9 October 1996 **Profs. S. Lee and M. Hopkins**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam I for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

1. Let $A = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 2 & 1 & 1 & 2 \\ 4 & 2 & 3 & 2 \end{bmatrix}$.

[12] **1a.** Give the LU-factorization of A.

[6] **1b.** Give a basis for the column space of A.

[16] **1c.** For what value of q is $Ax = \begin{bmatrix} 2 \\ 3 \\ q \end{bmatrix}$ solvable?

[8] **2a.** Let $s = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ and $t = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

Suppose we have a matrix A (4 rows, 4 columns) such that $Ax = s$ has no solution and $Ax = t$ has no solution.

The rank of A is at most (circle a number): 1 2 3 4.

[8] **2b.** Let $u = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 6 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}$.

Suppose we have a matrix B such that $Bx = u$ has no solution and $Bx = v$ has no solution. Is it also possible that $Bx = u + v$ has infinitely many solutions?
If 'yes', give a matrix B that satisfies these conditions.
If 'no', briefly state why the matrix B cannot exist.

3. Let $A = L \begin{bmatrix} 2 & -2 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ where A is symmetric and L is a unit lower triangular matrix. (Reminder: A unit lower triangular matrix has ones on the diagonal and zeros above the diagonal.)

[6] **3a.** Find a nonzero solution to $Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

[12] **3b.** Find the unit lower triangular matrix L.

[16] **3c.** Give the complete solution to $Ax = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$. Identify $x_{\text{homogeneous}}$.

[6] **4a.** Given $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & 1 \\ -3 & -9 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 3 \\ -1 & -1 & -1 \\ -3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find the inverse of $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 5 & -9 \\ 0 & 1 & -1 \end{bmatrix}$.

[10] **4b.** Can you find a matrix A for which $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is a basis for the column space,

and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a basis for the nullspace?

If 'yes', give the matrix A.
If 'no', explain why the matrix A cannot exist.