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14.123 Microeconomic Theory III  
Spring 2009

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## 14.123 Waiver Exam

Spring 2009, Peter Eso

The duration of the exam is 90 minutes, there are four questions. The difficulty of each question is indicated in terms of approximate time needed to solve them. Please use a separate bluebook for each question. Be brief but indicate your reasoning for maximum partial credit.

Q1 (15 min). Maurice is asked to choose between two lotteries:

- (A) Win 10,000Fr with 85% chance and lose 10,000Fr with 15% chance;
- (B) Win 10,000Fr with 75% chance (no gain, no loss with 25% chance).

He chooses the latter gamble, gamble B.

Independent of this choice (and with the same initial wealth, under the exact same circumstances—world peace, room temperature, humidity, etc.) he is asked to make another choice:

- (C) Lose 10,000Fr with 75% chance (nothing happens with 25% chance);
- (D) Lose 10,000Fr with 90% chance and win 10,000Fr with 10% chance.

He chooses the latter gamble, D over C.

(Q1.a) Is Maurice an expected-utility maximizer? (Are there expected-utility preferences over objective lotteries that explain his choices?)

(Q1.b) If he is, then exhibit an expected utility function that is consistent with his choices. If he is not, then formally describe the assumption of expected utility theory that his behavior violates.

Q2 (15 min). Johnny maximizes his expected utility using a logarithmic Bernoulli (or vNM-) utility function on his final wealth. That is, Johnny's expected utility from a random gamble  $\tilde{X}$  is  $E[\ln(w + \tilde{X})]$ , where  $w$  is his

initial wealth (money in his lock box) and the expectation is taken according to the distribution of  $\tilde{X}$ .

Johnny goes to the beach to play cards with the local kids. His winnings are proportional to the amount of money he carries with him. Denoting the amount of money that he decides to take with him by  $a$ , his stochastic wealth (after playing cards) is  $w + (\tilde{R} - 1)a$ , where  $w$  is his initial wealth, and  $\tilde{R}$  is a random variable with an expectation exceeding one.

(Q2.a) Johnny's mother institutes a new rule: Every morning, when Johnny leaves the house, she takes away half of the money that he carries and puts it back to his lock box. What will be the effect of this policy on Johnny's behavior? How much less or more will he gamble compared to the situation when he was not "taxed"?

(Q2.b) Give a brief, intuitive explanation for the result in part (a).

Q3 (30 min). Consider the following matrix game where Player 1 chooses rows and Player 2 chooses columns. Player 1's payoff is listed first.

|     |     |     |       |
|-----|-----|-----|-------|
|     | $L$ | $M$ | $R$   |
| $U$ | 3,1 | 1,2 | 0,0   |
| $M$ | 4,0 | 2,3 | 0,1   |
| $D$ | 0,0 | 1,0 | -1,-1 |

(Q3.a) Identify all *rationalizable outcomes* of this game. Does this set differ from the set of Nash equilibrium outcomes, and if so, how?

Now consider an infinitely-repeated version of this game where the players' discount factors are close to 1.

(Q3.b) Construct a subgame-perfect equilibrium of the infinitely-repeated game with average payoffs (3,1). Are these payoffs greater than the players' one-shot Nash equilibrium payoffs?

(Q3.c) The effects of repeated play are sometimes summed up by saying that repetition makes cooperation easier and/or more likely. Briefly discuss this interpretation in light of your previous answers.

Q4 (30 min). Consider the following signaling game.

Player 1 (he) is an employee with a college degree and has a privately-known type, either ‘Quant’ or ‘Poet’ ( $Q$  or  $P$ , respectively). He has the opportunity to pursue an MBA degree at MIT Sloan. If he gets an MBA, then his employer (Player 2) can promote him to Chief Financial Officer (CFO), which is a good match for the  $Q$  type, or Head of Human Resources (HR), which is the best use of a  $P$  type, or just keep him in his current position with a raise. Neither type of Player 1 likes to be promoted to the HR job. If Player 1 does not get an MBA, then he cannot be promoted.

The payoffs are summarized below. Nature chooses rows (Player 1’s type,  $Q$  or  $P$ , with 50-50% chance), Player 1 chooses matrices (No or MBA), and Player 2 chooses columns (conditional on MBA, either CFO, HR, or N). Payoffs are listed for Player 1 first.

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
|     |     | CFO | HR  | N   |
| $Q$ | 2,2 | 5,5 | 0,0 | 3,3 |
| $P$ | 2,2 | 0,0 | 1,5 | 3,3 |
|     | No  | MBA |     |     |

(Q4.a) Derive all pure-strategy perfect Bayesian equilibria of the game.

(Q4.b) Does the Intuitive Criterion (or equilibrium dominance) eliminate the equilibrium where neither type of Player 1 chooses ‘MBA’?

(Q4.c) Suppose Player 2 does not have action N (cannot keep the employee in his current position, even with a raise, in case he gets an MBA). Does the Intuitive Criterion eliminate pooling on ‘No’?