Recitation Notes 11

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1 Models of Turnover and their Empirical Predictions

1.1 Motivation

While this would certainly not do full justice to the theoretical papers we discussed in the lecture, our point of reference for looking at different models of turnover is a simple cross-sectional regression of earnings on education, experience, and tenure on the current job

$$Y_i = \alpha + S_i\beta + X_i\gamma + T_i\delta + \varepsilon_i$$

where we are mainly interested in estimating the returns to tenure, δ.
Under which circumstances will OLS a causal parameter (e.g. accumulation of firm-specific human capital through learning-by-doing)? From the point of view of the literature on job search, one major concern is that one important unobserved determinant of wages is the overall quality of the current match μ, which will be endogenous with respect to tenure if workers know about μ when they decide whether to quit or stay.
To fix thoughts, assume that in each period a worker receives a job offer μ_t which is drawn from the same distribution $F(\mu)$ and then decides whether to stay on his current job or move on. If there is no return to tenure, the observed average second period wage is going to be $\mathbb{E}[\mu_2|\mu_2 > \mu_1]$ for movers, and $\mathbb{E}[\mu_1|\mu_1 > \mu_2]$ for stayers. Since this expression is completely symmetric, and the μ_t are assumed to be independent draws from the same distribution,

$$\mathbb{E}\hat{\delta} = \mathbb{E}[\mu_1|\mu_1 > \mu_2] - \mathbb{E}[\mu_2|\mu_2 > \mu_1] = 0$$

which is the true return to tenure. Due to the symmetry of the situation, the two "selection" effects cancel out.
Now assume that there is some additional payoff to staying on the current job (most importantly the return to tenure), $s > 0$, so that stayers' wage in the second period becomes $w_2^S = \mu_1 + s$. Then a worker quits if and only if $\mu_2 > \mu_1 + s$, so that a cross-sectional regression estimates the return estimates[1]

$$\mathbb{E}\hat{\delta} = \mathbb{E}[\mu_1 + s|\mu_1 + s > \mu_2] - \mathbb{E}[\mu_2|\mu_2 > \mu_1 + s] = s + \mathbb{E}[\mu_1|\mu_1 + s > \mu_2] - \mathbb{E}[\mu_2|\mu_2 > \mu_1 + s] < s$$

[1] By symmetry,

$$\begin{aligned}
\mathbb{E}[\mu_1|\mu_1 + s > \mu_2] &= \mathbb{E}[\mu_2|\mu_2 + s > \mu_1] \\
&= \frac{\mathbb{E}[\mu_2|\mu_2 > \mu_1 + s]\mathbb{P}(\mu_2 > \mu_1 + s) + \mathbb{E}[\mu_2|\mu_1 + s > \mu_2 > \mu_1 - s]\mathbb{P}(\mu_1 + s > \mu_2 > \mu_1 - s)}{\mathbb{P}(\mu_2 + s > \mu_1)} \\
&\le \frac{\mathbb{E}[\mu_2|\mu_2 > \mu_1 + s]\mathbb{P}(\mu_2 > \mu_1 + s) + \mathbb{E}[\mu_2|\mu_2 > \mu_1 + s]\mathbb{P}(\mu_1 + s > \mu_2 > \mu_1 - s)}{\mathbb{P}(\mu_2 + s > \mu_1)} \\
&= \mathbb{E}[\mu_2|\mu_2 > \mu_1 + s]
\end{aligned}$$

so that our estimator for the return to tenure is downward biased.
So what kills the symmetry in the selection biases is the wedge s between the value of the current and the potential new job, so that ironically we will always get a biased estimate of the return to tenure when there actually *is* a (nontrivial) return to tenure. The two Jovanovic papers tell each their own story how the dynamics of endogenous turnover create an additional differential between the value of the current job and outside offers. In the Inspection Good model ("Jovanovic I" - Firm-Specific Capital and Turnover), the worker makes firm-specific investments (human capital) which discourage him from taking an outside job with comparable match quality. The Experience Good setup

since the math in that paper is a little bit more involved, I'll spend some time discussing

2 Matching and On-the-Job Search - Deconstructing Jovanovic II

The second Jovanovic (1979) model was designed to fit three stylized facts about wages and turnover:

1. quit rates decrease in tenure
2. quit rates decrease in the contemporaneous wage rate
3. wages increase with tenure

gradual learning about the quality of the current match (the "experience good" case)

You may remember from a finance class that this problem is basically a special case of finding the optimal exercise strategy for an American call option on the true value of the current job match with an expiration date at infinity at a strike price $\frac{Q}{r}$, the expected continuation value of switching to an outside firm.[2] The "underlying" in this model consists of the current posterior mean of the match quality μ, which as we saw in our discussion of the normal learning model, has a variance that is decreasing over time at a rate $\frac{1}{t}$.

2.1 Dynamic Decisions, Convexity, and Option Values

An *option* in the broadest sense is the possibility to take actions or make adjustments at a future point in time, possibly after getting more information on the true state of nature. Under basic rationality assumptions (in particular the requirement of time consistency, that the "future self" of an agent acts in the interest of the "present self"), the value of an option has to be positive.
There are two ways to see that. For one, there is a revealed preference argument: if the agent is free to maintain the status quo at any point in time, he will make an adjustment to his initial plan only if that makes him better off.
On the other hand, the possibility to act upon new information in the future makes the final payoff $V(\mu)$ as a function of the underlying random variable μ "more convex" in the following sense: say at $t = 2$ the agent learns the true value of μ and subsequently chooses an action $a_t \in A = \{a_1, a_2, \ldots\}$ with an associated payoff $U(a_t, \mu)$. Then at $t = 1$, his continuation value conditional on the true state of nature is $V(\mu) = \max_{a \in A} U(a, \mu)$. If U is (weakly) convex in μ for every action a, then so is V.[3] In some sense

[2] A *call* option is the right to buy an asset at a future point in time at a *strike price p* fixed in advance. An *American* option can be exercised at any date up to an expiration date T - in contrast to a *European* option, which can only be exercised *at* a particular date T.

[3] If for fixed a $U(a, \cdot)$ is convex in its second argument, this means that its epigraph $\text{epi}U(a, \cdot) := \{(\mu, u) | U(a, \mu) \leq u\}$ is convex. One can see that $\text{epi}(\max_{a \in A} U(a, \cdot)) = \bigcap_{a \in A} \text{epi}U(a, \cdot)$. Since a countable intersection of convex sets is also convex, this implies that $\max_{a \in A} U(a, \cdot)$ is convex.

even if the Us aren't convex, taking the maximum is going to make V "more convex" in some vague sense.
Now, the link between convexity and option values is given by Jensen's inequality, which you probably already have seen elsewhere:

Theorem 1 Jensen's Inequality *If $f(x)$ is a convex function, then for any random variable X,*

$$f(\mathbb{E}[X]) \leq \mathbb{E}[f(X)]$$

given that these expectations exist.

Definition 1 *For random variables M_1 and M_2, M_2 is called a* mean-preserving spread *of M_1 if we can write*

$$M_2 = M_1 + Y$$

where Y is some random variable with mean zero.

If M_2 is a mean-preserving spread of M_1, then for any convex function $f(x)$,

$$\mathbb{E}[f(M_1)] \leq \mathbb{E}[f(M_2)]$$

because by the law of iterated expectations and the definition of a mean-preserving spread

$$\mathbb{E}[f(M_2)] = \mathbb{E}\{\mathbb{E}[f(M_2)|M_1]\} = \mathbb{E}\{\mathbb{E}[f(M_1+Y)|M_1]\} \leq \mathbb{E}[f(\mathbb{E}[M_1+Y|M_1])] = \mathbb{E}[f(M_1)]$$

where Y is a random variable with mean zero, and the weak inequality comes from (the conditional version of) Jensen's Inequality.
In the normal learning model, the posterior mean (and therefore the worker's wage in a learning model) evolves according to

$$M_{t+1} = \frac{H_t M_t + h y_t}{H_t + h} = M_t + \frac{h(y_t - M_t)}{H_t + h}$$

The second term clearly has expectation zero (this is the martingale property of the posterior mean) and positive variance, so that learning induces a mean-preserving spread.

This means that the later you can still make adjustments, the better

2.2 Option Values in Continuous Time (not covered in recitation)

In this part, I am going to sketch a heuristic derivation of Itô's Formula, which is important in Jovanovic's experience good model (though the intuition for this comes from the discrete-time "toy" version we saw in the lecture). If you are more interested in this, there is a nice and intuitive introduction to continuous-time stochastic optimization on Maurice Obstfeld's website at Berkeley, and most advanced graduate textbooks on dynamic asset pricing should have good references about the topic.

In discrete time, we have already seen examples of processes $X_t = Z_1 + Z_2 + \ldots + Z_t$ for (not necessarily iid) random variables $Z_1, Z_2, \ldots$ - e.g. as shown above, the posterior mean in the normal learning model is a random walk starting at the prior mean M with increments $\frac{h(y_t - M_t)}{H_t + h}$ which are mean-zero and independent, but decrease in variance as the "stock" of information on μ increases.
This type of random walk can be rewritten as a difference equation

$$\Delta X_t := X_t - X_{t-1} = Z_t$$

If these innovations happen at very short time intervals Δt and increments are small in absolute value, we can take limits

$$dX_t = \mu_t dt + \sigma_t dW_t$$

A second-order Taylor expansion in X_t gives

$$\begin{aligned} dV(X_t) &\approx V'(X_t)dX_t + \frac{1}{2}V''(X_t)(dX_t)^2 \\ &= V'(X_t)[\mu_t + \sigma_t dW_t] + \frac{1}{2}V''(X_t)[\mu_t^2 + 2\mu_t\sigma_t dt dW_t + \sigma_t^2 dW_t^2] \\ &= \left[V'(X_t)\mu_t + V''\frac{\sigma_t^2}{2}\right]dt + \sigma_t dW_t \end{aligned} \tag{1}$$

chain rule, but not quite
Intuition for second-derivative term: Jensen's Inequality

References

[1] Jovanovic, B. "Firm-Specific Capital and Turnover." *Journal of Political Economy* **87**, no. 6 (1979): 1246-1260.

[2] Jovanovic, B. "Job Matching and the Theory of Turnover." *Journal of Political Economy* **87**, no. 5 (1979): 972-989.

[3] Obstfeld, M. "Dynamic Optimization in Continuous-Time Economic Models (A Guide for the Perplexed)." *working paper* (1992), http://www.econ.berkeley.edu/~obstfeld/ftp/perplexed/cts4a.pdf, and http://www.econ.berkeley.edu/~obstfeld/ftp/perplexed/cts4b.pdf