8 May 1996 **Prof. S. Lee**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam III for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

[6] **1a.** Give a 2 by 2 matrix B that is similar to $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$. (Do not use $B = A$.)

[6] **1b.** Give a 2 by 2 matrix B that is similar to $A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$. (Do not use $B = A$.)

[10] **1c.** Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 2 & 4 & 4 \end{bmatrix}$. Is $x^T Ax > 0$ for all nonzero vectors x?

If not, find a nonzero vector x for which $x^T Ax \leq 0$.

2. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$:

[4] **2a.** Find the eigenvalues of A.

[6] **2b.** Give a factorization $A = Q\Lambda Q^T$ where Q has orthonormal columns and Λ is a diagonal matrix.

[12] **2c.** As $t \to \infty$, what is the limit of $u(t)$ for $\frac{du}{dt} = -Au$ given the initial condition $u(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$?

3. Let $A = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$:

[8] **3a.** Find the eigenvalues and eigenvectors of the matrix A.

[14] **3b.** Let $u_{k+1} = A^{-1}u_k$. What is the limit of u_k as $k \to \infty$ given the initial condition $u_0 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$?

4. Suppose the following information is known about A:

$$A\begin{bmatrix}1\\2\\1\end{bmatrix}=6\begin{bmatrix}1\\2\\1\end{bmatrix}, \quad \text{and} \quad A\begin{bmatrix}1\\-1\\1\end{bmatrix}=3\begin{bmatrix}1\\-1\\1\end{bmatrix}, \quad \text{and} \quad A\begin{bmatrix}2\\-1\\0\end{bmatrix}=3\begin{bmatrix}1\\-1\\1\end{bmatrix}.$$

[9] **4a.** Find the eigenvalues of A.

[9] **4b.** Find the eigenvectors of A.

[16] **4c.** In each of these questions, you must give a correct reason (based on eigenvalues, eigenvectors, etc.) to get full credit.

(i) Is A a diagonalizable matrix?

(ii) Is A an invertible matrix?

(iii) Is A a Markov matrix?

(iv) Is A a projection matrix?

(v) Is A a symmetric matrix?