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Citation: Kewlani, G., and K. Iagnemma. "A Multi-Element generalized Polynomial Chaos approach to analysis of mobile robot dynamics under uncertainty." Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on. 2009. 1177-1182. ©2009 Institute of Electrical and Electronics Engineers.

As Published: <http://dx.doi.org/10.1109/IROS.2009.5354420>

Publisher: Institute of Electrical and Electronics Engineers

Persistent URL: <http://hdl.handle.net/1721.1/59443>

Version: Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

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A Multi-Element Generalized Polynomial Chaos Approach to Analysis of Mobile Robot Dynamics under Uncertainty

Gaurav Kewlani, Karl Iagnemma

Abstract — The ability of mobile robots to quickly and accurately analyze their dynamics is critical to their safety and efficient operation. In field conditions, significant uncertainty is associated with terrain and/or vehicle parameter estimates, and this must be considered in an analysis of robot motion. Here a Multi-Element generalized Polynomial Chaos (MEgPC) approach is presented that explicitly considers vehicle parameter uncertainty for long term estimation of robot dynamics. It is shown to be an improvement over the generalized Askey polynomial chaos framework as well as the standard Monte Carlo scheme, and can be used for efficient, accurate prediction of robot dynamics.

I. INTRODUCTION

A basic requirement for mobile robot systems operating in unstructured environments is the ability to rapidly and accurately predict their movement over rugged terrain. Most methods for analysis of robot dynamics rely on deterministic analysis that assumes accurate knowledge of vehicle and terrain parameters. In field conditions, however, mobile robots frequently have access to only sparse and uncertain terrain parameter estimates drawn from sensors such as LIDAR and vision. Moreover, robot parameters may be uncertain and/or time-varying due to, for example, fuel consumption and mechanical wear. There has also been little research that explicitly addresses the challenge of modeling robot motion over a given terrain region while considering these parametric uncertainties.

Numerous techniques can be employed to estimate the output(s) for processes that are subject to uncertainty, including interval mathematics, probabilistic methods, and fuzzy set theory, among others [1]-[3]. A traditional method for estimating the probability density function of a system's output response while considering uncertainty is the Monte Carlo method [4], [5]. This approach involves sampling values for each uncertain parameter from its uncertainty range (weighted by its probability of occurrence), followed by model simulation using this parameter set, then repeating this process many times to obtain the probability distribution of an output metric. A large number of simulation runs is generally required to obtain reasonable results, leading to a (usually) high computational cost. While structured sampling techniques such as Latin hypercube sampling and importance sampling can be used to improve computational efficiency, the gains may be modest for complex problems [6], [7].

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More recent approaches include the polynomial chaos approach (based on Wiener's theory of homogeneous chaos), and the stochastic response surface method (SRSM) [8][9]. Previous work by the authors has also focused on stochastic performance prediction of mobile robot mobility using SRSM [10], which has been shown to be more robust than the generalized polynomial chaos (gPC) method [9].

While gPC has been successfully applied to various problems, it has been shown to perform inadequately for problems concerning discontinuities induced by random inputs, and for long-term integration. In [11], the method has been successfully applied to approximate the solution of a stochastic ODE while showing exponential convergence; however, it has been shown that those optimal results hold only for short times [12]. For long-term integration, therefore, the gPC approximation to the analytical solution for a fixed polynomial degree q fails, resulting in increased error levels. These problems can be overcome through implementation of the Multi-Element generalized Polynomial Chaos (MEgPC) framework, which involves a decomposition of the random space, to yield more consistent results [12]. In this paper, an MEgPC approach to robot dynamic analysis is presented.

This paper is organized as follows. In Section 2, the gPC and MEgPC methods are briefly introduced. The latter approach is then applied to a simple stochastic system in Section 3 and its behavior is analyzed. This is followed by its application to robot models in Section 4. The effect of robot parameter uncertainty is studied and simulation results are compared for Monte Carlo, gPC and MEgPC approaches. It can be seen that efficient statistical mobility prediction can be achieved using the proposed techniques, and for long-term prediction, the MEgPC approach yields more accurate results compared to the gPC method.

II. UNCERTAINTY ANALYSIS TECHNIQUES

1. Generalized Polynomial Chaos

The gPC method involves representing inputs and outputs of a system under consideration via series approximations using standard random variables, thereby resulting in a computationally efficient means for uncertainty propagation through complex numerical models.

In this approach, the same set of random variables that is used to represent input stochasticity is used for representation of the output(s). For uniformly distributed random inputs, an equivalent reduced model for the output can be expressed in the form of a series expansion consisting of multi-dimensional Legendre polynomials of uniform random variables, as:

$$y = a_0 + \sum_{i_1=1}^n a_{i_1} \Gamma_1(\xi_{i_1}) + \sum_{i_1=1}^n \sum_{i_2=1}^{i_1} a_{i_1 i_2} \Gamma_2(\xi_{i_1}, \xi_{i_2}) + \dots \quad (1)$$

where y refers to an output metric, $\xi_{i_1}, \xi_{i_2}, \dots$ are i.i.d. uniform random variables, $\Gamma_q(\xi_{i_1}, \xi_{i_2}, \dots, \xi_{i_q})$ is the Legendre polynomial of degree q and $a_{i_1}, a_{i_1 i_2}, \dots$ are the corresponding coefficients.

For notational simplicity, the series can be written as:

$$y = \sum_{j=0}^{N_q} y_j \Phi_j(\xi) \quad (2)$$

where the series is truncated to a finite number of terms and there exists a correspondence between $\Gamma_q(\xi_{i_1}, \xi_{i_2}, \dots, \xi_{i_q})$ and $\Phi(\xi)$, and their corresponding coefficients.

The unknown coefficients can be determined by projecting each state variable onto the polynomial chaos basis (i.e. the Galerkin projection method) [11]. Another approach that is computationally more efficient is the probabilistic collocation method, where coefficient values are estimated from a limited number of model simulations, to generate the approximate reduced model [13]. If the governing equations are highly complex, the simplicity of the latter framework results in a faster algorithm, particularly for high dimensional problems.

A. Algorithmic Implementation

A summary of the gPC procedure, as applied to robotic dynamic analysis, is presented here. Further details can be found in [11].

a) Represent uncertain input parameters in terms of standard random variables. An uncertain terrain and/or vehicle parameter X_j can thus be written as:

$$X_j = \mu_j + \sigma_j \xi_j \quad (3)$$

where μ_j is the mean, σ_j is a constant (represents standard deviation when X_j is normally distributed) and ξ_j is a uniformly distributed random variable.

b) Express the model output (y) under consideration in terms of the same set of random variables as:

$$y = \sum_{j=0}^{N_q} y_j \Phi_j(\xi) \quad (4)$$

While for uniform random variables Legendre polynomials are used, different orthogonal polynomial basis functions are used for other types of probability distributions [10].

c) Estimate the unknown coefficients of the approximating series expansion. This is accomplished by computing the model output at a set of collocation points [9][13]. This results in a set of equations that can then be used to obtain the coefficient values. In the current analysis, the efficient collocation method (ECM) [9], is used.

d) Once the reduced order model is formulated (using orthonormal basis functions), the mean and variance can be directly obtained [14] as:

$$\mu = y_0 \quad (5)$$

$$\sigma^2 = \sum_{j=1}^{N_q} y_j^2 \quad (6)$$

The advantage of the gPC technique is that the number of model simulations is greatly reduced relative to more

conventional methods, thereby reducing computational cost. However, the technique is known to fail for long-term integration, losing its optimal convergence behavior and developing large error levels [12]. This behavior can be somewhat mitigated by increasing the expansion order, however, this approach is undesirable for several reasons. First, gPC computational cost generally increases with increasing polynomial order. More importantly, increasing the polynomial order only postpones error growth. For a fixed polynomial degree, error levels will become increasingly large over time. The MEgPC technique, however, has been shown to solve these long-term integration issues faced in the gPC framework [12]. This is briefly discussed below.

2. Multi-Element Generalized Polynomial Chaos

In [12], it has been shown that if the domain of random inputs is subdivided into multiple elements, the accuracy of stochastic solutions can be improved, especially for cases with discontinuities in stochastic solutions or for problems involving long-term integration. As a result, integration error at each time step can be reduced and the domain of solutions' discontinuity can be approximated more accurately within a smaller decomposed domain. Further, a (relatively) lower order polynomial can be used in each random element since the local degree of perturbation has been scaled down, thereby enhancing the accuracy of solutions for long term integration.

Thus, while the range of application of gPC is limited (since the polynomial order can not be increased arbitrarily high in practice), using MEgPC allows this range to be extended.

A. Decomposition of the Random Space

Let $\xi = [\xi_1, \xi_2, \dots, \xi_n]$ denote an n dimensional random input vector, where ξ_i is an independent, identically distributed uniform random variable, $U[-1,1]$. Next, decompose the domain of the random input into N non-intersecting elements. The domain of each element is contained within a hypercube, $[a^k_1, b^k_1] \times [a^k_2, b^k_2] \times \dots \times [a^k_n, b^k_n]$, where a and b denote the lower and upper bounds of the local random variables.

Then, define a local random vector within each element as $\xi^k = [\xi^k_1, \xi^k_2, \dots, \xi^k_n]$, and subsequently map it to a new random vector in $[-1,1]^n$: $\xi^k = g_k(\xi^k) = [\xi^k_1, \xi^k_2, \dots, \xi^k_n]$. This mapping is governed by the following relationship:

$$g_k(\xi^k): \xi^k_i = \frac{b^k_i - a^k_i}{2} \xi^k_i + \frac{b^k_i + a^k_i}{2} \quad (7)$$

Consequently, the gPC framework can be used locally to solve a system of differential equations, with the random inputs as ξ^k instead of ξ , to take advantage of orthogonality and related efficiencies by employing Legendre Chaos. The global mean and variance can then be reconstructed once local approximations of the mean and the variance are obtained.

B. Algorithmic Implementation

Decomposition of the random space can be done a priori, or adaptively. In the adaptive scenario, splitting of the random space occurs only when the local decay rate of the error of the gPC approximation η_k (see Equation 12) exceeds a threshold

value. The general procedure is briefly discussed below. Refer to [12] for further details.

Let the gPC expansion in random element k be given as:

$$y_k = \sum_{j=0}^{N_q} y_{k,j} \Phi_j(\xi) \quad (8)$$

The approximated global mean and variance can then be written as:

$$\bar{y} = \sum_{k=1}^N y_{k,0} V_k \quad (9)$$

where $V_k = \prod_{i=1}^N (b_{k,i} - a_{k,i})/2$, and

$$\bar{\sigma}^2 = \sum_{k=1}^N (\sigma_k^2 V_k + (y_{k,0} - \bar{y})^2 V_k) \quad (10)$$

where the local variance estimated by polynomial chaos (using orthonormal basis functions) is obtained as:

$$\sigma_k^2 = \sum_{j=1}^{N_q} y_{k,j}^2 \quad (11)$$

To include adaptive decomposition of the random space, first define η_k as:

$$\eta_k = \frac{\sum_{j=N_{q-1}+1}^{N_q} y_{k,j}^2}{\sigma_k^2} \quad (12)$$

Then, split a random element if the following criterion is satisfied:

$$\eta_k^\alpha V_k \geq \theta_1, \quad 0 < \alpha < 1 \quad (13)$$

where θ_1 is a suitable threshold parameter and α is a constant. Another parameter θ_2 may be used to choose the more sensitive random dimensions for decomposition, as in [12].

A critical numerical implementation involves assigning the initial condition after splitting the random dimension into multiple elements. This can be accomplished as follows:

First, represent the polynomial expansion of the current random field as:

$$\hat{y}(\xi) = \sum_{j=0}^{N_q} \hat{y}_j \Phi_j(\xi) \quad (14)$$

Next, let the expansion in the next level be denoted as:

$$\tilde{y}(\xi) = \tilde{y}(g(\xi)) = \sum_{j=0}^{N_q} \tilde{y}_j \Phi_j(\xi) \quad (15)$$

To calculate the N_q+1 coefficients in this new representation, choose an equal number of uniform grid points in $[-1,1]^n$, and solve the following linear system:

$$\begin{bmatrix} \Phi_{00} & \Phi_{10} & \dots & \Phi_{N_q 0} \\ \Phi_{01} & \Phi_{11} & \dots & \Phi_{N_q 1} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{0N_q} & \Phi_{1N_q} & \dots & \Phi_{N_q N_q} \end{bmatrix} \begin{bmatrix} \tilde{y}_0 \\ \tilde{y}_1 \\ \vdots \\ \tilde{y}_{N_q} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{N_q} \hat{y}_i \Phi_i(g^{-1}(\xi_0)) \\ \sum_{i=0}^{N_q} \hat{y}_i \Phi_i(g^{-1}(\xi_1)) \\ \vdots \\ \sum_{i=0}^{N_q} \hat{y}_i \Phi_i(g^{-1}(\xi_{N_q})) \end{bmatrix} \quad (16)$$

where $\Phi_{ij} = \Phi_i(\xi_j)$.

III. APPLICATION TO A SIMPLE STOCHASTIC SYSTEM AND STUDY OF CONVERGENCE

Consider a simple stochastic system: a first order linear ODE, described as:

$$\frac{dy}{dt} = -ky \quad \text{with } y_{t=0} = y_0 = 1 \quad (17)$$

Here, the decay rate coefficient k is considered to be a random variable, $k = \bar{k} + \sigma_k \xi$, with a constant mean ($\bar{k} = 1$) and standard deviation ($\sigma_k = 1$), and ξ is a standard uniform random variable. While the deterministic solution $y(t)$ for the ODE above is $y_0 e^{-kt}$, the mean of the stochastic solutions is given by:

$$\bar{y}_{exact}(t) = y_0 \int_S e^{-kt} f(k) dk = y_0 \frac{e^{-(\bar{k} + \sigma_k)t} (e^{2\sigma_k t} - 1)}{2\sigma_k t} \quad (18)$$

To study the rate of convergence, define the error as an L_2 norm difference between the estimated result and the reference solution, normalized by the L_2 norm of the latter. This relative error measurement for the mean is expressed as:

$$\varepsilon_{mean}(t) = \frac{\|\bar{y}(t) - \bar{y}_{exact}(t)\|_2}{\|\bar{y}_{exact}(t)\|_2} \quad (19)$$

where $\bar{y}(t) = E[y(t)]$

Next, the convergence behavior of MEgPC is analyzed, with varying values of the polynomial order P and number of elements N . For this analysis, the adaptive criterion is not applied, and instead the random space is decomposed according to the number of elements desired. The error in the mean is calculated at $t = 4$ s, and the results are similar to those obtained in [12].

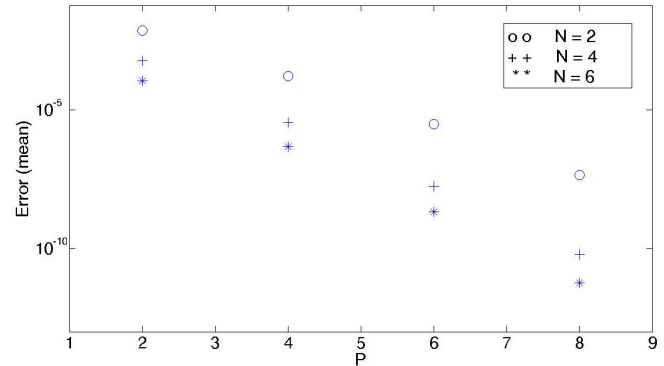


Fig. 1. Error convergence of MEgPC

In Figure 1, exponential convergence of MEgPC for various mesh sizes is shown. It can be observed that as the number of elements increases, not only does the error decrease, but the rate of convergence is higher as well.

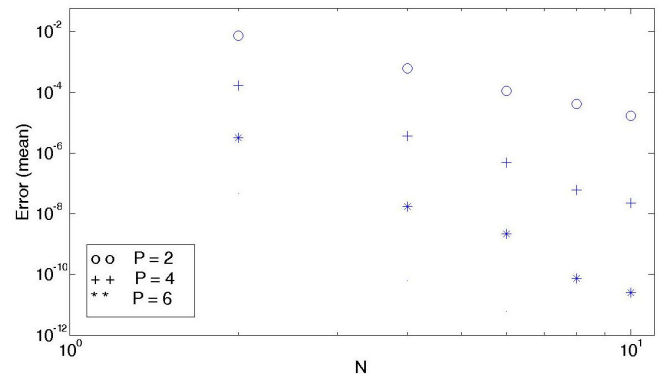


Fig. 2. Error convergence for MEgPC

In Figure 2, algebraic convergence of MEgPC in terms of the random element N is shown. A sufficiently large algebraic

index of convergence is seen, which indicates that random elements can influence the accuracy dramatically.

Next, the time evolution of the error for gPC and adaptive MEgPC approaches are compared at $t=4s$ (with parameters $P = 4$, $\theta_I=0.001$, $\alpha=0.5$). It can be seen that when the error of gPC crosses the threshold limit, it triggers decomposition of the random space and the accuracy is significantly improved.

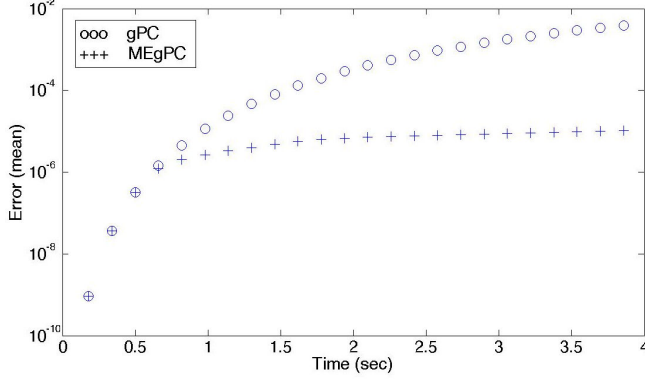


Fig. 3 Evolution of error for gPC and adaptive MEgPC

IV. APPLICATION TO MOBILE ROBOT DYNAMICS ANALYSIS

The MEgPC approach is here applied to the analysis of dynamic mechanical systems. We first study a two degree of freedom quarter-car model under uncertainty and then extend the analysis to include a three degree of freedom robot model traversing uneven terrain. While the exact stochastic solutions may be easy to obtain for simple systems, they may be difficult to obtain for large and complex systems such as those considered in the following analysis. For such scenarios, the exact solution can be replaced by a reference solution obtained from a Monte Carlo analysis.

1. Analysis of a Quarter-Car Model

Here a 2 DOF quarter car model of a vehicle suspension is considered (see Figure 4). The sprung mass, m_s , and the unsprung mass, m_u , are connected by a nonlinear spring of stiffness k_s , and a linear damper with damping coefficient c . The input is applied through a forcing function $z(t)$, to m_u , through a linear spring k_u . This represents the interaction of the quarter car system with the terrain. The governing equations for the quarter car system are given as:

$$m_s \frac{d^2 x_1}{dt^2} = -k_s (x_1 - x_2)^3 - c (\dot{x}_1 - \dot{x}_2) \quad (20)$$

$$m_u \frac{d^2 x_2}{dt^2} = k_s (x_1 - x_2)^3 + c (\dot{x}_1 - \dot{x}_2) + k_u (z(t) - x_2) \quad (21)$$

The displacements of the two masses under parametric uncertainty are analyzed, using the gPC and MEgPC approaches, and compared to results from a baseline Monte Carlo analysis with random sampling (SMC).

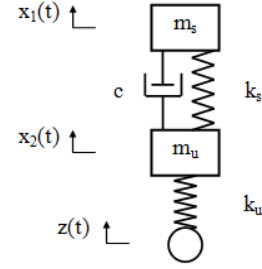


Fig. 4. Quarter-car model

Parametric uncertainty arises in the suspension stiffness. We consider the two springs to have uncertain spring constant values, uniformly distributed about a mean stiffness value. This can be represented as:

$$k_s = \mu_{k_s} + \xi_1 \sigma_{k_s} \quad (22)$$

$$k_u = \mu_{k_u} + \xi_2 \sigma_{k_u} \quad (23)$$

Displacements are then expressed as a series expansion of Legendre polynomials of standard uniform random variables ξ_1 and ξ_2 , and the system is analyzed for various terrain inputs. In general, the state may be expressed as:

$$\mathbf{X} = [x_1 \ x_2 \ \dot{x}_1 \ \dot{x}_2]^T \quad (24)$$

$$x_i(t, \boldsymbol{\xi}) = \sum_{j=0}^P x_{i,j}(t) \Phi_j(\boldsymbol{\xi}) \quad i=1,2 \quad (25)$$

$$\dot{x}_i(t, \boldsymbol{\xi}) = \sum_{j=0}^P \dot{x}_{i,j}(t) \Phi_j(\boldsymbol{\xi}) \quad i=1,2 \quad (26)$$

where $\boldsymbol{\xi} = [\xi_1, \xi_2]$

Stiffness parameter values used in this analysis are shown in Table 1.

TABLE 1
UNCERTAIN PARAMETERS IN QUARTER CAR MODEL

PARAMETER	μ	σ
k_s	400 N/m ³	40 N/m ³
k_u	2000 N/m	200 N/m

A. Simulation Results

For a step input (which models traversal over a bump), it is observed that parametric uncertainty causes significant variation in the resulting output of the sprung mass displacement (see Figure 5), thus indicating the importance of considering uncertainty during dynamic analysis.

The time profile obtained for the standard deviation of the displacement for the sprung mass is shown in Figure 6, for the gPC, MEgPC ($P = 3$, $\theta_I=0.001$, $\alpha=0.5$) and SMC methods.

We observe that, even for relatively short times, there is substantial difference between the predicted variance from the two techniques, with MEgPC yielding more precise results than gPC when compared to the baseline SMC analysis. Relative computation times for the different methods are shown in Table 2.

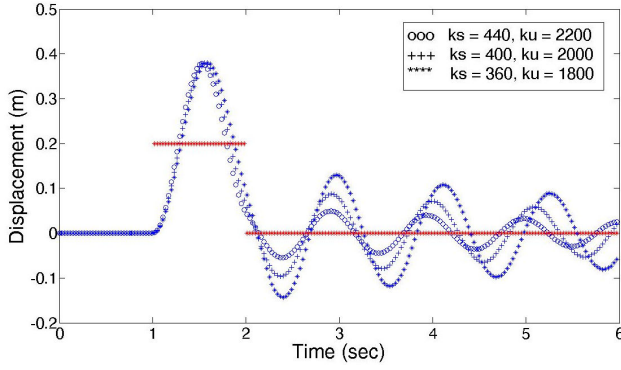


Fig. 5. Sprung mass displacement

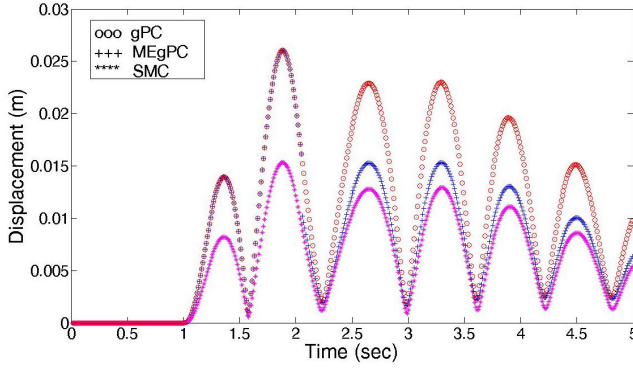


Fig. 6. Standard deviation of sprung mass displacement

TABLE 2
COMPUTATION TIME

METHOD	TIME TAKEN (s)
SMC (2000 runs)	5296.2 s
gPC	27.59 s
MEgPC	43.74 s

Next, the system is analyzed for a sinusoidal terrain input. As expected, inconsistency is found in the predictions made using the gPC approach (see Figure 7). This is, however, not the case with the MEgPC approach, which exhibits only a small bounded error over time.

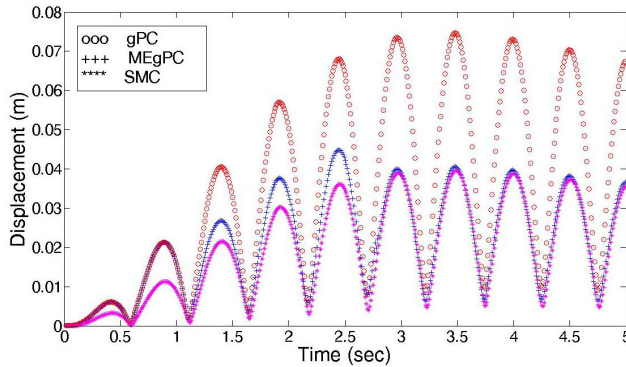


Fig. 7. Standard deviation of sprung mass displacement

2. Analysis of Dubins Vehicle

Here a three degree of freedom robot model (see Figure 8) is considered that includes lateral acceleration, yaw and roll dynamics, as in [15]. The roll and yaw moments of inertia are

represented by I_{xx} and I_{zz} respectively, m is the total vehicle mass, m_s is the sprung mass, V is the longitudinal velocity of the vehicle and δ represents the front wheel steering angle. The linearized equations for this model are given as:

$$\dot{\beta} = -\frac{GC}{mV}\beta + \left(-1 + \frac{KG}{mV^2}\right)\dot{\psi} + \frac{C_f G}{mV}\delta + \frac{m_s h M_s}{mV I_{xx}^o} + \frac{m_s^2 g h^2}{mV I_{xx}^o}\phi + \frac{G}{mV}\sum T_i \quad (27)$$

$$\ddot{\phi} = \frac{m_s g h}{I_{xx}^o}\phi + \frac{M_s}{I_{xx}^o} - \frac{m_s C h}{m I_{xx}^o}\beta + \frac{m_s K h}{m V I_{xx}^o}\dot{\psi} + \frac{C_f m_s h}{m I_{xx}^o} + \frac{m_s h}{m I_{xx}^o}\sum T_i \quad (28)$$

$$\ddot{\psi} = \frac{K}{I_{zz}}\beta - \frac{D}{V I_{zz}}\dot{\psi} + \frac{C_f l_f}{I_{zz}}\delta + \frac{1}{I_{zz}}\sum T_i l_i \quad (29)$$

where

$$C = C_f + C_r, K = C_r l_r - C_f l_f, D = C_f l_f^2 + C_r l_r^2, G = 1 + m_s^2 h^2 / (m I_{xx}^o), I_{xx}^o = I_{xx} + m_s h^2 (1 - m_s / m).$$

Here C_f and C_r are the cornering stiffness values of the lumped front and rear wheels, g is the gravitational acceleration, l_f and l_r are, respectively, the distances of the front and rear axles from the center of gravity, and T_i is the terrain force acting at each wheel.

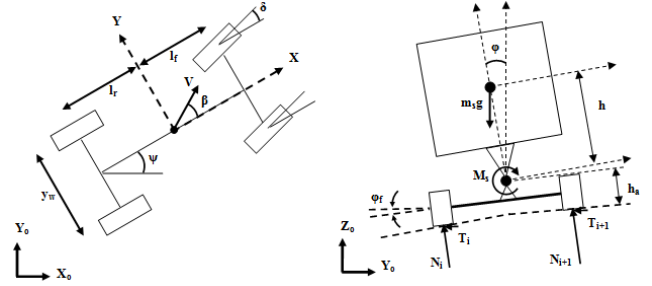


Fig. 8. Robot model for mobility analysis under uncertainty

The suspension moment is given as:

$$M_s = -k_f(\phi - \phi_f) - k_r(\phi - \phi_r) - b_f(\dot{\phi} - \dot{\phi}_f) - b_r(\dot{\phi} - \dot{\phi}_r) \quad (30)$$

where k_f and k_r are the stiffness values, b_f and b_r are the damping rates of the front and rear axles and ϕ_f and ϕ_r are the terrain roll angles. In these equations, lateral components of the terrain contact forces as well as the roll angles and rates due to the sloped terrain have been included. Details on calculating their values can be found in [15].

For measuring robot stability, a rollover coefficient is defined as in [16]. Using the principle of balance of moments and vertical forces, the rollover metric for the linear model under consideration is given as:

$$R = \frac{2m_s}{mgy_w}(h_a + h)(v(\dot{\beta} + \dot{\psi}) - h\dot{\phi}) \quad (31)$$

where h_a is the height of the roll axis above the ground and y_w is the track width. This may further be expressed in terms of the state space variables from the equations of motion above. For this metric, $|R| > 1$ indicates vehicle wheel liftoff and thus impending rollover.

In this analysis, the double-lane-change steering maneuver is considered as input and the roll angle evolution under vehicle parameter uncertainty is studied using the gPC, MEgPC ($P = 3, \theta_r = 0.05, \alpha = 0.5$) and SMC approaches, for motion over uneven terrain (represented using a combination of trigonometric functions). Here, roll stiffness parameters are

represented as polynomial chaos expansions, using Legendre polynomials of standard uniform random variables ξ_1 and ξ_2 .

The front and rear axle roll stiffness values are considered to be uniformly distributed about their mean values. This is represented as:

$$k_f = \mu_{k_f} + \xi_1 \sigma_{k_f} \quad (32)$$

$$k_r = \mu_{k_r} + \xi_2 \sigma_{k_r} \quad (33)$$

For the MEgPC approach, the state variables can be represented as:

$$X_i(t, \xi) = \sum_{j=0}^p X_{i,j}(t) \Phi_j(\xi), \quad i = 1 \text{ to } 7 \quad (34)$$

where $\xi = [\xi_1, \xi_2]$.

The roll stiffness parameter values considered in the study are shown in Table 3.

TABLE 3
UNCERTAIN VEHICLE PARAMETERS IN ROLLOVER ANALYSIS

PARAMETER	μ (Nm/rad)	σ (Nm/rad)
k_f	30×10^3	4×10^3
k_r	30×10^3	4×10^3

A. Simulation Results

Using the expansions in (34), a spectral stochastic analysis [16] is performed to obtain the time evolution of the roll angle standard deviation. It can be seen that the prediction from the gPC approach differs substantially from the MEgPC and SMC results.

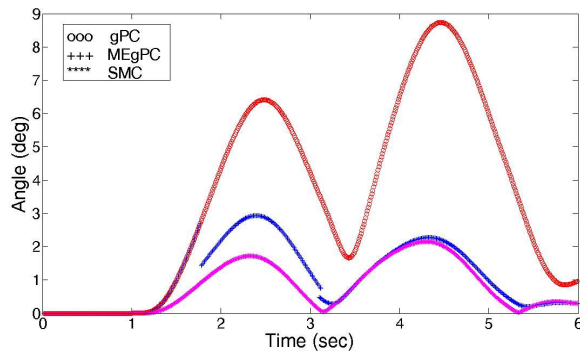


Fig. 9. Prediction of standard deviation of roll angle

Next, the time evolution of the standard deviation of R is studied for a sinusoidal input (see Figure 10). It can again be observed that there is significant difference in the predictions, for the two polynomial chaos-based techniques.

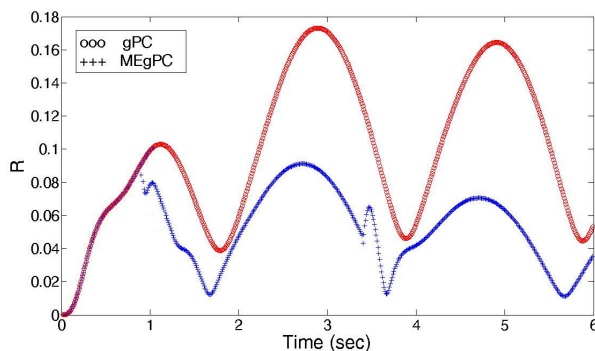


Fig. 10. Prediction of standard deviation of rollover metric

Relative computation times for the methods are shown below in Table 4.

TABLE 4
COMPUTATION TIME

METHOD	TIME TAKEN (s)
SMC (2000 runs)	7076.2 s
gPC	35.83 s
MEgPC	192.38 s

V. CONCLUSION

This paper has presented an approach to mobile robot dynamics prediction based on the MEgPC framework, while explicitly considering uncertainty in robot parameter estimates. Simulation results show that the method represents a significant improvement over the Monte Carlo technique in terms of computational cost, and over the gPC method in terms of accuracy of long-term predictions. The approach can be applied to various applications, such as mobility prediction and path planning under uncertainty.

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