18.06 Hour Exam I 8 March 1996 Prof. S. Lee

Your Recitation Leader:

Your Name:

Your Recitation Time:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these 7 pages. No calculators or notes.
Point values (total of 100) are marked on the left margin.

[4] **1a.** Are the vectors $\begin{bmatrix} -2 \\ -1 \\ 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -8 \\ 2 \\ -2 \\ 1 \end{bmatrix}$ linearly independent?

Are these vectors perpendicular to each other? Explain.

[4] **1b.** Are the vectors $\begin{bmatrix} -2 \\ -1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 8 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ 1 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \\ 4 \end{bmatrix}$ a basis for $\mathbf{R}^4$? Explain.

[8] **1c.** Are the vectors $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ 2 \\ 1 \end{bmatrix}$ a basis for the subspace described by the 3-dimensional plane $x_1 + 2x_2 + 3x_3 + 6x_4 = 0$? Explain.

[10] **1d.** Find the value for q for which the vectors $\begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 12 \\ 10 \end{bmatrix}, \begin{bmatrix} q \\ 3 \\ 1 \end{bmatrix}$ do **not** span $\mathbf{R}^3$.

2. Consider the following 3×3 matrix, $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & 1 \\ -3 & -9 & -1 \end{bmatrix}$.

[10] **(a)** Find the missing entries of $A^{-1} = \begin{bmatrix} 4 & 3 & 3 \\ \alpha_{21} & -1 & -1 \\ \alpha_{31} & \alpha_{32} & -1 \end{bmatrix}$.

[10] **(b)** Using the result in part(a), find the inverse of $\begin{bmatrix} 3 & 0 & 1 \\ 5 & 1 & 2 \\ -9 & -1 & -3 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 1 & 5 & 0 & -3 \\ 2 & 10 & 1 & -4 \\ -1 & -5 & 1 & 5 \end{bmatrix}$.

[6] **(a)** Give an LU-factorization of A.

[6] **(b)** Give a basis for the column space of A.

[6] **(c)** Give a basis for the row space of A.

[6] **(d)** Give a basis for the nullspace of A.

[6] **(e)** Give the complete solution (general solution) to the equation $Ax = b$ in the form $x = x_{\text{particular}} + x_{\text{homogeneous}}$ where

$$b = \begin{bmatrix} 6 \\ 4 \\ -14 \end{bmatrix}.$$

4. The following information is known about A:

$$A\begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} \quad \text{and} \quad A\begin{bmatrix} 3 \\ -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -18 \\ 9 \end{bmatrix}.$$

In fact, Ax is always some multiple of $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

[6] **(a)** What is the dimension of the nullspace of A?

[4] **(b)** What is the dimension of the row space of A?

[6] **(c)** What is the dimension of the left nullspace of A?

[8] **(d)** Give a nonzero solution to $Ax = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.