COMMUNICATION SCIENCES

AND

ENGINEERING

XIX. STATISTICAL COMMUNICATION THEORY*

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A. WORK COMPLETED

1. STATIC AND DYNAMIC ANALYSIS OF A FEEDBACK-CONTROLLED TWO-STATE MODULATION SYSTEM

This study has been completed by J. E. Schindall. In May 1967, he submitted the results to the Department of Electrical Engineering, M. I. T., as a thesis in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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2. A STUDY OF NOISE IN A TWO-STATE AMPLIFIER

This study has been completed by L. R. Poulo. In May 1967, he submitted the results to the Department of Electrical Engineering, M. I. T., as a thesis in partial fulfillment of the requirements for the degree of Master of Science.

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3. A NONLINEAR RECORD SCRATCH FILTER

This study has been completed by C. M. Cooper. In May 1967, he submitted the results to the Department of Electrical Engineering, M. I. T., as a thesis in partial fulfillment of the requirements for the degree of Bachelor of Science.

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4. MEASUREMENT OF SYMMETRY OF AMPLITUDE PROBABILITY DISTRIBUTION OF RANDOM NOISE

This study has been completed by B. A. Twickler. In May 1967, he submitted the

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results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Bachelor of Science.

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5. THE IMPULSE RESPONSE OF THE PINNA

This study has been completed by A. E. Roland. In May 1967, he submitted the results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Master of Science.

J. D. Bruce

6. AN INVESTIGATION OF "ELECTROPHONIC HEARING"

This study has been completed by F. P. Tuhy, Jr. In May 1967, he submitted the results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Master of Science.

J. D. Bruce

7. THE PINNAE'S ROLE IN MONAURAL LOCALIZATION

This study has been completed by J. L. Walker. In May 1967, he submitted the results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Master of Science.

J. D. Bruce

8. HARD-LIMITING OF SINUSOIDS IN GAUSSIAN NOISE

This study has been completed by B. H. Ashton. In May 1967, he submitted the results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Bachelor of Science.

J. D. Bruce

9. SINGLE-BIT SAMPLING THROUGH TWO-STATE MODULATION

This study has been completed by K. E. Schoman, Jr. In May 1967, he submitted the results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Bachelor of Science.

J. D. Bruce

10. THE DELAY OF ANALOG SIGNALS BY MONOSTABLE SWITCHING CIRCUITS

This study has been completed by A. D. Pitegoff. In May 1967, he submitted the results to the Department of Electrical Engineering, M.I.T., as a thesis in partial fulfillment of the requirements for the degree of Bachelor of Science.

D. E. Nelsen

B. INVESTIGATION OF SWITCHING JITTER IN PHYSICAL ELECTRONIC SWITCHES

1. Introduction

This report summarizes the work of this group on the investigation of switching jitter in physical switching circuits. The basic problem is to learn the mechanism whereby switching-time randomness, or jitter, arises in switches such as the tunnel diode, Schmidt trigger, neon bulb or other regenerative circuits. We began the investigation with a detailed study of jitter in the tunnel diode switch. A model was obtained for that circuit which predicts the jitter statistics over a wide range of operating conditions.[1,2] The methods developed for modeling this process are now being applied to the investigation of the jitter mechanism in other kinds of switching circuits. Measurements of jitter statistics were also made for a neon bulb switch, but a model describing that jitter process has not yet been obtained.[3] We are now investigating the jitter process in regenerative transistor circuits. The over-all progress and some of the more significant results of these investigations will be briefly described in this report.

2. Basic Problem

The problem can be more completely described as follows. If a ramp is applied to the input of a physical switching circuit such as a Schmidt trigger or tunnel diode switch, switching will occur approximately when the ramp crosses some threshold that is characteristic of the circuit. If this experiment is repeated many times, the switching instants will be found to be randomly distributed. This time randomness, or jitter, is frequently a limitation in systems involving switching circuits, such as analog-to-digital converters, switching-type modulators, and trigger circuits. The object of our investigation is to determine the mechanism that causes this switching-time jitter. Thus we hope to be able to predict the jitter statistics for given circuits and to minimize jitter by optimal circuit design.

3. Model for Tunnel Diode Switch

We approached the over-all problem by investigating in detail the tunnel diode switching circuit. This particular switch was chosen because of its stability, wide bandwidth, and relatively well understood circuit model. A model was obtained that predicts the statistics of the jitter over a wide range of operating conditions. This work has been described in detail elsewhere.[1,2]

Several aspects of this work are significant with respect to the general problem of describing jitter in arbitrary switching systems. First, a successful approach to the modeling of jitter, if only in one kind of switch, was found. By using approaches similar to this, we are now trying to obtain models for jitter in other kinds of switching circuits. Second, a "switching equation" that describes the relation between noise in the circuit and jitter statistics was obtained. This equation has the form

$$\frac{dx}{dt} + f(x) = t + n(t;N_o), \tag{1}$$

where $f(x) = -x^2$, $n(t;N_o)$ is white noise with spectral height N_o, x is the tunnel diode voltage, and t is time. The solution x(t) has the property that it goes to infinity at some finite time T_s which can be interpreted to be the switching time. With noise present ($N_o > O$), T_s is a random variable with statistics dependent upon N_o. This equation was solved for the statistics of T_s with the help of a computer.[1,2] Equations similar to this have been obtained for transistor circuits that we are now investigating. We hope to be able to define the general form of switching equations that result for arbitrary switching systems. These equations will be discussed in more detail later.

Another significant result is that this equation predicts a power-law relationship between σ_T (the standard deviation of T_s) and ramp slope, α, of the form

$$\sigma_T \sim \frac{1}{\alpha^p} \quad \text{where } p = \frac{5}{6}.$$

Measured values of p were close to this theoretical value of 5/6.

This power-law behavior is interesting because it has also been measured for transistor switches that are under investigation. In these measurements, however, p took on values between 75 and 1.1, depending on the circuit parameters. These measurements were typically made for between two and four decades of α variation.

4. Investigation of Jitter in a Neon Bulb Switch

James C. Stafford, in his Master's thesis investigation,[3] attempted to obtain a model for jitter in a neon bulb switch. The jitter standard deviation and distribution were measured as a function of several circuit parameters. We have not been able to obtain a model for the jitter from these data, or from other models of the breakdown process.

It was not clear that he isolated the breakdown process experimentally, since it is highly dependent upon gas temperature. Thus for measurements of jitter statistics with ramp slope as a parameter, the rate at which the experiment was performed, and the voltages at which the ramp (of preset slope) was started and finished affected the gas temperature, which, in turn, strongly influenced the jitter statistics. By placing a radiation source next to the bulb, the effects of this variation were lessened, because of a large increase in the number of photoelectrons. The switching process was sufficiently complicated to warrant instead investigation of other, simpler processes as a step toward understanding the general jitter problem.

5. Jitter in Regenerative Transistor Circuits

We are, at present, investigating jitter in regenerative transistor switching circuits. These circuits are essentially amplifiers with positive feedback around them. Common two-transistor switches are the Schmidt trigger and the flip-flop.

One approach, a theoretical one, has been to examine the kinds of switching equations that result for various two-transistor combinations. The other approach, which is being carried out in conjunction with the first, is to determine experimentally the dependence of the jitter standard deviation upon ramp slope and circuit parameters. By combining these two approaches, we hope to be able to determine the circuit parameters that are most important, and hence the kinds of approximations that will be most useful in analyzing the switching equations.

a. Switching Equations for Two-Transistor Switches

We have been investigating the types of switching equations that result for two-transistor regenerative switches. These equations have been obtained from the equilibrium equations that hold when the circuit is at the threshold on the verge of switching. The task of reducing the set of equilibrium equations can often be simplified, since one transistor is usually in the active region when the other is at the threshold between cutoff or saturation and the active region. The active transistor can then be approximated by an instantaneous, constant-gain, linear amplifier. The transistors were modeled by the complete (including the effects of space-charge layer capacitance) charge control equations.[4] These equations describe the large-signal dynamic transistor behavior. They also include the nonlinear effects that cause the threshold. Noise was accounted for by placing appropriate shot- and thermal-noise sources in the circuit.

In its most general form, the switching equation will be a fourth-order nonlinear differential equation. An approximation can be made, however, which reduces the equation to a first-order equation. If the fourth-order equation is linearized in the vicinity of the threshold, it will be observed that the system function corresponding to the linearized differential equation has four poles. One of these poles is at a very low frequency, on the real axis of the x-plane, crossing from the left-half plane to the right as regeneration increases. The other three poles are relatively stationary at high frequencies. Thus one pole dominates the system behavior near the threshold. This means that the system can be approximated by a first-order equation in this region. We are not sure how valid this approximation is for arbitrary two-transistor circuits, but it is certainly possible to design circuits for which it is valid.

By making the approximation above, equations of the form

$$\frac{dx}{dt} + f(x) = t + n(t) \tag{2}$$

result. In this case, x is one of the charge storage components, f(x) is a function that is convex downward (for the tunnel diode, $f(x) = -kx^2$), n(t) is noise resulting from the presence of shot noise and thermal noise in the circuit, and t is time.

This equation is very similar to Eq. 1 obtained for the tunnel diode, the only difference being the form of f(x). Instead of being a simple parabola, f(x) is considerably more complicated, depending on several circuit parameters. Solution of the equation by computer was practical for the simple, parabolic f(x) for the tunnel diode, but for this case we are reluctant to begin numerical analyses until we learn more about the relationship between the shape of f(x) and the actual jitter statistics by means of experimental measurements.

6. Applications in Which Switching-time Jitter Is a Limitation

Two students have just completed thesis projects concerning applications in which switching jitter was a limitation.

Louis R. Poulo, in his Master's thesis investigation,[5] studied the noise behavior of a type of two-state modulator. This modulator converts a continuous, analog input signal to a two-state form in which all signal information is contained in the positions of the transitions between the two states. Noise is introduced in the modulator by jitter in the positions of these transitions. Jitter arises because the position of each transition is determined by a switching circuit that detects the intersection of a ramp with a threshold. Poulo investigated, theoretically and experimentally, the tradeoffs between output noise and modulator parameters for a fixed switching circuit.

The other investigation, performed by Alexander D. Pitegoff as his Bachelor's thesis,[6] was concerned with a means for digitally delaying a two-state signal by means of a cascaded string of monostable multivibrators. One of the limitations of this system was jitter introduced in the delay of each monostable circuit. We measured the jitter standard deviation as a function of delay length for a single monostable circuit. Then Pitegoff investigated tradeoffs that resulted (for this type of monostable delay) between delay length, jitter, and the number of stages.

D. E. Nelsen

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