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Labor Economics I
Matching and Wage Growth

Consider a two period matching model. Each period, every worker gets an offer ϵ_i, independently drawn from some fixed distribution, which, for simplicity, has mean zero. The value of the new offer is observed before the worker has to decide whether or not to take the new job. This match specific wage component is the only source of wage variation, so there is no wage growth due to experience or specific capital. Thus, the true (structural) wage profile is flat. Any observed wage growth will be due to sorting. To evaluate the bias arising from this in OLS regressions, it is useful to figure out the quantity $E(\epsilon \mid X, T)$, i.e. the expectation of the current ϵ for a worker with experience X and tenure T. Note that the current wage is $\epsilon = \epsilon_{X-T+1}$.

In the first period, everybody is in their first job, so

$$E(\epsilon \mid 1, 1) = 0$$

At the beginning of the second period everybody receives a new independent draw from the same distribution. Workers with offers $\epsilon_2 > \epsilon_1$ will switch jobs. Thus,

$$E(\epsilon \mid 2, 1) = E(\epsilon_2 \mid \epsilon_2 > \epsilon_1)$$

Similarly for stayers

$$E(\epsilon \mid 2, 2) = E(\epsilon_1 \mid \epsilon_1 > \epsilon_2)$$

Notice that the right hand sides of these two equations are symmetric and since ϵ_1 and ϵ_2 are independent, we have

$$E(\epsilon_2 \mid \epsilon_2 > \epsilon_1) = E(\epsilon_1 \mid \epsilon_1 > \epsilon_2)$$

$$\Longrightarrow E(\epsilon \mid 2, 1) = E(\epsilon \mid 2, 2)$$

which says that the second period wages of movers and stayers are the same in the model without wage growth on the job. Put differently, conditional on experience ϵ is independent of tenure. The symmetry underlying the result is illustrated in Figure (1).

Intuitively, there are two offseting effects. First, movers tend to have worse offers the first period. Secondly, movers tend to have better offers the second period. In this model, the two effects are of the exact same magnitude, that's what makes the second period expected wages equal.

Now, consider per period true wage growth on the job, due to, say, specific training or learning-by-doing, which is β. This makes the condition for moving more stringent, it implies

$$E(\epsilon \mid 2, 1) = E(\epsilon_2 \mid \epsilon_2 > \epsilon_1 + \beta)$$

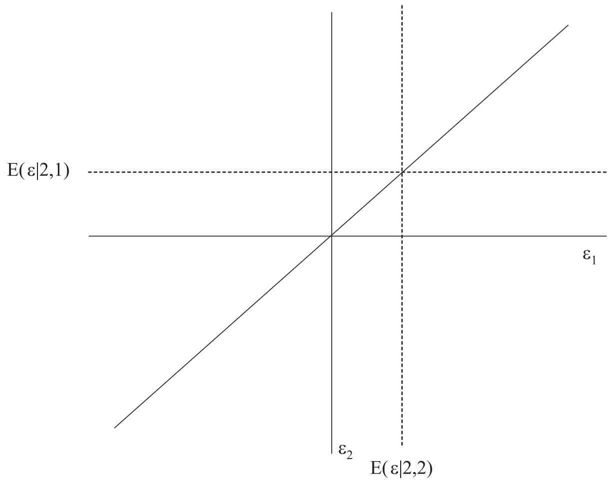

FIGURE 1

Second period wages for stayers are

$$E(\epsilon \mid 2,2) = E(\epsilon_1 \mid \epsilon_1 + \beta > \epsilon_2) = E(\epsilon_1 \mid \epsilon_1 > \epsilon_2 - \beta)$$

Notice that these two equations are not symmetric anymore. The wage growth effect introduces a wedge which makes moving less likely. This implies that we are now taking expectations of ϵ_2 for movers over a smaller region and expectations ϵ_1 for stayers over a larger region. Thus

$$E(\epsilon_2 \mid \epsilon_2 > \epsilon_1 + \beta) > E(\epsilon_1 \mid \epsilon_1 > \epsilon_2 - \beta)$$

$$\Longrightarrow E(\epsilon \mid 2,1) > E(\epsilon \mid 2,2)$$

This is most easily seen in the same type of graph again (Figure 2).

In terms of the two effects mentioned above, movers still may have below average offers in period one but this effect is muted because β enhances the value of the first period job (even for low values of ϵ_1). On the other hand, since β makes the condition for moving more stringent, second period offers for movers must have been particularly good. The introduction of a positive β mutes the first effect (which is negative) and enhances the second (which is positiv). This is why we know that movers must have a higher ϵ conditional on experience. The true tenure effect will therefore be underestimated in an OLS regression:

$$E\widehat{\beta} = \beta + E(\epsilon \mid 2,2) - E(\epsilon \mid 2,1) < \beta$$

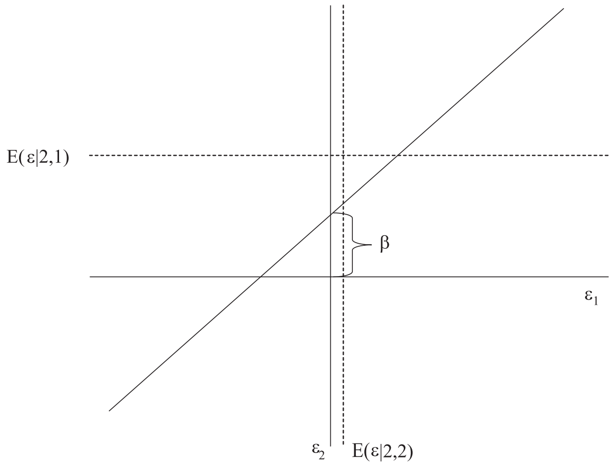

FIGURE 2

The Normal Learning Model[1]

In many models in the matching and labor contract literature match quality or productive ability is not immediately revealed. A convenient tool for modeling uncertainty about productivity is the *normal learning model*. Suppose that output y is a noisy function of true ability η, i.e. $y = \eta + \epsilon$ with $\eta \sim N(M, 1/H)$ and $\epsilon \sim N(0, 1/h)$. The quantities H and h, (i.e. one over the variances) are refered to as precision and turn out to be a useful way to represent uncertainty in the model. Assume that η and ϵ are independent. Suppose a firm hires a worker from a random pool of applicants. The worker's ex ante expected productivity is M. We want to know the firm's (and possibly the worker's) updated belief after one period of production, i.e. we want to know the conditional density $f(\eta \mid y)$. Using Bayes' Rule

$$f(\eta \mid y) = \frac{f(\eta, y)}{f(y)} = \frac{f(y \mid \eta)\ f(\eta)}{\int f(y \mid \eta)\ f(\eta) d\eta}$$

Using the normal density function

$$f(\eta) = \sqrt{\frac{H}{2\pi}} e^{-\frac{1}{2}H(\eta - M)^2}$$

$$f(y \mid \eta) = f(\eta + \epsilon \mid \eta) = f(\epsilon) = \sqrt{\frac{h}{2\pi}} e^{-\frac{1}{2}h(y-\eta)^2}$$

[1]This section is drawn from notes by Bob Gibbons.

The rest is pure algebra.

$$f(y \mid \eta)\ f(\eta) = \sqrt{\frac{h}{2\pi}}\sqrt{\frac{H}{2\pi}}e^{-\frac{1}{2}\left\{h(y-\eta)^2+H(\eta-M)^2\right\}}$$

Concentrating on the exponent for a second

$$h(y-\eta)^2 + H(\eta - M)^2 = (H+h)\eta^2 - 2\eta(HM+hy) + HM^2 + hy^2$$

Taking the first two terms on the right hand side and completing the square yields

$$(H+h)\left[\eta - \frac{HM+hy}{H+h}\right]^2 - (H+h)\left[\frac{HM+hy}{H+h}\right]^2$$

so that we can rewrite

$$\begin{aligned} h(y-\eta)^2 + H(\eta - M)^2 &= (H+h)\left[\eta - \frac{HM+hy}{H+h}\right]^2 \\ &\quad \underbrace{-(H+h)\left[\frac{HM+hy}{H+h}\right]^2 + HM^2 + hy^2}_{\equiv K(y)} \end{aligned}$$

Notice that this separates the equation into two terms, one involving η and the other one independent of η (denoted $K(y)$). This yields

$$f(y \mid \eta)\ f(\eta) = \sqrt{\frac{h}{2\pi}}\sqrt{\frac{H}{2\pi}}\sqrt{\frac{2\pi}{H+h}}e^{-\frac{1}{2}K(y)} \cdot \sqrt{\frac{H+h}{2\pi}}e^{-\frac{1}{2}(H+h)\left[\eta-\frac{HM+hy}{H+h}\right]^2}$$

Substituting into Bayes' formula

$$\begin{aligned} f(\eta \mid y) &= \frac{\sqrt{\frac{h}{2\pi}}\sqrt{\frac{H}{2\pi}}\sqrt{\frac{2\pi}{H+h}}e^{-\frac{1}{2}K(y)} \cdot \sqrt{\frac{H+h}{2\pi}}e^{-\frac{1}{2}(H+h)\left[\eta-\frac{HM+hy}{H+h}\right]^2}}{\int \sqrt{\frac{h}{2\pi}}\sqrt{\frac{H}{2\pi}}\sqrt{\frac{2\pi}{H+h}}e^{-\frac{1}{2}K(y)} \cdot \sqrt{\frac{H+h}{2\pi}}e^{-\frac{1}{2}(H+h)\left[\eta-\frac{HM+hy}{H+h}\right]^2}d\eta} \\ &= \frac{\sqrt{\frac{H+h}{2\pi}}e^{-\frac{1}{2}(H+h)\left[\eta-\frac{HM+hy}{H+h}\right]^2}}{\int \sqrt{\frac{H+h}{2\pi}}e^{-\frac{1}{2}(H+h)\left[\eta-\frac{HM+hy}{H+h}\right]^2}\ d\eta} \\ &= \sqrt{\frac{H+h}{2\pi}}e^{-\frac{1}{2}(H+h)\left[\eta-\frac{HM+hy}{H+h}\right]^2} \end{aligned}$$

where the first step uses the fact that $K(y)$ is independent of η, and the second step results from the fact that we are dealing with a proper normal density function which integrates to one. Thus we found

$$\eta \mid y \sim N\left(\frac{HM+hy}{H+h}, \frac{1}{H+h}\right)$$

It is easy to extend this to t-period updating. Denote the conditional mean

$$M_1 = E(\eta \mid y) = \frac{HM + hy}{H + h}$$

and the conditional precision

$$H_1 = H + h$$

Analogously,

$$M_t = E(\eta \mid y_1, ..., y_t) = \frac{H_{t-1}M_{t-1} + hy_t}{H_{t-1} + h} = \frac{HM + h\sum_1^t y_i}{H + th}$$

and

$$H_t = H + th$$

Learning about Match Quality

We can use the normal learning model to study the two period matching model where match quality is not observed ex ante. Thus, assume that output in job j is given by $y_{jt} = \eta_j + \epsilon_t$. Assume that η has mean zero and precision one, while ϵ has mean zero and precision h. If the worker gets all the surplus from the match and the wage is set at the beginning of each period then the wage on job j in the first period is

$$w_{j1} = E(\eta_j) = 0$$

If there is no growth of wages due to specific capital then the worker will move to a new job whenever

$$w_{j2} = E\left(y_{j2}|y_{j1}\right) = E\left(\eta_j|y_{j1}\right) = \frac{hy_{j1}}{1 + h} < 0$$

or $y_{j1} < 0$.
Use $E(w \mid X, T)$ again to denote the conditional expectation of the wage of a worker with experience X and tenure T. An average mover earns in the second period just the unconditional mean of the match qualities

$$E(w \mid 2, 1) = 0$$

while a stayer is expected to make

$$\begin{aligned} E(w \quad | \quad & 2, 2) = E\left[E(y_2 \mid y_1) \mid y_1 > 0\right] \\ = \quad & \frac{h}{1 + h}E(y_1 \mid y_1 > 0) > 0 \end{aligned}$$

so we have

$$E(w \mid 2, 2) > E(w \mid 2, 1)$$

Stayers earn more than movers. The matching model with learning implies that OLS will overestimate the tenure coefficient (conditioning on experience). As we add wage growth β on the job the condition for switching jobs becomes again more stringent. A worker switches if

$$w_{j2} = \frac{hy_{j1}}{1+h} + \beta < 0$$

or

$$y_{j1} < -\beta \frac{1+h}{h}$$

Notice that match quality becomes completely revealed as $h \longrightarrow \infty$. In this case, the model (almost) collapses back to the case were matches were an inspection good. The only difference is that here it still takes a period to learn about match quality, so there is no selection effect due to second period wages for movers as in the model in the first section of the handout. Here, movers all earn zero in the second period. On the other hand, as $h \longrightarrow 0$ output is completely random and there is no match specific component anymore. Thus, it makes never sense to change jobs. If some worker changed jobs accidentally (or for some exogenous reason) they would loose the job specific wage component β. So we would find in the case of $h = 0$ and completely exogenous moving that

$$E(w \mid 2,2) - E(w \mid 2,1) = \beta$$

OLS would estimate true returns to tenure. For positive and finite h

$$E(w \mid 2,2) - E(w \mid 2,1) > \beta$$

so that OLS still overestimates the return to tenure. This follows from

$$\begin{aligned} E(w \mid 2,2) &= E\left(w_2 \middle| \frac{hy_1}{1+h} > -\beta\right) = E\left(\frac{hy_1}{1+h} + \beta \middle| \frac{hy_1}{1+h} > -\beta\right) \\ &= \frac{h}{1+h} \underbrace{E\left(y_1 \middle| y_1 > -\beta \frac{1+h}{h}\right)}_{>0} + \beta > \beta \end{aligned}$$

This means that Topel's procedure of identifying a lower bound on the true return to tenure crucially depends on his assumption of matches being inspection goods. If matches are experience goods he identifies an upper bound to the returns to tenure. If both aspects are present (i.e. there is some ex ante knowledge about the match quality but this knowledge is imperfect) we can presumably not put any bounds on the bias in β anymore. (You could work that out as an exercise. For example, let productivity on the each job still be $y_{jt} = \eta_j + \epsilon_t$. After the first period, the worker receives a signal $s_2 = \eta_2 + u$ on his productivity in a new job.)