10 April 1996 **Prof. S. Lee**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam II for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

[6] **1a.** Give a 3 by 3 matrix A with the properties: $A^T = A^{-1}$ and $\det A = 1$.
(A is not allowed to be a diagonal matrix.)

[6] **1b.** Give a 3 by 3 matrix A with the properties: $A^T = A$ and $A^2 = A$ and $\operatorname{rank}(A) = 1$.
(A is not allowed to be a diagonal matrix.)

[6] **1c.** Suppose A is a 5 by 3 matrix with orthonormal columns. Evaluate the following determinants:

$\det(A^T A) =$ ____________

$\det(AA^T) =$ ____________

$\det(A(A^T A)^{-1} A^T) =$ ____________

[6] **1d.** What value(s) of x give $\det A = 0$, where

$$A = \begin{bmatrix} x & 2 & 3 \\ -x & x & 0 \\ 3 & 2 & 5 \end{bmatrix}?$$

2. Suppose $Ax = b$ has the complete solution $x = \underbrace{\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}}_{x_{\text{particular}}} + \underbrace{x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}}_{x_{\text{homogeneous}}}$,

where $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

[8] **2a.** How many linearly independent rows does A have? Explain.

[8] **2b.** Give a basis for the row space of A.

3. The projection matrix

$$P = \frac{1}{50}\begin{bmatrix} 34 & -20 & -12 \\ -20 & 25 & -15 \\ -12 & -15 & 41 \end{bmatrix}$$

projects vectors onto a 2-dimensional plane in $\mathbf{R}^3$.

[8] **3a.** $PP^T =$ ____(give the 3 by 3 matrix)____.

[10] **3b.** Give a nonzero vector c such that $Pc = c$.

[12] **3c.** Find a matrix A such that $A(A^TA)^{-1}A^T = P$.

4. The vectors $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -10 \\ 0 \\ 1 \end{bmatrix}$ are linearly independent and span the left nullspace of a matrix A.

[8] **4a.** Give an orthonormal basis for the left nullspace of A.

[10] **4b.** Find the vector p (in the column space of A) that is closest to $b = \begin{bmatrix} -2 \\ 6 \\ -20 \end{bmatrix}$.

[12] **4c.** Find the projection matrix P that projects all vectors onto the column space of A.