

IV. QUANTUM ELECTRONICS

A. Nonlinear Phenomena

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1. SHORT LASER PULSES: PASSIVE MODE-LOCKING SOLUTION

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We have developed a theory of forced mode locking in the frequency domain.¹ With some modifications of this theory, it is possible to obtain what we believe is the first closed-form solution for saturable absorber mode locking.

As in forced mode locking, we treat mode locking by a saturable absorber as a form of injection locking. This means that the equivalent voltage of each cavity mode (representing the E field at a standing-wave maximum) is to be balanced by the injection-locking voltage produced through interaction of the equivalent cavity current with the saturable absorber. We replace the mode spectrum with a continuum; i. e., if the cavity mode spacing is $\Delta\omega$, and n is the mode number counting from the laser medium line center where $n = 0$, then $\omega \equiv \lim_{n \rightarrow \infty, \Delta\omega \rightarrow 0} n\Delta\omega$ is the (continuous) index of the cavity mode. Then the normalized reactance of the cavity mode, x_c , is also a function of ω because modes at a different "distance" (in frequency space) from the laser line center will oscillate in general at different detunings from cavity resonance. The normalized negative resistance of the laser medium is taken as $r \left[1 - \omega^2 / \omega_M^2 \right]$, where ω_M is a measure of the medium linewidth, and the Lorentzian denominator has been expanded. By the Kramers-Krönig relations, a reactance has to be associated with the laser medium, $-jr \frac{\omega}{\omega_M}$. The parameter r is assumed to depend on the power $r = r_0 / (1 + P/P_s)$, since it pertains to a homogeneously broadened laser line. All impedances are normalized to the equivalent (empty) cavity-mode resistance at resonance. Then the fundamental equation is

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$$S * I = \left[1 + jx_c(\omega) - r \left(1 - \frac{\omega^2}{\omega_M^2} - j \frac{\omega}{\omega_M} \right) \right] I. \quad (1)$$

On the left-hand side we have written the equivalent injection voltage produced by the saturable absorber driven by the equivalent cavity current I . We have indicated this voltage symbolically as a convolution because in some cases of interest this is what it is. The analysis presented in the sequel is rigorously applicable to a traveling-wave ring laser. $|I(t)|^2$ may be normalized to give the power in a single traveling wave. When adapted to a standing-wave cavity laser with the thin saturable absorber located near one of the mirrors the absorber is exposed to simultaneous excitation of both countertraveling waves, and hence is exposed to twice the power that there would be in a single traveling wave. The rate equation for the population difference n between the lower and upper levels of the saturable absorber (the standing-wave laser requires another factor of 2 in front of $|I(t)|^2$) is

$$\frac{\partial n}{\partial t} = -\frac{n - n_e}{T_1} - 2\sigma \frac{|I(t)|^2}{\hbar\omega A} n. \quad (2)$$

Here T_1 is the relaxation time of the absorbing medium, n_e is the equilibrium population difference, σ is the optical cross section, and A is the cross section of the laser mode. If the thickness of the absorbing medium is θ and the length of the cavity is ℓ , then we may define a Q associated with the absorbing medium.

$$\frac{\omega_o}{Q_a} = \frac{2\sigma n_e \theta}{\hbar\omega_o \ell}, \quad (3)$$

where ω_o , the resonance frequency of a cavity mode, is set equal to the laser line center frequency. This Q is a function of intensity and is obtained from Eqs. 2 and 3 as a solution of the differential equation. If the relaxation time T_1 of the absorber is fast compared with the rate of change of the intensity, we may assume that the population difference is an instantaneous function of intensity, and hence we obtain an approximate expression for the instantaneous Q of the saturable absorber.

$$\begin{aligned} \frac{1}{Q_a} &= \frac{c\theta}{\omega_o \ell} \frac{2\sigma n_e}{\hbar\omega_o} \left[1 - \frac{|I(t)|^2}{P_a} \right] \\ &\equiv \frac{1}{Q_a^o} \left[1 - \frac{|I(t)|^2}{P_a} \right], \end{aligned} \quad (4)$$

where we have defined the saturation power for the absorber as

$$P_a = \frac{\hbar\omega_o A}{2\sigma T_1}. \quad (5)$$

The time-independent part of the Q of the absorber may be incorporated in the cavity loss. In the time domain the time-dependent part contributes an equivalent injection voltage on the left-hand side of (1) which is the product of the absolute square of the equivalent current times the current. In the frequency domain the product is replaced by a convolution; hence, the choice of notation in Eq. 1.

We now turn to an adaptation of Eq. 1 in order to make it soluble in the case of a fast saturable absorber. The right-hand side of Eq. 1 contains a complex coefficient. It is clear that the imaginary parts of the coefficient may be balanced by assuming that

$$x_c(\omega) = - \frac{r_o}{1 + \frac{P}{P_s}} \frac{\omega}{\omega_M}. \quad (6)$$

This means that the cavity modes farther and farther away from the laser medium line center are more and more detuned from cavity resonance. Once we assume that (6) holds, we are left with an equation possessing a real coefficient on the right-hand side and a convolution operator on the left-hand side. Since the convolution operator has a particularly simple form in the time domain, we transform the right-hand side of the remainder of (1) into the time domain. We obtain the differential equation

$$\frac{Q}{Q_a^o} \frac{|I(t)|^2}{P_a} I(t) = \left[1 - r \left(1 + \frac{1}{\omega_M^2} \frac{d^2}{dt^2} \right) \right] I. \quad (7)$$

We assume a solution of the form

$$I(t) = \frac{A_o}{\cosh\left(\frac{t}{\tau}\right)}. \quad (8)$$

Note that A_o^2 is related to the power. Indeed if we assume that the repetition rate of the pulses is T , we have

$$\text{Power} = \frac{1}{T} \int_{-\infty}^{\infty} \frac{A_o^2}{\cosh^2\left(\frac{t}{\tau}\right)} dt = \frac{2\tau}{T} A_o^2. \quad (9)$$

Introducing (8) in (7) and balancing the terms in the hyperbolic secant and in the cube

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of the hyperbolic secant, we obtain two equations:

$$\kappa \left(\frac{P}{P_s} \right) = \frac{r}{\omega_M \tau}, \quad (10)$$

where we have used (9) in defining the coefficient

$$\kappa \equiv \frac{Q}{4Q_a} \frac{P_s}{P_a} \omega_M T,$$

and

$$1 - r = \frac{r}{2} \frac{1}{\omega_M \tau}. \quad (11)$$

Equations 10 and 11 supplemented by the dependence of the negative resistance r upon power P ,

$$r = \frac{r_o}{1 + \frac{P}{P_s}}, \quad (12)$$

yield three equations for the three unknowns P , r , and the pulsewidth T . From Eq. 11, we note that r is less than unity. This means that in the presence of a saturable absorber the laser is below threshold with respect to the linear loss (loss in absence of laser power). The laser oscillates because the bleaching of the absorber reduces the loss below the linear loss. The equivalent injection-locking voltages have to be equal to the difference between the voltages produced by the positive and negative impedances. Since this combination of impedances is below threshold over the entire mode spectrum, the injection voltages are all of the same sign. This finding has to be contrasted with forced mode locking.¹ When the center portion of the spectrum is above threshold, the wings are below threshold and the equivalent injection-locking voltages change sign with the progression from the center of the line to the edge.

Equation 11 may be used to eliminate r , and from the two remaining equations (12) and (10) we obtain two equations for P/P_s and $\omega_M \tau$:

$$\frac{1}{r_o} \frac{P}{P_s} = \frac{\frac{1}{r_o \kappa}}{\omega_M \tau + \frac{1}{\omega_M \tau}} = \left(1 - \frac{1}{r_o} \right) + \frac{1}{\omega_M \tau}. \quad (13)$$

These equations may be solved graphically, as shown in Fig. IV-1. It is clear that under certain conditions no mode-locking solutions are found. Indeed that happens for a fixed

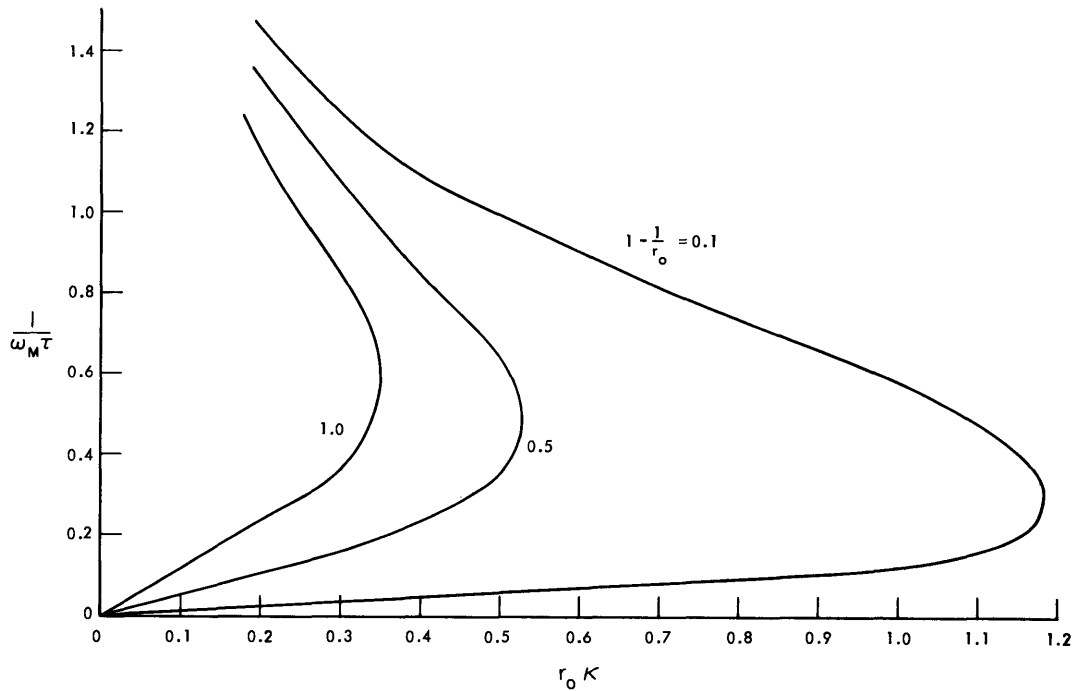


Fig. IV-1. Inverse pulse length as a function of absorber Q , with excess gain as a parameter.

excess gain parameter $1 - 1/r_0$ when κ becomes too large; that is, the Q of the absorber becomes too small or its saturation power P_a becomes too small. The saturable absorber is too "overpowering," the pulse wants to become too high and too short, and the laser medium cannot adjust itself to it. In general, two solutions are obtained.

Since we have found two solutions in general corresponding to different pulsewidths and intensities, we would expect at least one of these solutions to be unstable. This matter will be investigated further.

References

1. H. A. Haus, "A Theory of Forced Mode Locking," Memorandum No. 3, Research Laboratory of Electronics Quantum Electronics Group, January 1974 (unpublished).

