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14.123 Microeconomic Theory III
Spring 2009

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14.123 Problem Set

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Due on Friday, March 13, 2009

Starred parts () are optional*

Q1. Consider the following signaling game.

Player 1 (he) is an employee with a college degree and has a privately-known type, either ‘Quant’ or ‘Poet’ (Q or P , respectively). He has the opportunity to pursue an MBA degree at MIT Sloan. If he gets an MBA, then his employer (Player 2) can promote him to Chief Financial Officer (CFO), which is a good match for the Q type, or Head of Human Resources (HR), which is the best use of a P type, or just keep him in his current position with a raise. Neither type of Player 1 likes to be promoted to the HR job. If Player 1 does not get an MBA, then he cannot be promoted.

The payoffs are summarized below. Nature chooses rows (Player 1’s type, Q or P , with 50-50% chance), Player 1 chooses matrices (‘No’ or ‘MBA’), and Player 2 chooses columns (conditional on ‘MBA’, either ‘CFO’, ‘HR’, or ‘N’). Payoffs are listed for Player 1 first.

		CFO	HR	N				
Q	<table border="1"><tr><td>2,2</td></tr></table>	2,2	<table border="1"><tr><td>5,5</td></tr></table>	5,5	<table border="1"><tr><td>0,0</td></tr></table>	0,0	<table border="1"><tr><td>3,3</td></tr></table>	3,3
2,2								
5,5								
0,0								
3,3								
P	<table border="1"><tr><td>2,2</td></tr></table>	2,2	<table border="1"><tr><td>0,0</td></tr></table>	0,0	<table border="1"><tr><td>1,5</td></tr></table>	1,5	<table border="1"><tr><td>3,3</td></tr></table>	3,3
2,2								
0,0								
1,5								
3,3								
	No	MBA						

- (a) Derive all pure-strategy perfect Bayesian equilibria of the game.
- (b) Prove that the Intuitive Criterion does *not* eliminate the equilibrium where both types of Player 1 choose ‘No’. Prove the same about the D1 criterion.

(c) Suppose Player 2 does not have action ‘N’ (cannot keep the employee in his current position, even with a raise, in case he gets an MBA). Does the Intuitive Criterion eliminate pooling on ‘No’? How about ‘D1’?

*(d) Check if the pooling equilibrium on ‘No’ is stable in the original game.

Q2. MWG 13.C.6 (Consider a market for loans to finance investment projects...)

Q3. Suppose there are n buyers for a single object. The buyers have private values θ_i drawn i.i.d. according to continuous pdf $f > 0$ on $[\underline{\theta}, \bar{\theta}]$. The buyers are risk averse with zero initial wealth, so i 's utility is $u(\theta_i - b)$ when he gets the object and pays b , and $u(0)$ otherwise; we normalize $u(0) = 0$ and assume $0 < u'(\cdot) < \infty$, $u''(\cdot) < 0$.

(a) Write down the differential equations that characterize the symmetric equilibrium bid functions in the First-Price Auction (FPA) and the All-Pay Auction (APA) and show that the bid functions are strictly increasing.

Rules of the First-Price Auction: Simultaneously submit bids; the bidder with the highest bid wins and pays his own bid. All-Pay Auction: simultaneously submit bids; the highest bid wins, all bidders pay their bids. Ignore tie-breaking as the distribution of valuations is atomless.

(b) Prove that risk-averse buyers prefer the symmetric equilibrium of the First-Price Auction to the symmetric equilibrium of the All-Pay Auction in this model.

Hint: Write down a $U^M(\theta_i, \theta'_i)$, the deviation payoff of bidder i with valuation θ_i bidding as if his valuation was θ'_i in mechanism $M = FPA$ and $M = APA$, respectively. Use the Envelope Theorem to calculate the slope of $V^M(\theta_i) \equiv U^M(\theta_i, \theta_i)$, the (indirect) utility in mechanism M . Then use Lemma 2 (from Lecture 10 or Milgrom and Weber (1982)).