IX. ELECTRODYNAMICS OF MEDIA

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1. ELECTROMAGNETIC WAVES

Jin Au Kong

In our studies of electromagnetic waves we have the following objectives: examination of subsurface probing and communication with a dipole antenna, observation of remote sensing of the Earth, investigation of wave optics as applied to optical communication systems, and advancement of our understanding of the fundamental issues concerning electromagnetic waves.

Publications during 1975 and publications in press that are supported by the Joint Services Electronics Program are listed.[1-9] In geophysical subsurface probing we have found that the horizontal magnetic dipole is more effective than the horizontal electric dipole.[1] We have also investigated the application of the radio-frequency interferometry method to stratified anisotropic media.[2] In remote sensing we studied microwave thermal emission from stratified media with nonuniform temperature profiles[3] and calculated brightness temperatures with nonuniform temperature distributions.[4] We investigated scattering and emission properties of bounded random media[5,6] with applications to both active and passive remote sensing (see Part II, Sec. IX-B). In applied optics we studied wave behavior at boundary surfaces by a dispersion analysis.[7] We calculated photon production in anisotropic crystals by Čerenkov radiation.[8,9] We also developed the modal theory for electro-optical modulators in integrated optical systems (see Part II, Sec. IX-A).

We shall consider all kinds of dipole configurations in the study of subsurface probing and communication, and evaluate their performance. In remote sensing we shall develop composite models that incorporate surface roughness, nonuniform temperature distributions, and subsurface scattering effects. The study of wave optics will be directed toward the application of electro-optical and nonisotropic material in integrated optical components. Spatially periodic media will be investigated in detail. Our approach to problems is to aim at obtaining analytical expressions or computational procedures that are simple and can be readily calculated on a computer. The computer system MACSYMA has been used to assist in symbolic manipulations, as well as seminumerical computations.

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6. L. Tsang and J. A. Kong, "Microwave Remote Sensing of a Two-Layer Random Medium" (to appear in IEEE Trans. on Antennas and Propagation).

7. J. A. Kong, "Dispersion Analysis of Reflection and Transmission by a Plane Boundary — A Graphical Approach," Am. J. Phys. 43, 73-76 (1975).

8. J. A. Kong, "Čerenkov Radiation in Anisotropic Crystals," 1975 Annual Meeting, Optical Society of America, Boston, Massachusetts, October 21-24, 1975.

9. J. A. Kong, "Quantum Theory of Čerenkov Radiation in Uniaxial Crystals" (to appear in Phys. Rev. D).

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2. PASSIVE REMOTE SENSING OF THE EARTH WITH MICROWAVES

Jin Au Kong, David H. Staelin

In passive remote sensing of the Earth, we direct our attention toward microwave emission properties of natural surfaces, taking into account the various surface and sub-surface features. We calculated brightness temperatures of half-space random media with nonuniform temperature profiles[1] and investigated the problem of microwave thermal emission from stratified media.[2] Employing a two-layer random media model, we studied scattering and emission properties[3-4] (see also Part II, Sec. IX-B). A controlled model tank experiment has been carried out to simulate profiles of the nonuniform temperature distributions of soil moisture (see Part II, Sec. IX-C). We are now applying calculated theoretical results to the interpretation of the experimental data and we expect to develop an inversion scheme. Spacecraft and satellite data, when available, will also be interpreted with the aid of the theoretical models. Computer-simulated experiments are now under way. Our objective is to develop theoretical models that are close to reality for various circumstances, and still are mathematically tractable and simple to use.

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2. L. Tsang, E. Njoku, and J. A. Kong, "Microwave Thermal Emission from a Stratified Medium with Nonuniform Temperature Profile" (to appear in J. Appl. Phys.).

3. L. Tsang and J. A. Kong, "Microwave Remote Sensing of Bounded Random Media," USNC/URSI-IEEE 1975 Annual Meeting, Boulder, Colorado, October 20-23, 1975.

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IX. ELECTRODYNAMICS OF MEDIA

A. MODAL THEORY FOR ELECTRO-OPTICAL GRATING MODULATORS

Ray S. Chu, Jin Au Kong, Donald L. Lee

A popular configuration for electro-optical modulators used in integrated optics has periodic electrodes placed on the surface of a thin-film waveguide made of electro-optical material (Fig. IX-1). When voltages are applied to the electrodes, the thin-film waveguide becomes spatially modulated with periodicity equal to that of the electrodes. Guided light is diffracted after passing through the modulated region. Experiments have been performed with light normally incident upon the periodic medium,[1,2] as well as incident at the Bragg angle.[3] The measured results were interpreted with well-known theories applicable either in the Raman-Nath regime[4] or in the Phariseau limit.[5,6] When both limits cannot be applied, Klein and Cook[6] devised a numerical solution in which they approximated differential equations by difference equations. The check of the experimental results with these theories has not been satisfactory, especially when the modulation voltage is large.
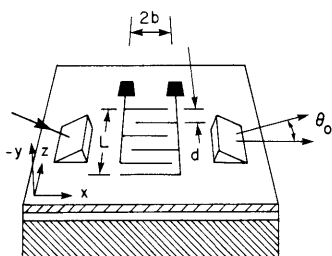

Fig. IX-1.

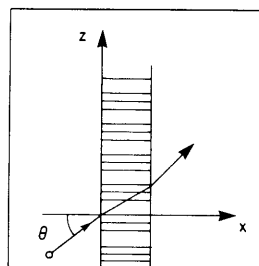Electro-optical grating modulator experiment.

Fig. IX-2.

Diffraction by a slab periodic medium.

These theories suffer two drawbacks. First, they neglect reflections at the two boundaries upon entering and exiting the modulated region. The boundary effects become more important as the modulating voltage is increased. Second, they discard terms involving second derivatives. In this report we propose modifications in theory to remedy these defects. We extend the modal theory[7,8] which provides the most rigorous approach to the problem of diffraction by a periodically modulated medium.

We assume a model of a slab periodic medium of thickness 2b (Fig. IX-2) whose permittivity takes the form:

$$\epsilon(z) = \epsilon_r \left[1 - M \cos \frac{2\pi z}{d} \right],$$

where d is the periodicity of the modulation, M is the index of modulation, and ϵ_r is the relative permittivity in the absence of modulation, i.e., M = 0. Under the assumption that $M\lambda/d \ll 1$, the transverse modal function $\phi_\nu(z)$ for both TE and TM waves in the modulating region satisfies the Mathieu differential equation.[7]

$$\frac{d^2 \phi_\nu(z)}{dz^2} + (\pi/d)^2 \left(p_\nu - 2q \cos \frac{2\pi}{d} z \right) \phi_\nu(z) = 0,$$

with $p_\nu = (d/\pi)^2 \left(\epsilon_r k^2 - \xi_\nu^2 \right)$ and $q = 2M\epsilon_r(d/\lambda)^2$, where the x-component wave numbers ξ_ν are found from p_ν which in turn is determined from the characteristic equations for Mathieu functions. The boundary conditions at x = ±b are then matched with assumed Floquet wave solutions outside the modulated regions. The problem is then reduced to the solution of properly truncated matrix equations.

In Fig. IX-3 we compare our results with those obtained by Klein and Cook
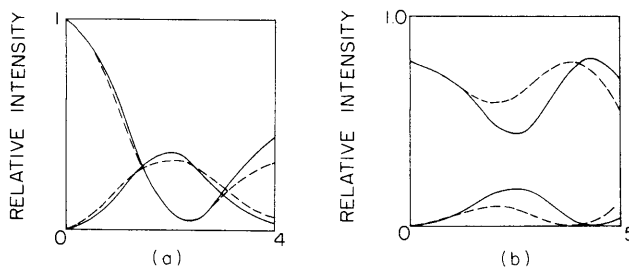

Fig. IX-3. Comparison of our results with those of Klein and Cook (dashed curves): (a) Q = 2, (b) Q = 7.
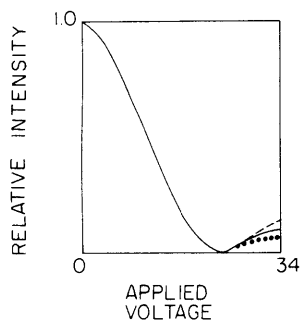

Fig. IX-4.

Comparison of our results with Raman-Nath theory (dashed curve) and with experimental data (dotted curve) at normal incidence.
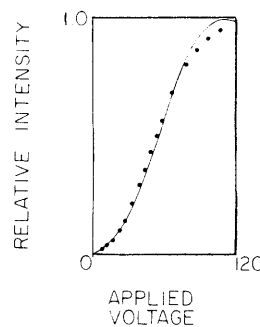
Fig. IX-5.

Comparison of our results with experimental data (dotted curve) at Bragg angle incidence.

with $Q = 2$ and $Q = 7$, where $Q = 4\pi\lambda b/d^2 \sqrt{\epsilon_r}$. In Fig. IX-4 we compare our results
with the Raman-Nath theory and with experimental results[1] at normal incidence cor-
responding to $Q = 0.14$. In Fig. IX-5 we compare our results with data at the first Bragg
angle incidence.[3] Note in particular that the maximum value is not unity, in contrast
with Phariseau's theory which predicts a sinusoidal distribution with unit amplitude.

We must note that in the actual situation the periodic medium is not truly homoge-
neous in the y direction; instead, the dielectric constant decreases as y increases and
even curves around at large distances away from the electrodes. A boundary-value prob-
lem approach to a model of this kind is extremely difficult. Thus we propose an integral
equation approach that includes changing in the dielectric constant inside the integral.
The advantage of the integral formulation is that, since perturbation technique is applied,
the effect of inhomogeneity in the permittivity can be treated.

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Leung Tsang, Jin Au Kong

In microwave remote sensing of the Earth or other planets, theoretical models are essential for interpretation of the collected data. The effects of absorption and scattering have long been recognized as dominant factors in both active and passive sensing. Using the model of a half-space random medium with laminar structure, Gurvich, Kalinin, and Matveyev[1] investigated thermal radio emission in Antarctic regions. The problem of scattering by a medium with a small random fluctuating part in permittivity was studied by Stogryn.[2] England[3] examined emission darkening caused by isotropic point scatterers in a half-space medium. The problem of microwave thermal emission from a half-space random medium has been solved by using a radiative transfer approach.[4,5] Random media can usually be treated by radiative transfer methods that deal with energy fluxes and by the renormalization method that deals directly with field quantities. The renormalization method gives rise to the Dyson equation for the mean field and the Bethe-Salpeter equation for the covariance of the field.[6-8] In solving these equations, the bilocal approximation is usually applied to the Dyson equation, which is then solved by mathematical techniques such as the Fourier transform method. A ladder approximation is made on the Bethe-Salpeter equation, which is solved by the method of iteration.[6,7] In the case of multiple-wave scattering the method of iteration involves solving many integrals and leads to complicated results after one or two iterations. Under the assumptions of far-field interaction and incoherence among waves in different directions, radiative transfer equations have been derived from Bethe-Salpeter equations to study multiple scattering.[9-14]
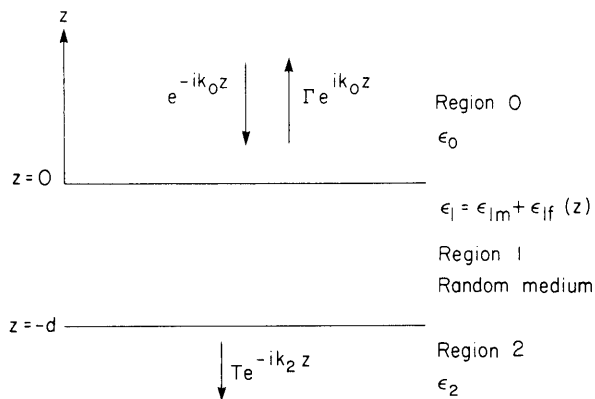

Fig. IX-6. Geometrical configuration of the problem.

In this report we investigate the problem of scattering and emission of microwaves by a slab random medium with a laminar structure bounded by a different dielectric on each side. We employ the nonlinear,[15,16] rather than the more popular bilocal, approximation to the Dyson equation. A two-variable expansion technique[17,18] is applied to obtain the zeroth-order mean Green's function, which then gives rise to a set of modified radiative transfer (MRT) equations from the Bethe-Salpeter equation. They are modified because correlation effects among waves in different directions are included.

Consider a two-layer random medium with boundaries at $z = 0$ and $z = -d$ (Fig. IX-6) with a permittivity

$$\epsilon_1 = \epsilon_{1m} + \epsilon_{1f}(z) \tag{1}$$

$$\langle \epsilon_{1f}(z) \rangle = 0. \tag{2}$$

The emissivity is calculated to be $e = 1 - r$ with

$$r = \frac{\left| R_{01} + R_{12}\, e^{i(\eta_1+\eta_2)d} \right|^2}{|D|^2} + \frac{t_{01}^2}{|D|^2}$$

$$\times \left\{ \frac{(1-f_2 r_{12})\left(f_2 - r_{12}\, e^{-(\kappa_1+\kappa_2)d}\right) - (f_2-r_{12})\left(1 - f_2 r_{12}\, e^{-(\kappa_1+\kappa_2)d}\right) e^{-2\alpha d}}{(1-f_2 r_{01})(1-f_2 r_{12}) - (f_2-r_{01})(f_2-r)\, e^{-2\alpha d}} \right\}, \tag{3}$$

where

$$\kappa_1 = 2\eta_1'' = \kappa_a + \frac{\kappa_s}{2} + \frac{\kappa_s}{2}\, \frac{D_1}{|D|^2}\, \frac{\left(3 + 8k_{1m}^2 \ell^2\right)}{\left(1 + 4k_{1m}^2 \ell^2\right)}$$

$$\kappa_2 = 2\eta_2'' = \kappa_a - \frac{\kappa_s}{2} + \frac{\kappa_s}{2}\, \frac{D_1}{|D|^2}\, \frac{\left(3 + 8k_{1m}^2 \ell^2\right)}{\left(1 + 4k_{1m}^2 \ell^2\right)}$$

$$L_1 = \frac{1}{i2k_{1m}D}$$

$$D = 1 + R_{01}R_{12}\, e^{i(\eta_1+\eta_2)d}$$

$$D_1 = 1 - r_{01}r_{12}\, e^{-(\kappa_1+\kappa_2)d}$$

$$f_2 = \frac{a - \kappa_a}{a + \kappa_a}$$

$$a = \kappa_e (1-\tilde{\omega})^{1/2} \left(1 - \frac{\tilde{\omega}}{2} p_f + \frac{\tilde{\omega}}{2} p_b\right)^{1/2}$$

$$\eta_1 - \eta_2 = i\frac{\kappa_s}{2}$$

$$\eta_1 + \eta_2 = 2k_{1m} - k_{1m}^3 \delta \ell L_1 \left\{ \frac{3 - i4k_{1m}\ell}{1 - i2k_{1m}\ell} + R_{10}R_{12}\left(\frac{3 + i4k_{1m}\ell}{1 + i2k_{1m}\ell}\right) e^{i(\eta_1+\eta_2)d} \right\}$$

$$k_s = \frac{\kappa_s\left(1 + 2k_{1m}^2\ell^2\right)}{1 + 4k_{1m}^2\ell^2}$$

$$K_s = \frac{D_1}{|D|^2} k_s$$

$$\kappa_e = \kappa_a + K_s,$$

with $\tilde{\omega} = K_s/\kappa_e$ the scattering albedo, R_{01} and R_{12} the Fresnel reflection coefficients, $r_{01} = |R_{01}|^2$ and $r_{12} = |R_{12}|^2$ the reflectivities, and $t_{01} = 1 - r_{01}$.

We now consider the following special cases:

Case 1. A half-space random medium. We let $d \to \infty$, and (3) becomes

$$r = r_{01} + \frac{t_{01}^2 f_2}{1 - r_{01}f_2} \tag{4}$$

which is identical to the emissivity derived from a phenomenological radiative transfer approach.[5]

Case 2. The second layer a perfect conductor. We have $R_{12} = -1$ and $r_{12} = 1$. Equation 3 becomes

$$r = \left| \frac{R_{01} - e^{i(\eta_1+\eta_2)d}}{1 - R_{01}e^{i(\eta_1+\eta_2)d}} \right|^2 + \frac{t_{01}^2 \left\{ f_2 - e^{-(\kappa_1+\kappa_2)d} + \left(1 - f_2 e^{-(\kappa_1+\kappa_2)d}\right) e^{-2ad} \right\}}{\left\{1 - f_2 r_{01} + (f_2 - r_{01}) e^{-2ad}\right\} \left|1 - R_{01}e^{i(\eta_1+\eta_2)d}\right|^2}. \tag{5}$$
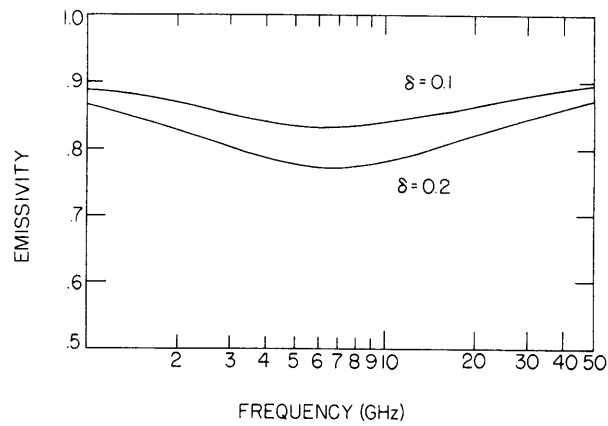
Fig. IX-7.   Emissivity of a half-space medium with $\epsilon_1' = 3.2\,\epsilon_0$, $\epsilon_1'' = 0.16\,\epsilon_0$, $\ell = 2$ mm.
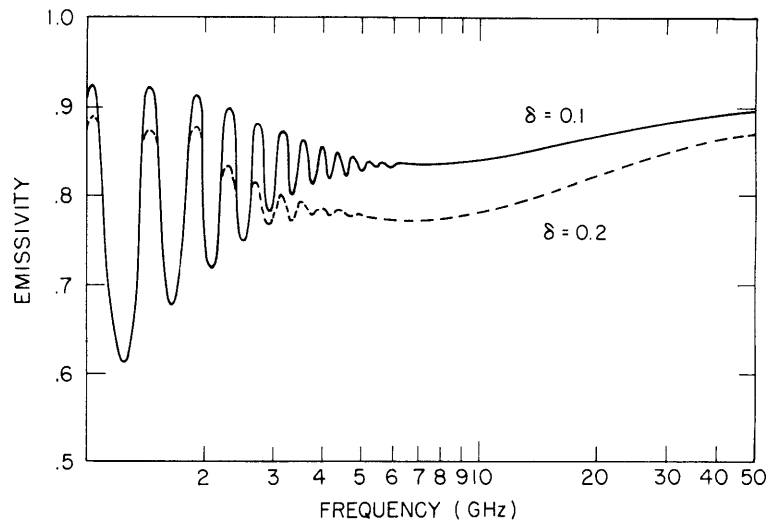
Fig. IX-8.   Emissivity of a two-layer random medium with $\epsilon_1' = 3.2\,\epsilon_0$, $\epsilon_1'' = 0.16\,\epsilon_0$, $\ell = 2$ mm, $\epsilon_2 = 81\,\epsilon_0$, and d = 20 cm.

It is interesting to note that as $\tilde{\omega} \to 1$, we have $a \to 0$, $f_2 \to 1$, and from (5) $r \to 1$. Thus all the incident power is reflected. In Figs. IX-7 and IX-8 we illustrate the emissivity for a half-space random medium and a slab random medium with $\delta = 0.1$ and 0.2. We see that in the half-space case the null in emissivity occurs at $4k_{1m}^2 \ell^2 = 1$ because of scattering. In the slab case scattering dampens the interference pattern and decreases emissivity in general. The existence of the interference patterns depends on the location of the subsurface and the extinction loss of the random medium.

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C. OBSERVATIONS OF MICROWAVE THERMAL EMISSION FROM CONTROLLED TARGET AREAS

California Institute of Technology (Contract 953524)

Eni G. Njoku

An approach for calculating the microwave thermal emission from stratified media with nonuniform temperature profiles has recently been described.[1] To provide experimental validity for this approach, and to collect reliable data for a proposed inversion scheme, a series of ground-based radiometric observations were made of microwave emission from a layer of sand above a reflector plate. The sand was contained in a wooden sandbox with its base covered by reflective aluminum sheets. The parameters that were varied during the observations were layer depth, temperature profile of the sand, and the moisture profile. Measurements of vertically polarized emission were made at frequencies 0.675 GHz, 10.69 GHz, and 31.4 GHz, at an observation angle of 30°.

Figure IX-9 shows the emissivities deduced from experimentally observed brightness temperatures, as the dry sand layer depth was increased in steps of 1 cm or greater $[e_N(d) = e(d)/e(\infty)]$. Theoretical curves are superimposed on the data plots, the indicated values of complex dielectric constant being used. The interference fringes were too closely spaced to be resolved by the 1 cm depth increments at 10.69 GHz and 31.4 GHz; hence, only the fringe envelopes are shown in these cases.

Temperature profiles were introduced in a full 3 ft depth of sand by heating the sandbox underneath, and from the natural heating of the sun in a few inches at the top. During a period of several days physical temperatures of the sand were recorded at various depths as the sand was heated underneath. The results are shown in Fig. IX-10. Radiometric brightness temperatures that were measured during this heating cycle are shown in Fig. IX-11.

Moisture profiles were produced by sprinkling the surface of the sand with water, and allowing the moisture to seep downward by gravity. Moisture content as a function of depth was obtained by extracting core samples, dividing them into 2-inch intervals, and weighing the samples before and after they had been dried in an oven at 120°C. Twelve brightness temperature measurements were made, each with corresponding moisture and temperature profiles measured as we have just described. These are shown in Fig. IX-12. Illustrative moisture and temperature profiles for experiments 1, 5, and 8 are shown in Fig. IX-13.

The data measurements presented here show qualitative agreement with theoretical expectations. The properties of the emitting medium (depth, temperature, dielectric constant), are well known; hence, the use of the data for comparison with theory and
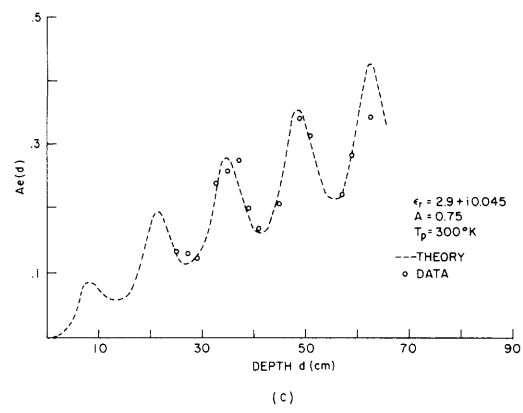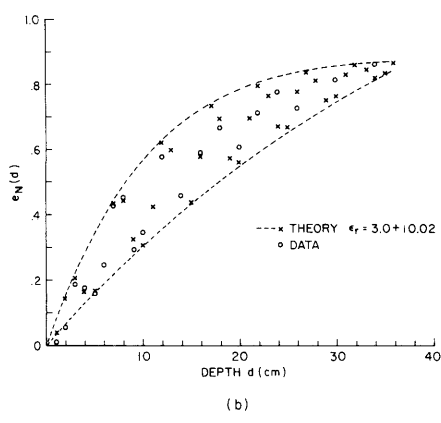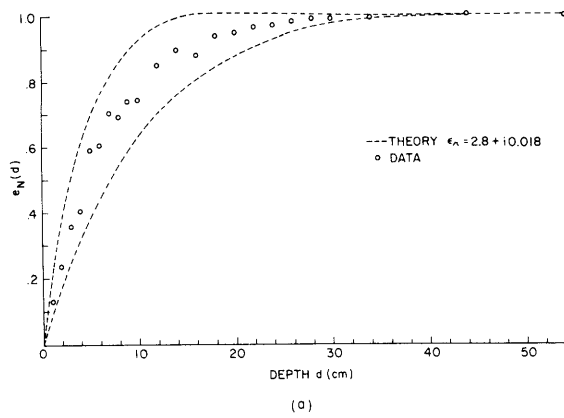
Fig. IX-9. Emissivity as a function of depth at three frequencies:
(a) 31. 4 GHz, (b) 10. 69 GHz, (c) 0. 675 GHz.
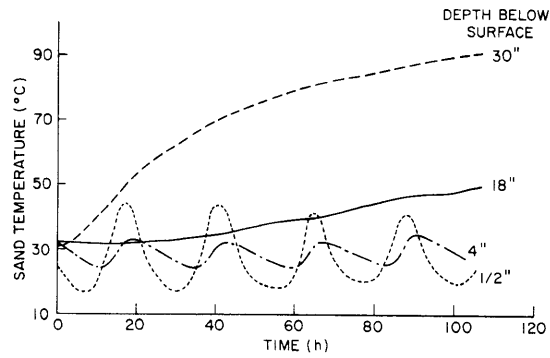
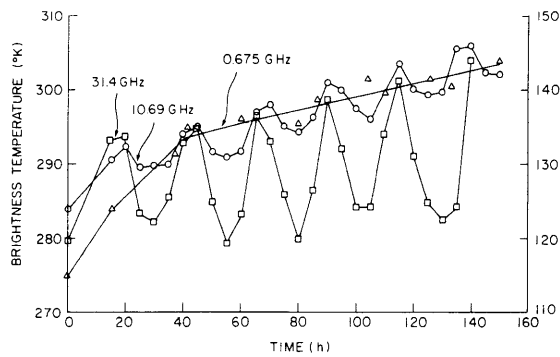Fig. IX-10. Sand temperature vs time.

Fig. IX-11.

Observed brightness temperature vs time.
Left scale: frequencies 31.4 GHz, 10.69
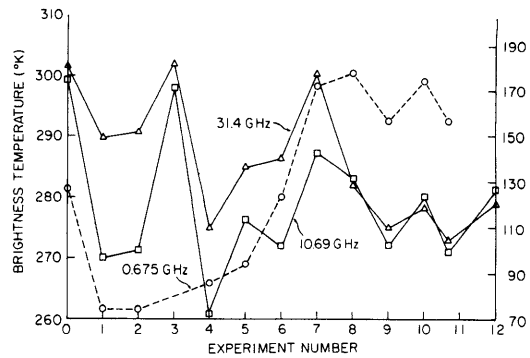GHz. Right scale: frequency 0.675 GHz.

Fig. IX-12.

Moisture profile experiments. Left
scale: frequencies 31.4 GHz, 10.69 GHz.
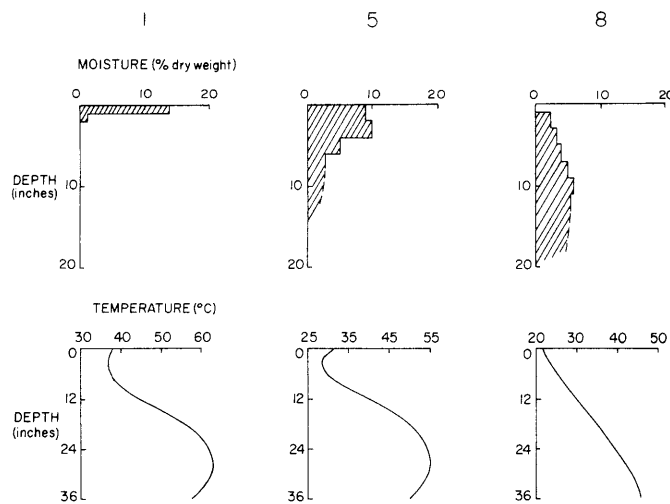Right scale: frequency 0.675 GHz.

Fig. IX-13. Moisture and temperature profiles for
experiments 1, 5, and 8.

for formulating inversion procedures was facilitated. Work is proceeding in this direction.

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