MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Practice Final for 18.06: Linear Algebra

Recitation Instructor:

Your Name:

Recitation Time:

Lecturer: **S. Lee**

Grading

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TOTAL:

Point values (total of 200) are marked on the left margin.
No calculators or notes. Please work carefully, and check your intermediate results whenever possible.

1. Let $A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$.

[4] **1a.** Give a basis for the row space of A.

[4] **1b.** Give a basis for the nullspace of A.

[4] **1c.** Give a basis for the column space of A.

[4] **1d.** Do the solutions to $Ax = \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix}$ form a subspace?

If so, what is the dimension of this subspace.

2. Write down a matrix with the specified properties, or explain why no such matrix exists:

[6] **2a.** A matrix whose column space is spanned by $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$,

and row space is spanned by $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

[6] **2b.** A matrix whose column space has a basis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and row space has a basis $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

[6] **2c.** A matrix whose row space contains $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, nullspace contains $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, and whose determinant is -1.

[20] **3.** In $\mathbf{R}^3$, what point on the plane described by $x_1+x_2-x_3 = 0$ is closest to $b = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$?

[20] **4.** Find a matrix B such that $B^T B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 8 & 8 \\ 2 & 8 & 12 \end{bmatrix}$.

[24] **5.** Give an orthonormal basis for the column space of $A = \begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$.

6. Let $A = \begin{bmatrix} 1 & -2 & v_1 \\ -2 & 1 & v_2 \\ -2 & -2 & v_3 \end{bmatrix}$, where the third column of A is the vector $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$.

If possible, find a vector v that gives A the following properties.

[6] **6a.** A vector v that makes A a singular matrix.

[6] **6b.** A vector v that makes A an invertible matrix.

[6] **6c.** A vector v that makes A a matrix with orthogonal columns.

[6] **6d.** A vector v that makes A a symmetric positive definite matrix.

7. Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. The eigenvalues of A are $\lambda_1 = 4$ and $\lambda_2 = 2$.

[6] **7a.** Find $u(t)$ given that $\frac{du}{dt} = Au$ and $u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

[6] **7b.** Find $u(t)$ given that $\frac{du}{dt} = Au$ and $u(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

[12] **7c.** Find e^{At}.

8. Suppose that A has eigenvalues 0, 1, 2, with respective eigenvectors u, v, w.

[6] **8a.** Describe the nullspace, column space, and row space of A in terms of u, v, w.

[6] **8b.** Find all solutions to $Ax = v - w$ in terms of u, v, w.

[6] **8c.** Prove that A is not an orthogonal matrix.

9. Let $A = \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix}$.

[4] **9a.** Find the eigenvalues of A.

[10] **9b.** Find the steady-state limit of the vector $u_k = A^k \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, as $k \to \infty$.

[10] **9c.** Find the limit of A^k, as $k \to \infty$.

10. Let $A = \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}$, where $i = \sqrt{-1}$.

[6] **10a.** Give the LU factorization of A.

[6] **10b.** If possible, find a real matrix B that is similar to A.