MIT 14.123 (2009) by Peter Eso
Lecture 11: Global Games

1. A Contribution Game
2. Carlsson & van Damme (ECMA, 1993)
3. Morris & Shin (1998)

Read: Assigned reading (C-vD'92, M-S'98)

Motivation

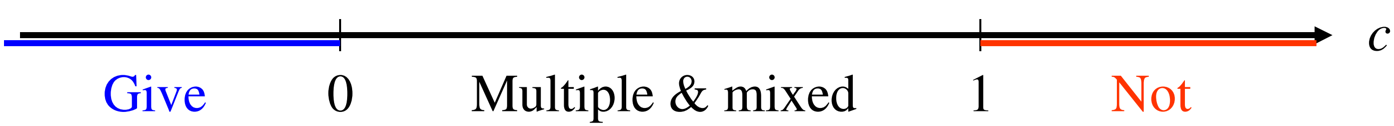
- Economic phenomena where outcomes vary in otherwise similar environments (apparent multiplicity of equilibria?):
  - Joint investment / partnerships
  - Bank runs
  - Currency attacks
  - Electoral competition …
- Explanation: Strategic complementarity and multiple equilibria; coordination and/or coordination failure.
- Global games approach: Introducing incomplete information (small noise in players' perception of the game's payoffs) makes the game dominance solvable. The unique Bayesian equilibrium exhibits "tipping point" features, matches data.

1. A Contribution Game

- Each player can contribute endowment c to joint project.
- 1 util for each player is generated iff both contribute.

	Give	Not
Give	1, 1	0, c
Not	c, 0	c, c

- If $c > 1$, then 'Not' is strictly dominant strategy $\Rightarrow$ (N,N).
- If $c < 0$, then 'Give' is strictly dominant $\Rightarrow$ (G,G).
- If $c \in (0,1)$ then multiplicity: strict Nash equilibria (G, G), (N, N), symmetric mixed equilibrium with Prob(G) = c .



Multiple Equilibria

- If $c < 1$, then (G,G) is the Pareto-optimal strict eqm.
- Would anything explain coordinating on (N,N)?

$c \in (0,1)$	Give	Not
Give	1, 1	0, c
Not	c, 0	c, c

- In equilibrium (G,G), each player's deviation loss is $1-c$. In equilibrium (N,N), the deviation losses are c for each.
- DEF: An equilibrium is risk dominant if the product of the players' deviation losses is the greatest. (See Harsanyi & Selten (1988)).

  In 2×2 symmetric games, a strategy is part of the risk dominant equilibrium iff best responds to the other player's 50-50% mixing.

- Risk-dominance contradicts Pareto-dominance if $c \in (\frac{1}{2}, 1)$.

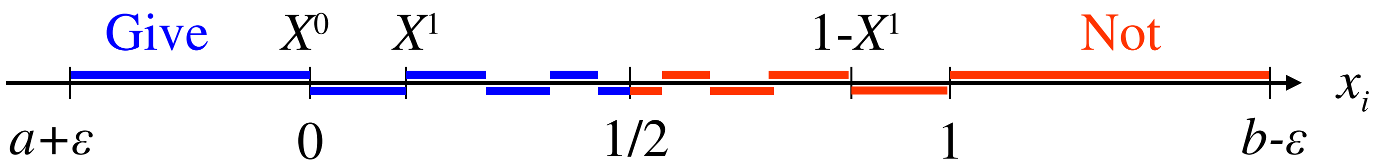
Global Game Perturbation

- Assume c is a random draw from $[a,b] \supset [0,1]$.
- Player i observes x_i uniform random on $[c-\varepsilon, c+\varepsilon]$, $\varepsilon>0$ small.

c random	Give	Not
Give	1, 1	0, c
Not	c, 0	c, c

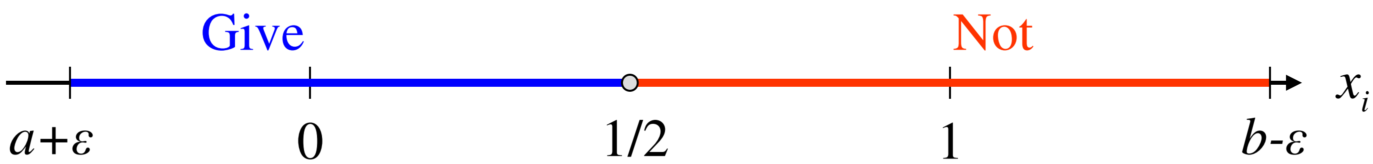
- The players' signals about the "payoff state" are independent conditional on c, but not independent unconditionally (affiliated!).
- Perturbation proposed by Carlsson & van Damme (ECMA 1993). [Compare to: Each player's payoff in each cell is independently perturbed; learn own payoff only. ⇒Harsanyi's Purification Thm.]
- When player i learns x_i, he makes inference about the state as well as the other's signal, and both play Bayesian equilibrium.

Iterated Dominance

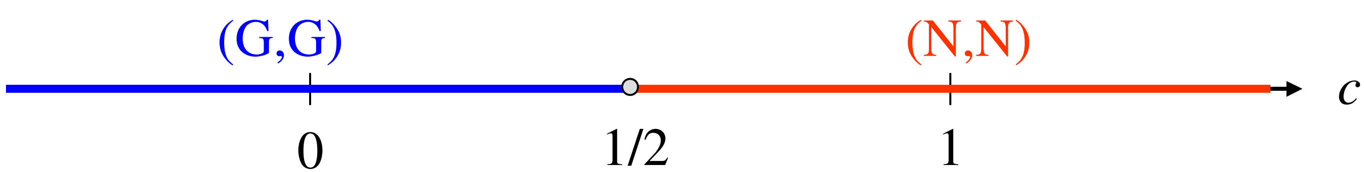


- If $x_i \in [a+\varepsilon, b-\varepsilon]$, then i believes c is uniform on $[x_i-\varepsilon, x_i+\varepsilon]$, and that x_j is uniform on $[x_i-2\varepsilon, x_i+2\varepsilon]$. Expected payoff from 'N' is x_i .
- Let $X^0 = 0$. If $x_i < X^0$, then 'N' is strictly dominated for i (negative payoff), while if $x_i > 1 - X^0$, then 'G' is strictly dominated for i.
- Suppose j plays 'G' $\forall x_j < X^k$, $k = 0,1,\ldots$. Then, $x_i \in [X^k, X^k + 2\varepsilon)$ believes j plays 'G' with prob. $> ½ - (x_i - X^k)/4\varepsilon$, so 'G' dominates 'N' for x_i if $½ - (x_i - X^k)/4\varepsilon > x_i \Leftrightarrow x_i < X^{k+1} = (2\varepsilon + X^k)/(1+ 4\varepsilon)$.
- Similarly: If 'G' is dominated $\forall x_i > 1-X^k$ then so is $\forall x_i > 1-X^{k+1}$.
- $X^{k+1} > X^k$ iff $X^k < ½$, so $\lim_{k\to\infty} X^k = ½$.

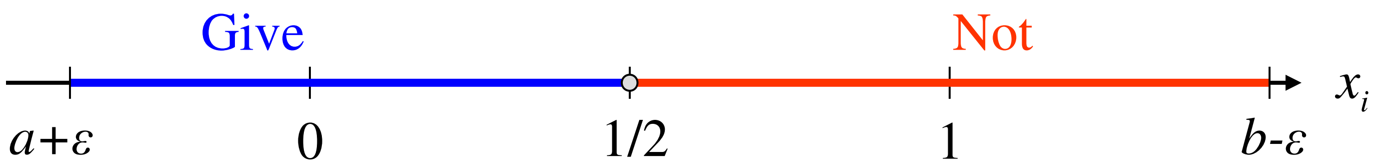
Outcome



- In the perturbed game (global game à la Carlsson & van Damme), there is a unique Bayesian equilibrium by iterated dominance: Play 'G' if $x_i < ½$, 'N' if $x_i > ½$, anything at $x_i = ½$.
- Iterated dominance in the global game selects the equilibrium in accordance with risk dominance (even against payoff dominance).
- Players do not always coordinate on the risk dominant outcome in the game they play (=given c). They do so if ε is small given c.



Why Risk Dominance?



- c = ¾, so (G,G) ↦ (1,1) Pareto-dominates (N,N) ↦ (c,c), yet we expect coordination on (N,N) in the global game for $\varepsilon < ¼$.
- Why does player i play ‘N’ with $x_i \in (1/2,1)$?
- Since ‘N’ is dominant for $x_j > 1$, i expects j to “switch” to playing ‘N’ at some $x_j^* \leq 1$. But then i would rather switch to ‘N’ earlier, at some $x_i^* \in (x_j^*-2\varepsilon, x_j^*)$. By symmetry $x_i^* = x_j^*$; unraveling.
- Even if $x_1 = x_2 = c =$ ¾, the only thing commonly known about c is that it belongs to [a,b].

2. Carlsson & van Damme (1993)

- 2×2 games, such that (A,A) and (B,B) are both Nash equilibria.
- (A,A) is risk dominant: $(u_{11}-u_{21})(v_{11}-v_{12}) > (u_{22}-u_{12})(v_{22}-v_{21})$.
- Define $g_1^a = u_{11} - u_{21}$, etc.
- (A,A) risk dominant iff $g_1^a g_2^a > g_1^b g_2^b$.

	A	B
A	u_{11}, v_{11}	u_{12}, v_{12}
B	u_{21}, v_{21}	u_{22}, v_{22}

	A	B
A	g_1^a, g_2^a	0, 0
B	0, 0	g_1^b, g_2^b

- Let $\underline{s}_j$ be j's strategy (Pr(A)) such that i is indifferent against $\underline{s}_j$.
- (A,A) risk dominant: $\underline{s}_1 + \underline{s}_2 < 1$.

Dominance vs Risk Dominance

	A	B
A	g_1^a, g_2^a	0, 0
B	0, 0	g_1^b, g_2^b

- Dominance region:

$$D_i^a = \{(u,v) | \; g_i^a > 0, \; g_i^b < 0\}.$$

- Risk-dominance region:

$$R^a = \{(u,v) | \; g_1^a > 0 \; g_2^a > 0; \; g_1^b, \; g_2^b > 0 \Rightarrow \underline{s}_1 + \underline{s}_2 < 1\}.$$

- Next, introduce global game model, where
  1. Nature selects a game from a set with parametrized payoffs.
  2. Each player observes the parameters with some noise.
  3. Players choose simultaneous actions.
  4. Payoffs determined by players' choices & payoff parameters.

Global Games Theorem

- Payoff parameter $\theta \in \Theta \subseteq \mathbb{R}^m$; Θ open; (u,v) are continuously differentiable functions of θ with bounded derivatives.
- Prior on θ has density $h > 0$, continuously diff'able, bounded.
- Each player i observes a signal $x_i = \theta + \varepsilon\eta_i$ where η_i is a bounded random variable independent of θ with a continuous density.
- THM: Suppose x is on a continuous curve $C \subseteq \Theta$, such that $(u(c),v(c)) \in R^a \; \forall c \in C$, and $(u(c),v(c)) \in D^a$ for some $c \in C$.

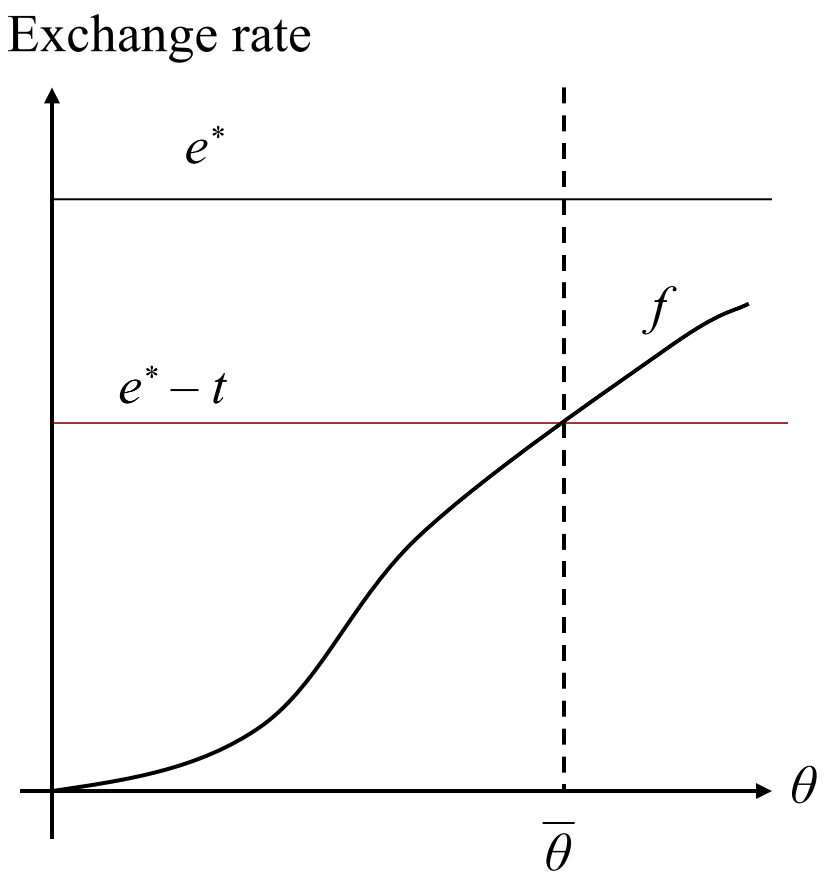
  Then, A is the only rationalizable action at x when ε is small.
- Moral: The global games perturbation selects the risk-dominant equilibrium in general 2×2 games with two pure equilibria.

3. Application: Currency Attacks

Morris, S., and H. S. Shin: "Unique equilibrium in a model of self-fulfilling currency attacks", AER 1998

- Fundamental: $\theta \in [0,1]$ uniform; the higher the better.
- Competitive exchange rate: $f(\theta)$; f is strictly increasing.
- Exchange rate is initially pegged at $e^* \geq f(1)$.
- A continuum (unit mass) of speculators, who either
  - Attack, which costs $t > 0$, or
  - Not attack.
- Government observes ratio of attackers; defends the peg or not.
- The exchange rate is e^* if defended, $f(\theta)$ otherwise.

Speculator's Payoff



- Attack, not defended:

$$e^* - f(\theta) - t$$

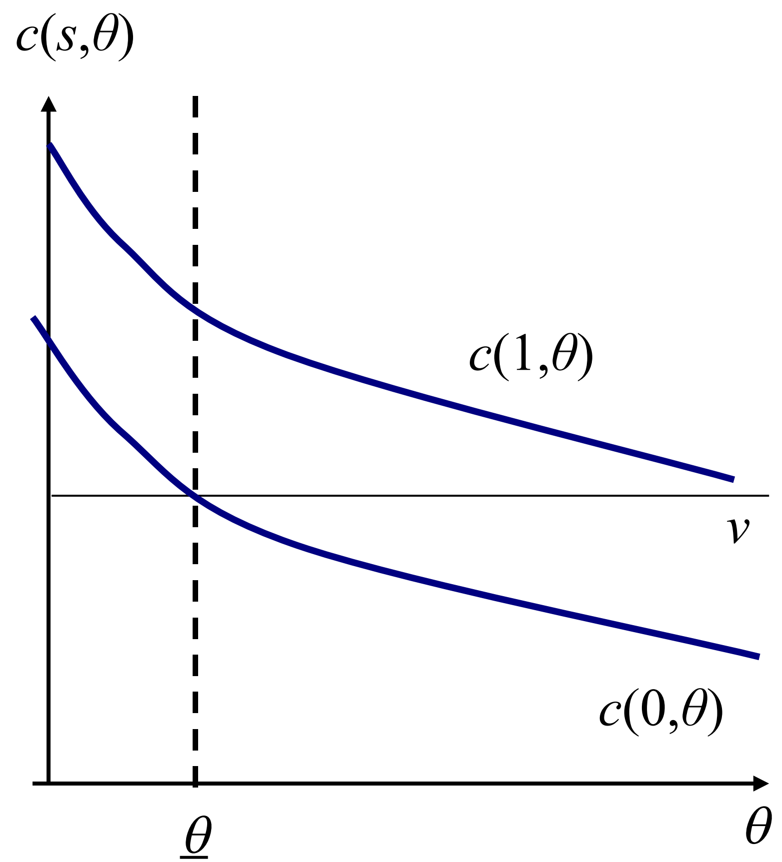
- Attack, defended:

$$-t$$

- No attack: 0

Government's Payoff

- Value of peg = v.
- Cost of defending is $c(s,\theta)$, where s is the share of speculators who attack.
- c is increasing in s and decreasing in θ.
- Gov't observes s and θ. Defends the peg iff $v > c(s,\theta)$, and abandons it otherwise.



Speculators Commonly Know θ

$\theta < \underline{\theta}$	Attack	Not
Attack	$e^* - f(\theta) - t$	$e^* - f(\theta) - t$
Not	0	0

$\theta < \overline{\theta}$	Attack	Not
Attack	$e^* - f(\theta) - t$	$-t$
Not	0	0

$\underline{\theta} < \theta < \overline{\theta}$	Attack	Not
Attack	$e^* - f(\theta) - t$	$e^* - f(\theta) - t$
Not	0	0

- Payoffs are the same for any two speculators; payoff of the player choosing rows is shown in each matrix.

Global Game Perturbation

- Assume θ is uniform on [0,1].
- Each speculator i gets signal $x_i = \theta + \eta_i$, where the η_i's are iid uniform on $[-\varepsilon, \varepsilon]$ with $\varepsilon > 0$ small.
- The distribution of η_i's is common knowledge.
- Government sees s and θ. It defends the peg iff $v > c(s,\theta)$. Let $a(\theta)$ be lowest s where G abandons the peg: $v \equiv c(a(\theta),\theta)$.
- $a(\underline{\theta}) = 0$, increasing for $\theta > \underline{\theta}$.
- Let s = ratio of speculators that attack. Speculator payoff:

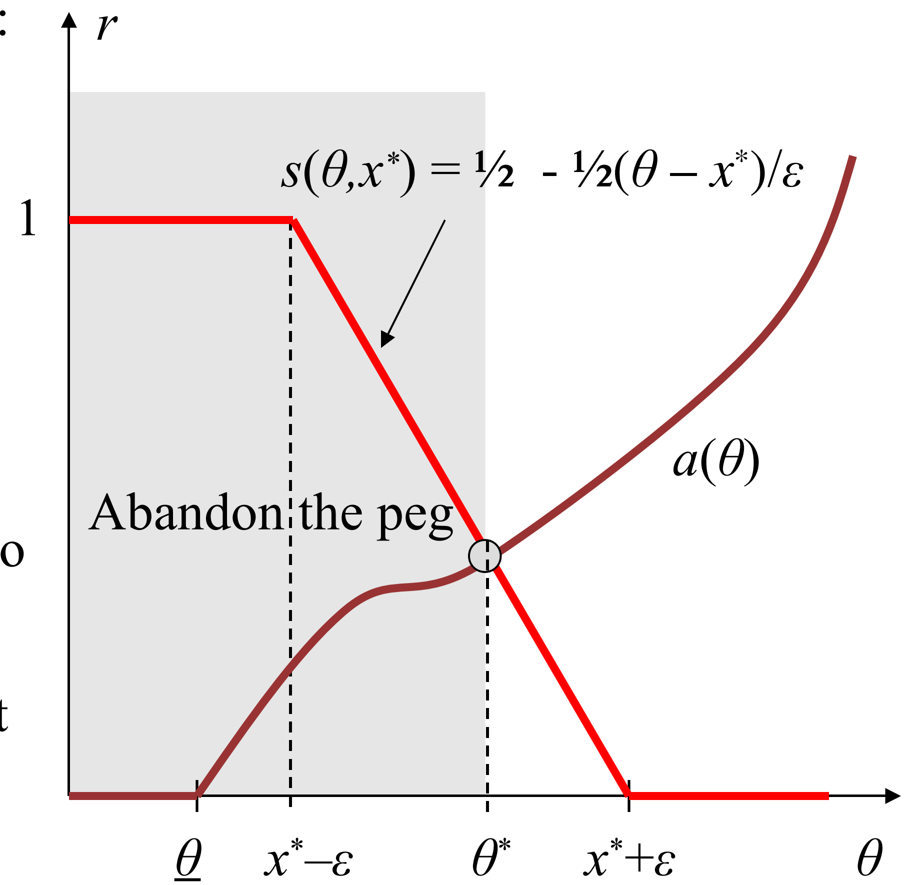
  $u(\text{Attack}, s, \theta) = e^* - f(\theta) - t$, if $s \geq a(\theta)$; $-t$ otherwise.

  $u(\text{Not}, s, \theta) = 0$.

Unique Equilibrium

- Speculators' eqm strategy: Attack iff $x_i \leq x^*$.
- $s(\theta,x^*) = \Pr(x_i \leq x^* \mid \theta)$ $= ½ – ½(\theta – x^*)/\varepsilon$.
- Two conditions pin down x^* as well as θ^* (in fig.):

1) At $\theta=\theta^*$, $s(\theta^*,x^*) = a(\theta^*)$, so $x^* = \theta^* – \varepsilon[1 - 2a(\theta^*)]$.
2) Speculators are indifferent to attack at $\theta=\theta^*$, hence $[1 – a(\theta^*)](e^* - f(\theta^*)) = t$.



Conclusions

- θ observed with small noise: "Attack" very likely iff $\theta < \theta^*$.
- "Risk dominance" in this game:
  - Suppose all strategies equally likely: s uniform on [0,1].
  - Expected payoff from Attack $(1-a(\theta))(e^*-f(\theta)) - t$
  - Attack is "risk dominant" iff $(1-a(\theta))(e^*-f(\theta)) > t$
  - Cutoff value θ^*: $(1-a(\theta^*))(e^*-f(\theta^*)) = t$.
- Comparative statics: How θ^* varies with t, e^*, c (cost scale):
  - θ^* decreases in t
  - θ^* increases in e^*
  - θ^* increases in cost parameter