4 December 1996 **Profs. S. Lee and M. Hopkins**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam III for Course 18.06: Linear Algebra

Recitation Instructor:

Your Name:

Recitation Time:

Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

[10] **1a.** If possible, find an invertible matrix M such that $M^{-1}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}M = \begin{bmatrix}1&1&1\\1&2&2\\1&2&2\end{bmatrix}$.

If it is not possible, briefly state why M cannot exist.

[10] **1b.** For what real values of c (if any) is $A = \begin{bmatrix} -1 & c & 2 \\ c & -4 & -3 \\ 2 & -3 & 4 \end{bmatrix}$ a symmetric positive definite matrix?

2. Let $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$, where $i = \sqrt{-1}$.

[8] **2a.** Find the eigenvalues and eigenvectors of A.

[8] **2b.** Find A^{-1}.

[8] **2c.** Find A^{100}.

3. Let $A = \begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix}$.

[8] **3a.** Find the eigenvalues and eigenvectors of A.

[12] **3b.** $\det(e^{At}) =$ ________________________.

[12] **3c.** For the differential equation $\frac{du}{dt} = Au$, give a nonzero initial vector $u(0)$ such that

$$u(t) \to \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ as } t \to \infty.$$

4. Suppose the following information is known about a matrix A:

$$A\begin{bmatrix}2\\0\\0\end{bmatrix}=\begin{bmatrix}2\\4\\-6\end{bmatrix},\quad A\begin{bmatrix}0\\-1\\0\end{bmatrix}=\begin{bmatrix}-2\\-4\\6\end{bmatrix}\text{ and } A \text{ is symmetric.}$$

The questions below refer to any matrix A with the above properties.

[8] **4a.** Is A invertible? Explain.

[8] **4b.** Does A have linearly independent eigenvectors? Explain.

[8] **4c.** Give a specific example of a matrix A with the above properties and whose eigenvalues add up to zero.