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Labor Economics I
Problem Set 3

1. **(Signaling):** In this problem, you are asked to work through a model that combines signaling with productive aspects of schooling. There are two types of agents: "high" and "low" ability. Education (e) is continuous and observed, but individual ability (and output) is not. The labor productivity for the "low" type is $y_l(e) = \alpha_1$ and the cost of education is $c_l(e) = \frac{3}{2}e^2$. For the "high" type, output and education costs are $y_h(e) = \alpha_1 + \alpha_2 e$ and $c_h(e) = e^2$, respectively.

   (a) Solve for the most efficient separating equilibrium of the signaling game.

   (b) Show that the high type does not have an incentive to deviate from your proposed equilibrium strategy.

   (c) Does the high type's investment in education differ from what would have obtained in the perfect-information case? Why or why not?

   (d) Suppose now that $c_l(e) = 10e^2$. Does the high type's investment in education differ from what would have obtained in the perfect-information case? Why or why not?

   (e) Suppose again that $c_l(e) = \frac{3}{2}e^2$ and furthermore suppose that there is a compulsory schooling requirement of $\underline{e}$. Characterize the most efficient separating equilibrium. Does the high type invest in education more or less in this case than in (a)? Explain why.

   (f) Characterize the equilibrium if $y_l(e) = y_h(e) = \alpha_1 + \alpha_2 e$; $c_l = \frac{3}{2}e^2$; and $c_h = e^2$. Why does the equilibrium differ from the one in (a)?

   (g) Does a separating equilibrium exist when $y_l(e) = \alpha_0 + \alpha_2 e$; $y_h(e) = \alpha_1 + \alpha_2 e$; $c_l = c_h = e^2$ with $\alpha_0 < \alpha_1$?

   (h) Compute the observed return to schooling in part (a).

2. **(Training With Labor-Market Frictions):** Consider the following economy. At $t = 0$, the firm decides how much to invest in its employee's general skills. The cost of an investment τ is $c(\tau)$, which is incurred by the firm. A worker with general skills τ produces $1 + \tau$ output in period $t = 1$. At this point, he can also move to a different firm where his wage will be $1 + \tau - \theta$ where θ is the cost of moving to a different firm. θ is a random variable, drawn from a uniform distribution $[0, 1]$, and is the private information of the worker (i.e., the firm does not observe it). The exact sequence of events is as follows: at $t = 0$, the firm chooses τ and makes a wage offer (w) to the worker; next, the worker, knowing her own θ, decides whether to quit or to stay.

   (a) Characterize the firm's wage offer as a function of τ. In particular, is $w'(\tau)$ positive, negative, zero, or ambiguously determined? Why?

   (b) Solve for the firm's level of training and wage offer that maximize expected profit. Explain why the firm is not investing in τ?

(c) Suppose now that the worker can finance his own training investment. Solve for the worker's choice of training and the firm's wage offer. How does the worker's choice of training compare to the first best training level?

(d) Suppose again that the worker cannot finance her training, but that her wage, if she quits the firm, is given by $1 + \tau(1 - \theta)$. Explain why the mobility cost might take this form. Solve for τ and w. Why is the firm investing in training in this case? Contrast these results with those obtained in part (b). How does the training level chosen by the firm compare to the first best level?

3. **(General and Specific Training Investments):** This question asks you to think about a three-period training model. Consider the following timeline:
In period 1, firm-specific investments in human capital are made by the worker.
In period 2, investment in general human capital is made by the firm.
In period 3, the firm makes a wage offer and workers decide whether to stay at the firm or work in a competitive labor market outside the firm.
Assume that the production function has the following form:

$$f(\tau, s) = (1 + \tau)(1 + s)$$

in which τ is general human capital and s is specific human capital. The production function outside the firm is

$$g(\tau, s) = 1 + \tau$$

Finally, the cost of general human capital, incurred by the firm in period 1, is τ^2, and the cost of specific human capital is s^2 and is incurred by the worker in period 2.

(a) What is the wage offer the firm will make to the worker in period 3? Explain.

(b) Assume that the firm cannot invest in any general human capital in period 2 (or ever). Solve for s and w. Interpret.

(c) Assume instead that the worker cannot invest in specific human capital. Solve for τ and w. Interpret.

(d) Now, assume that both parties *can* the make investments as described above. What incentive does the firm have to invest in general human capital? What incentive does the worker have to invest in specific human capital? (HINT: use backwards induction.)

(e) Explain how and why your answer would change if $f(\tau, s) = 1 + \tau + s$. Why is this the case?

4. **(Firm Specific Investments):** Consider the following worker-firm relationship. At $t = 0$, the risk neutral worker decides how much to invest in his firm-specific skills, s. The cost of an investment s is $c(s)$, which is incurred by the worker. A worker with specific skills s produces

$$1 + s - \eta$$

output in period $t = 1$, where η is a random variable, drawn from a uniform distribution $[0, 1]$, and is the private information of the firm (i.e., the worker does not observe it).

Before production takes place, the worker can also move to a different firm where his wage will be $1+\theta$ where θ is a random variable, drawn from a uniform distribution $[0,1]$, and is the private information of the worker (i.e., the firm does not observe it). θ is realized at $t=1$. If the worker quits, the firm receives a payoff of 0.

(a) Consider the following sequence of events as follows: at $t=0$, the worker chooses s. Then the firm, observing η makes a take-it-or-leave-it wage offer w to the worker; next, the worker, knowing his own θ, decides whether to quit or to stay.

1. Determine the quit behavior of the worker as a function of θ and the wage offer, w. Given this, characterize the wage rate offer that a firm would make as a function of the specific skill investment of the worker and its own productivity, $w(s,\eta)$ (Hint: if you wish, you can ignore corner solutions to simplify the discussion).
2. Determine the equilibrium probability that the worker will be employed with this firm as a function of her investment, $p(s)$, and the average equilibrium wage function $w(s)$.
3. Determine the equilibrium investment of the worker, s^* (Assume that $c(s)$ is sufficiently convex so that you can simply look at first-order conditions).

(b) Now consider a different sequence of events: at $t=0$, the worker chooses s. Then after observing θ, he makes a wage offer w to the firm; next, the firm, knowing its η, decides whether to employ the worker or lay him off.

1. Determine the wage rate offer that the worker would make as a function of the specific skill investment and his own outside productivity, $w(s,\theta)$.
2. Determine the equilibrium probability that the worker will be employed with the firm as a function of its investment, $\tilde{p}(s)$.
3. Determine the equilibrium investment of the worker, $\tilde{s}$.

(c) Compare $\tilde{s}$ to s^*. What is the intuition why workers invest in specific human capital in these models? Why does the level of investment differ in the way it does between the two versions of the model?

(d) Discuss what other wage setting institutions might be preferable to the two discussed here (naturally, θ and η are still private information, thus non-contractible). For example, may a contract specifying a fixed wage rate, $\bar{w}$, with the option for the firm to fire to worker and the worker to quit, be preferable? (Hint: even though the wage is constant, the worker will invest in specific skills. Why is that?).