14.123 Microeconomic Theory III
Spring 2009

14.123, Spring 2009 (Peter Eso)
Midterm Exam

Five questions for 90 minutes. The number in parentheses before each question represents available points; these points also add up to 90. Please be brief, yet show your work. Good luck, and have a great spring break!

(10) 1. (a) Describe the Independence Axiom for preferences over simple and reduced compound lotteries with objective (given) probabilities.

(b) A decision maker is risk averse and maximizes expected utility over simple and reduced compound lotteries. Show that s/he prefers the lottery that pays \$20 with 5/9 chance and \$10 with 4/9 chance to the lottery that pays \$30 with 5/18, \$20 with 1/9, \$10 with 5/18, and \$5 with 3/9 chance.

(20) 2. Assume that F and G are two cumulative distribution functions (cdfs) with positive densities f and g (respectively) on a compact interval $[A, B] \subset \mathbb{R}$ with $A < 0 < B$.

(a) Give two equivalent definitions for what it means that F dominates G in the first-order stochastic sense. (Do not prove equivalence.)

(b) Give a definition for F dominating G in the monotone likelihood ratio (MLR) sense.

(c) A risk-averse expected-utility maximizer with vNM utility u (on final wealth) and fixed initial wealth w_0 can invest in an asset with net return $\tilde{x}$. (If she invests α, her final wealth is $w_0 + \alpha\tilde{x}$.) Suppose that her optimal investment when $\tilde{x}$ is distributed according to cdf F is α^*. Prove that her investment is lower than α^* if $\tilde{x}$ is distributed according to cdf G such that F dominates G in the MLR sense.

(10) 3. (a) In a finite game with two players, simultaneous actions, define what it means for a Nash equilibrium to be trembling-hand perfect.

(b) In one sentence, say how the stability of this equilibrium differs from its trembling-hand perfection.

(25) 4. There are two identical objects for sale to two potential buyers.

Valuations: Each buyer draws two random numbers that are iid uniform on [0,1]. His valuation for the first unit of the good is the maximum of the two random variables; his valuation for the second unit is the minimum.

That is, buyer i's valuation for one unit of the good is $v_i(1) = \max\{x_i^1, x_i^2\}$ while his valuation for two units is $v_i(2) = x_i^1 + x_i^2$ where x_i^1 and x_i^2 are iid uniform on [0,1].

Sales mechanism: The goods are sold in a uniform-price auction defined as follows. Both bidders submit two bids; the submitter(s) of the highest two bids win and pay a unit price equal to the third highest bid. (That is, if you win two units, you pay this price twice.)

(a) Show that it is an equilibrium for the buyers to submit bids $\{v_i(1), 0\}$, $i = 1, 2$. In words: Each buyer submits his valuation for the first unit truthfully, and submits a zero bid for the second unit. (This is called "demand reduction" in the uniform-price auction and holds more generally).

(b) How to modify the rules of the auction to induce buyers to submit bids equal to their (marginal) valuations for each unit? (Provide a brief answer, do not prove that truthful bidding is equilibrium in the mechanism you propose.)

(25) 5. Consider the following two-person, symmetric, normal-form game:

	A	B
A	5,1	0,0
B	4,4	1,5

What is the highest symmetric average discounted payoff level sustained in a subgame-perfect equilibrium, if this game is played

(a) once

(b) twice

(c) three times

(d) infinitely many times?

Please, support your numerical answers with clear reasoning; and explicitly construct strategy-profiles in each case that sustain the highest possible symmetric payoffs (on average over time, with discounting).

In (a)–(c) assume the discount factor is 1 (i.e., use simple average, per-period payoffs); in (d) assume the discount factor is $\delta < 1$, but as close to 1 as we want. When the game is repeated, both players observe the actions taken by both of them in all previous rounds of play. (If a player uses a mixed strategy, only the realized action is observed, not the mixing probabilities.)