| \gamma \frac{u_1}{\hat{u}_1(\gamma)} + (1-\gamma)\frac{u_2}{\hat{u}_2(\gamma)} \leq 2 \right\}$$

for some arbitrary choice of $\gamma \in [0,1]$.
Now note that $U \subset U'(\tilde{\gamma})$ because U is convex and the gradient of $\sum_{i=1}^2 \gamma_i \log(\hat{u}_i)$ is equal to the slope of the frontier of $U'(\tilde{\gamma})$, which also equals $\left[\frac{\tilde{\gamma}}{\hat{u}_1(\tilde{\gamma})}, \frac{1-\tilde{\gamma}}{\hat{u}_2(\tilde{\gamma})}\right]$ *draw graph to illustrate this: by the implicit function theorem (or an intuitive argument based on the optimality of $(\hat{u}_1, \hat{u}_2)$), the gradient of the criterion function is orthogonal to the tangency to U at the Nash solution, so that by convexity the whole set U must lie below that tangency, which in turn is the boundary of U' as we constructed it.*
Now consider the choice rule $g(\cdot)$ on a simple linear utility possibility set

$$U'' := \left\{ u \in \mathbb{R}^2 \middle| u_1 + u_2 \leq 1 \right\}$$

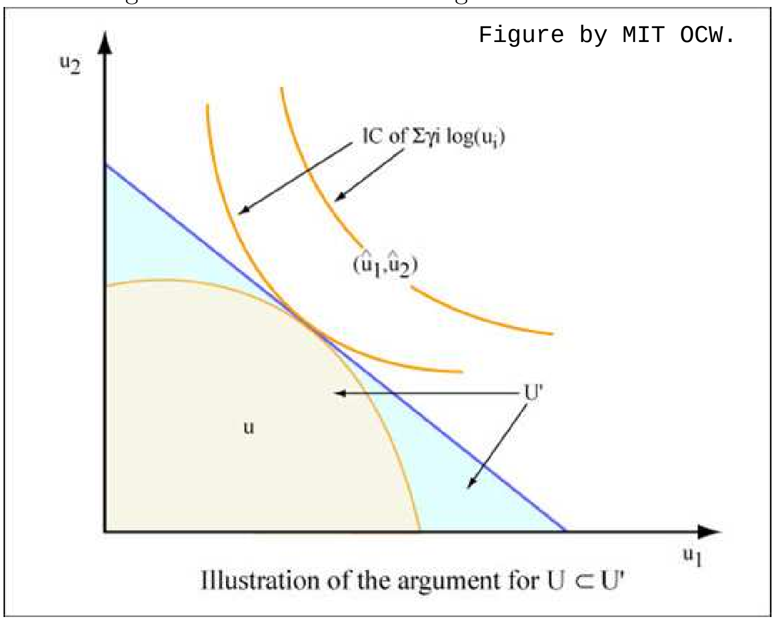
Individual rationality implies that within the set U'', each individual must get a utility level between 0 (the threat point) and 1 (the maximal amount to be split among both players). The Pareto criterion implies that the choice rule $g(\cdot)$ must give $g(0, U'') = [\tilde{\gamma}, 1-\tilde{\gamma}]$ for some value $\tilde{\gamma} \in [0,1]$. By the invariance property on U, $g(0, U'(\tilde{\gamma})) = [\hat{u}_1, \hat{u}_2] = f_N(0, U)$ (where $b_i = \frac{\gamma_i}{\hat{u}_i}$).
Since $U \subset U'$ and $g(0, U') \in U$, we can use (IIA) to arrive at $g(0, U) = g(0, U'(\tilde{\gamma})) = f(0, U)$, so that $g(\cdot)$ is identical with the Nash bargaining solution $f(\cdot)$. QED

The intuition for this result is relatively straightforward: only a bargaining rule that awards each party a constant *share* of total surplus, regardless the overall size of the surplus, can be invariant with respect to linear transformations. Looking at the maximand which generates the Nash bargaining rule, this works off the same property of the Cobb-Douglas function which produces constant budget shares if it shows up as a utility function (i.e. with elasticity of substitution equal to minus 1, income and substitution effects cancel out).

2.3 Interlude: Efficient Bargaining with Inefficient Outcomes?

Now after this discussion of classical bargaining you should be puzzled how this actually relates to the obvious inefficiency in the Acemoglu and Pischke setting - after all we imposed Pareto efficiency as an axiom in order to justify the Nash bargaining solution!
The resolution of this "paradox" is that once the parties sit at the negotiating table, outcomes will in fact be efficient *conditional* on agents' choices at preceding stages. The true source of the problem is that the parameters of the bargaining problem - the outside options $(\underline{u}_1, \underline{u}_2)$ and the bargaining weights γ_i - may not be fixed *ex ante*, but players may (and will) make otherwise suboptimal choices and even engage in damaging activities to enhance their ex-post bargaining position.
In the classical version of this problem, agents try to skew the threat point in their favor, but there are also models e.g. in intra-household bargaining in which members manipulate the bargaining weights through labor supply decisions and by changing the structure of the household's assets (though typically there's an equivalent formulation in terms of changes in the status quo).

Figure 2: Illustration of the argument for $U \subset U'$



3 The Hold-Up Problem - Investment in Physical Capital

Regardless of which view we take on the ex-post bargaining problem, the central problem for the incentives to invest ex ante is that the investment does not only change the size of the overall surplus (i.e. the "total pie"), but also the individual threat points/outside options $(\underline{u}_1, \underline{u}_2)$. In the Acemoglu and Pischke model on provision of general skills, the firm's threat point is at zero (no production, note that the cost of skill provision is already sunk at that stage) and therefore doesn't depend on the level of training. The problematic side is that of the worker, since the outside option also increases with training (notice that the way I'm putting the problem differs from the classical Becker setup in which the reference case is no wage compression at all). Ideally we'd like to e.g. fix wages ex ante, constrain the worker to stick with the firm, or have a constant wage floor which is binding at all training levels, all in order to keep the worker's "status-quo" (i.e. the outside wage) flat with respect to the training level, in which case the firm would appropriate a fraction β of the surplus (in which case the problem remains that the investment in skills is sunk at the bargaining stage and the firm can't appropriate the full surplus from training).
Contract theorists like to frame this question as the optimal assignment of property or residual control rights. I.e. the party with property rights on assets can walk of with the assets if the relationship breaks down in the bargaining stage, which will typically increase its incentive to invest. Of course in many setups, nobody opts out in equilibrium, but the threat is real and changes the bargaining outcome.

Now, do example from Daron's notes pp.163-166 ff.

The problem in the training example is that transfer of residual control rights over general skills from the worker to the firm isn't feasible by the nature of the problem, so that we may be stuck with the worse institutional design no matter what, and this is what makes the investment in skills scenario special.