XXII. COGNITIVE INFORMATION PROCESSING*

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A. ACQUIRED ASYMMETRY OF SHAPE LEARNING

Using geometrically transformed texts, we have been able to show that humans learn to decode them differentially.[1] Mathematically equivalent transformations impose different burdens, both with respect to speed of decoding (as measured by the amount of time taken to read pages in a single geometrical transformation), and with respect to the rate of learning. The order of difficulty of the transformations shows that rotation in the plane of the page is easiest to master, while inversion on a horizontal axis and reflection about a vertical axis are far more difficult. Furthermore, there is a negative correlation between speed of initial learning and rate of acquisition of skill – the easiest transformation shows the least improvement.

These results imply that learned sequences of visual scanning, in conjunction with learned preferences for shapes of differing orientation, powerfully affects the subjects' ability to decode the texts. To learn whether the asymmetrical preferences are specific characteristics of the nervous system or reflect special learning was the motivation for the following experiment.

The experiment was performed in Israel with native speakers of Hebrew who had virtually no familiarity with languages read from left to right. Their reading skill with Hebrew (and for some of them with Arabic, too) was at or near the level of college freshmen. For reasons that are irrelevant to this report, it was difficult to obtain

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subjects, and the few whom we finally got could be tested only on a restricted set of materials, because of various time commitments. The array of 8 transformations that we have used previously was therefore reduced to 6, and testing time itself reduced to 5 days. Despite the small number of subjects, the results are quite clear.

The geometrical operations performed on the text were identical with those previously performed with English,[1] except for the 2 missing examples. Hebrew, as all Semitic languages, is read from right to left. A rotation in the plane of the page of Hebrew, consequently, yields a geometry that is opposite, but symmetrical, to that of English, and similarly with all other transformations that we have used. That is, identical geometrical transformations were performed on different bases (English and Hebrew) which are left-right inversions of each other. On each of the 5 days, each of the 6 students read aloud one page of Hebrew text in each of the 6 transformations. The results of the testing are shown in Fig. XXII-1, time being represented in minutes on

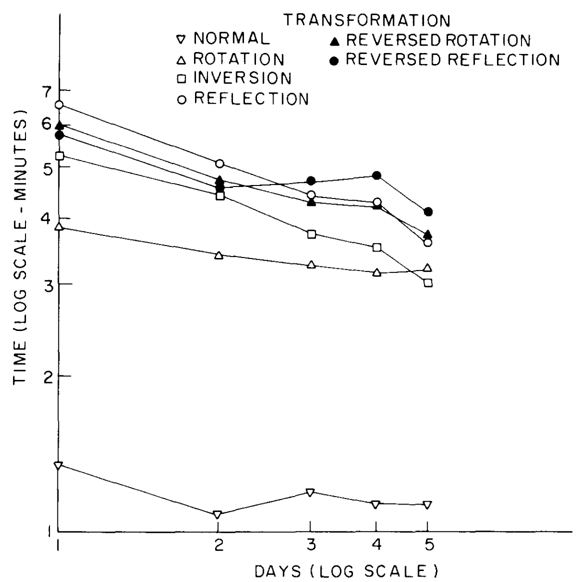

Fig. XXII-1. Results of tests.

the ordinate, and successive pages (days) on the abscissa. Very little improvement occurs in the reading of normally oriented Hebrew; rotation in the plane of the page is the easiest of the transformations, followed by inversion and reflection. In these respects, the data are identical with those obtained from native readers of English for the same transformations. The results show clearly, therefore, that the relative difficulty with the transformations, which had previously been found with native readers of English is not due to specific biases for orientation in the visual system. The difficulties rather seem to be due to biases induced by a well-practiced scanning "program" upon the ability to utilize other such "programs" (decoding strategies). If the biases were specific to orientations, the results from the Hebrew readers would have shown inversion to be far easier than reflection, in order for them to be consistent with those obtained from English readers. The fact that it is the relative orientation that orders the difficulty with the transformations reveals clearly that the biases are acquired.

P. A. Kolers

References

1. P. A. Kolers, A. C. Boyer, and K. F. Rosenthal, "Protracted Practice on Decoding Spatially Transformed Text," Quarterly Progress Report No. 78, Research Laboratory of Electronics, M. I. T., July 15, 1965, pp. 229-231.

B. TRANSMISSION AND CODING OF COLOR PICTURES

Efficient schemes for transmission and coding of color pictures have been investigated by real-time processing of color television signals.[1] Subjective effects of random noise, PCM quantization noise, delta-modulation noise, and sampling and filtering degradations in color signals were evaluated by direct viewing of kinescope displays.[2] Such real-time displays are not only superior in quality to photographs obtained by computer simulation process but also show frame-to-frame degradations, thereby making evaluation more meaningful.

The blocks of a color picture transmission system are shown in Fig. XXII-2. The

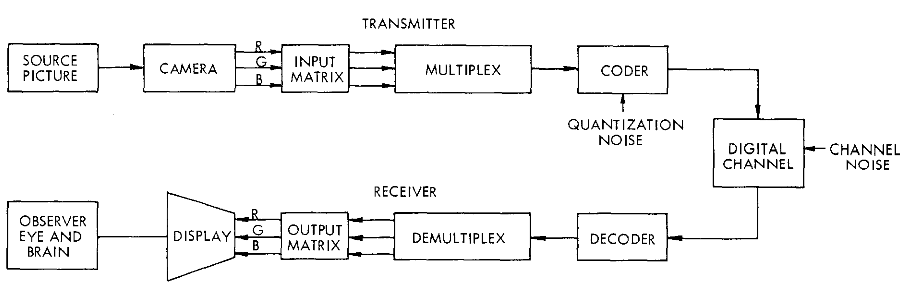

Fig. XXII-2. Color picture transmission system.

source pictures (standard SMPTE and other color transparencies) are scanned by the RCA TK-27 color camera to generate Red (R), Green (G), and Blue (B) signals of 2-MHz bandwidth. These signals may be matrixed, if desired, and multiplexed suitably to give a single analog signal. The analog signal is encoded into a digital stream and transmitted. At the receiver, the digital signal is decoded back to its original analog form, demultiplexed, and displayed on the TM21-B RCA color monitor to reconstruct the original picture. This picture is viewed by the human observer through his eye and evaluated in his brain.

Time-division multiplexing may be more efficient and convenient if the signal is to be transmitted digitally. Thus sequential transmission of color television signals was considered as an alternative approach to matrixing and subcarrier multiplexing. For this purpose, a time-division multiplexing unit was constructed for sampling and multiplexing the color signals at variable bit rates. The two codecs (coder and decoder) that were used primarily in the experiments were the 111 Mb/sec solid-state PCM facility[3] of the High Speed PCM Terminal Department of Bell Telephone Laboratories, Inc., for standard PCM, and the Ball Brothers Research Corporation Codec[4] for the delta modulator. The performance of the codecs and the multiplexing equipment was checked and insured by appropriate noise-loading tests. The noise performance of the multiplexer was at least 10 dB better than a 9-bit PCM coder.

Figure XXII-3 compares the bound on signal-to-quantization-noise ratio for

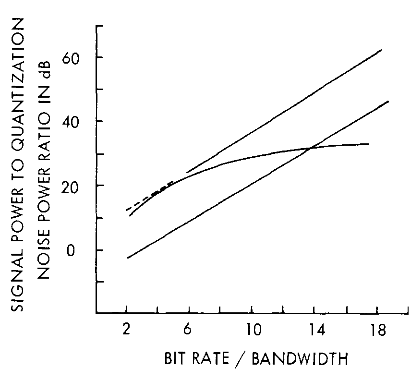

Fig. XXII-3. Comparison of DPCM, ΔM, and standard PCM systems for Gaussian signals.

differential PCM (DPCM), delta modulation (ΔM), and standard PCM systems for Gaussian signal inputs.[5] This figure indicates how efficient the coding scheme is at the desired signal-to-noise ratio (SNR), or alternatively the best SNR obtainable at the desired bit rate. The SNR given is the rms signal-to-rms noise ratio. To calculate the peak-to-peak signal-to-rms-noise ratio, we have to add 18-20 dB, the amount depending on the statistics of the television signal.[6] Thus to obtain $(SNR)_{pp}$ of 54, 46, 38, and 29 dB by ΔM, we need bit rates of 16, 8, 4, and 2 times the bandwidth; for standard PCM this figure is approximately 12, 10, 7, and 4 times the bandwidth, respectively. Although for standard PCM, the SNR in dB increases linearly with bit rate, for ΔM it increases only logarithmically. Thus for higher SNR standard PCM would be more efficient than ΔM, and for lower SNR, ΔM would be superior. The quality of a television picture cannot be judged, however, by its SNR alone. Though these analytical results and physically measurable tests are often helpful in making gross judgments, it is the perceptual significance of transmission distortion that is the crucial factor in determining the merit of a transmission system. Only a subjective test can lead to a meaningful judgment in the final evaluation of picture quality.

Evaluation and subjective tests consisted of assigning a comment on the degradation perceived on a high-quality color monitor display. The comments were chosen from a comment scale ranging from 'not perceptible' to 'extremely objectionable,' as indicated in Figs. XXII-5, XXII-7 and XXII-8. The results were viewed by several trained observers in a dark studio with a viewing distance of 4 times the picture height (5 ft). The screen highlight brightness was kept at 25 ft L (85 cd/m^2), and from ten to twelve slides with widely varying color, tones, content, and details were used for the tests. The arithmetic mean of comments from all observers and on all pictures was taken for the final results. The standard deviation in comments for observers was, on the average, 0.5, while that for pictures was 1.0. Thus for accurate evaluation of color picture-transmission schemes not only a large number of observers but also a large number of widely varying pictures should be used.

To study subjective effects of random noise in the three color channels, the experimental arrangement of Fig. XXII-4 was used. The SNR can be varied by the attenuators

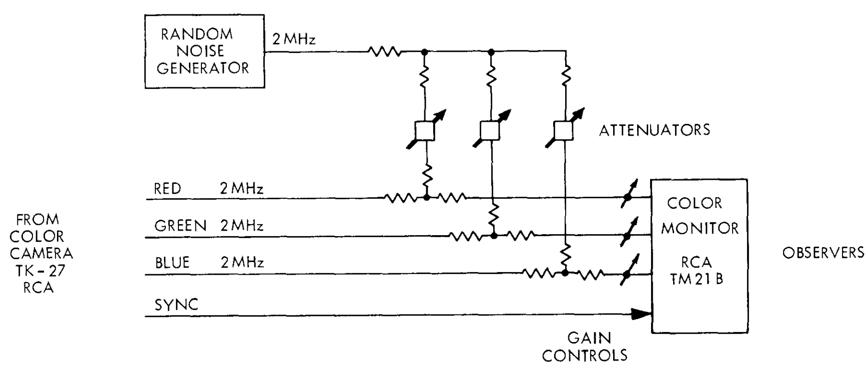

Fig. XXII-4. Experimental arrangement for evaluating the effects of random noise.

for each of the channels. Noise added in this form is perfectly correlated. To observe the effects of uncorrelated noise, delays of 20 and 40 μsec (1/3 and 2/3 of line scanning time) were added to the noise path of two of the channels. For evaluating the subjective effects of degradation by random noise for the channels individually, the SNR in two of the channels were maintained at a level of 55 dB, while the SNR of the third channel varied. The results are shown graphically in Fig. XXII-5. There is an approximate 10-dB difference in the SNR required for the blue, red, and green signals for the same transmission objective (just perceptible, not objectionable, etc.). With noise present in all three channels together, correlated noise, which appears more as luminance

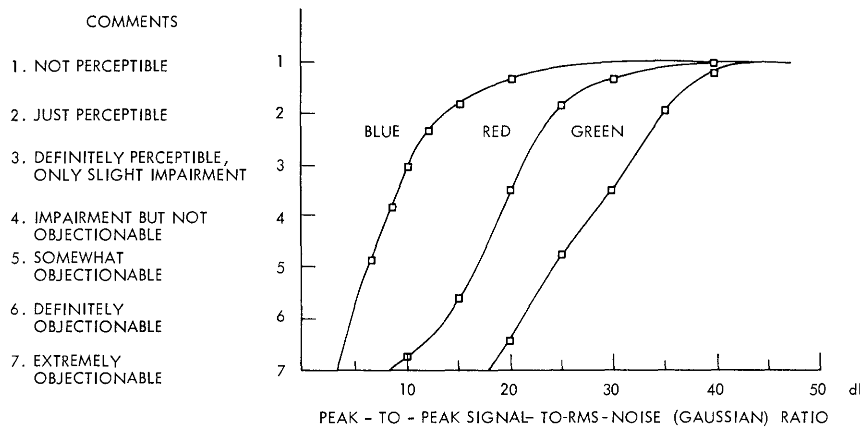

Fig. XXII-5. Subjective effects of random noise in color channels.

degradations, is slightly more degrading than uncorrelated noise, which appears more as true degradations (see Table XXII-1).

Subjective effects of delta-modulation noise and PCM quantization noise in the three color channels were evaluated by individually coding and decoding each of the color

Table XXII-1. Effects of correlated and uncorrelated random noise.

Source	SNR (db)			Mean Comment
	Green	Red	Blue	
Correlated	35	25	15	3.5
Correlated	37	27	17	3.1
Correlated	40	30	20	2.1
Uncorrelated	39	28	10	4.1
Uncorrelated	36	25	14	4
Uncorrelated	40	30	18	2.3
Uncorrelated	40	30	20	1.8
Uncorrelated	42	31	21	1.4
Uncorrelated	43	32	24	1

signals while directly transmitting the other two (unimpaired analog signals) (see Fig. XXII-6). The results for delta modulation are summarized in Fig. XXII-7. The bit rates for transmission by ΔM that are required for a given transmission objective may be determined from Fig. XXII-7. For a transmission objective of 4 (impairment but not objectionable) the bit rates required by the green, red, and blue signals are 9, 8, and 4 mB/sec, respectively. ΔM is slightly more degrading subjectively than

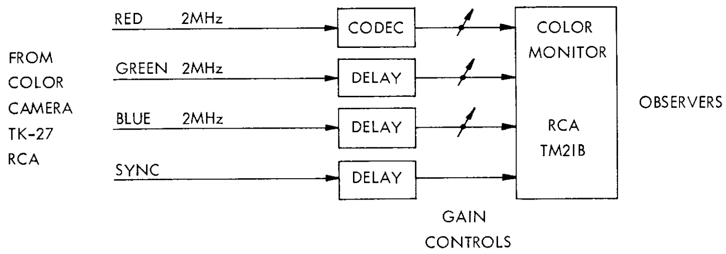

Fig. XXII-6. Experimental arrangement for evaluating the effects of modulation noise and quantization noise.

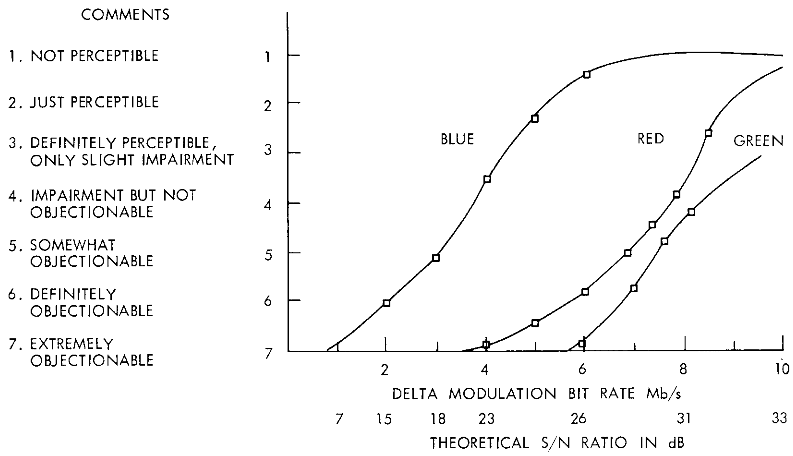

Fig. XXII-7. Subjective effects of ΔM noise in color channels.

random noise for the same SNR.

In transmission of signals by standard PCM much higher SNR are required, as PCM quantization noise is subjectively most annoying. The subjective effects of PCM quantization noise are shown graphically in Fig. XXII-8. For a comment of 4 the quantization bits required for the green, red, and blue signals are 5, 5, and 4, respectively. This corresponds to SNR of 40, 40, and 34 dB, which is much higher than the 32, 31, and 23 dB required by delta modulation. Not only is delta modulation more efficient in terms of SNR but also at the low SNR involved delta modulation requires a lower bit rate for transmission than standard PCM, as shown in Fig. XXII-3. The crossover point when standard PCM becomes more efficient than delta modulation for the same SNR is at 50 dB peak-to-peak signal-to-rms noise ratio.

Three different kinds of filters were used for bandlimiting the color signals, and the subjective effect on the composite picture was evaluated. The filters were the Thomson and the overshoot filters which were identified by their 20-dB down point, and

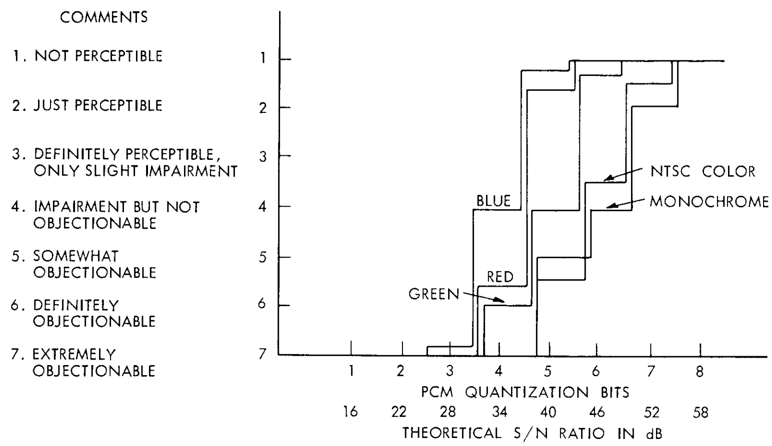

Fig. XXII-8. Subjective effects of PCM quantization noise in color signals.

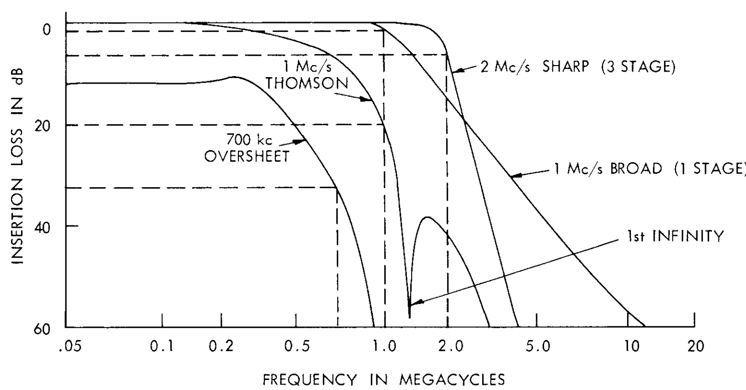

Fig. XXII-9. Filter characteristics.

Table XXII-2. Lowpass filtering of color signals.

Blue		Red		Green	Mean Comment
Thomson	0.7 MHz	Thomson	1.5 MHz	Original 2 MHz	4
Thomson	1	Thomson	1.5	Original 2	2
Thomson	1	Thomson	1	Original 2	2.5
Thomson	1	Broad	1	Original 2	2
Overshoot	0.7	Broad	1	Original 2	2.5
Overshoot	0.7	Overshoot	1.5	Original 2	4.5

the broad filter, which was identified by its design frequency, f_c. The filter characteristics are shown in Fig. XXII-9. The overshoot filter acts as a crispener, by overemphasizing the steepness of transition at edges. It achieves edge enhancement by addition of the inverted second derivative (or approximate) of the image signal to the image signal.[7] The overshoot filter also produces a ring, because of its sharp cutoff characteristics. The comments obtained about the composite picture when the indicated filters were used are shown in Table XXII-2. The ringing caused by the overshoot filter is not perceptible in the blues, but is objectionable in the reds and results in a false chroma edge.

To determine the subjective effects of degradations introduced during sampling, the color signals were sampled at variable rates by the time-division multiplexing unit built for this purpose. Fig. XXII-10 shows the results of sampling the individual color channels at different sampling rates. In these experiments only one signal was sampled and the other two transmitted directly. The filter losses at the sampling rate and half the sampling rate were computed and the results are shown in Table XXII-3.

The efficiency of transmission by standard PCM can be improved by judicious choice of sampling frequency. The idle bands generated in the spectrum of a television signal, because of the scanning process, can be shared by frequency-interleaving[8] it with the image band around the sampling frequency, thereby minimizing foldover distortion. When such is the case, we no longer need to sample at the Nyquist rate of twice the bandwidth, but at a much lower rate. Thus an 800-kHz blue signal can be sampled at a frequency of 1.15 MHz (see Fig. XXII-10) with almost no perceptible degradations. The filter losses required for the blue, red, and green signals are indicated to be approximately 4, 7, and 12 dB at half the sampling frequency, and 16, 36, and 44 dB at the sampling frequency. For monochrome signals comparable results are 16-19 dB at half the sampling rate, and 38 dB at the sampling rate.[9] A comparison of bit rates required for sequential transmission of color signals by delta modulation and standard PCM is shown in Table XXII-4 for different transmission objectives.

For the NTSC signal transmitted by standard PCM (quantized to 6 bits and sampled at 9 MHz) the bit rate required is 54 Mb/sec. This represents a transmission objective of 3-4. Taking into account the fact the combined degradation in the picture from all three signals will make it subjectively worse (this effect is approximately one step on the comment scale), we need approximately 26 Mb/sec for transmission by delta modulation and 33 Mb/sec for transmission by standard PCM for the same transmission objective in a sequential transmission system. The picture in the last case will not be as sharp, however, because the green signal transmitted is of 2-MHz quality. For a sharper picture a bit rate only slightly higher would be required. Thus for digital transmission of color signals, time-division multiplexing would be more efficient than frequency-division multiplexing by a considerable amount. The bit rate required

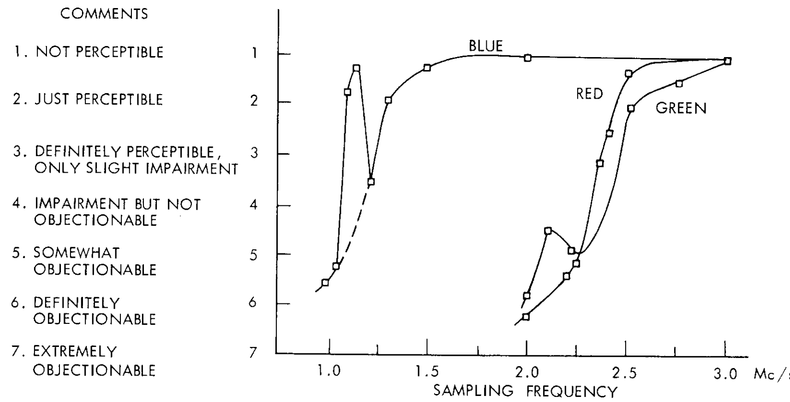

Fig. XXII-10. Subjective effects of sampling degradations in color channels.

Table XXII-3. Sampling of signals and filter insertion losses.

Signal	Sampling Rate (Mc/sec)	Combined Filter Loss at Half Sampling Rate	Output Filter Loss at Sampling Rate	Mean Comment
Blue	1.0	1.5	7	5.5
	1.05	1.8	8.5	5.2
	1.10	2.0	10	1.7
	1.15	2.2	10.5	1.2
	1.20	2.3	11	3.6
	1.30	2.6	12	1.8
	1.50	4	16	1.2
	2.0	17	26	1
Red	2.0	7	11	6.2
	2.20	8	22	5.4
	2.25	9	27	5.1
	2.35	9.5	30	3.1
	2.40	10	34	2.5
	2.50	11	36	1.3
	3.0	16	66	1
Green	2.0	7	11	5.8
	2.10	7.5	17	4.5
	2.25	9	28	4.8
	2.5	11	36	2
	2.75	12	44	1.5
	3.0	16	50	1

Table XXII-4. Comparison of standard PCM and delta modulation.

Transmission Objective	Delta Modulation				Bit Rates Required (Mb/sec) Standard PCM									
					Green			Red			Blue			
	Green	Red	Blue	Total Bit Rate	S. R.	Q. B.	B.R.	S. R.	Q. B.	B.R.	S. R.	Q. B.	B.R.	Total Bit Rate
1-2	16	10	6	32	3	6	18	2.5	5	12.5	1.5	5	7.5	38
2-3	12	8.5	5	25.5	2.5	6	15	2.35	5	11.75	1.25	5	6.25	33
3-4	9	8	4	21	2.4	6	14.4	2.30	5	11.50	1.2	5	6	31.9
4-5	8	7	3	18	2.3	5	11.5	2.25	5	11.25	1.1	4	4.4	27.2
5-6	7	6	2.5	15.5	2.25	5	11.25	2.0	4	8	1	4	4	23.3

S.R. – Sampling rate in MHz

Q.B. – Quantization bits

B.R. – Bit rate in Mb/sec

for transmission of the NTSC signal by delta modulation was not determined, but it is suspected to be much higher, as the NTSC signal is very sensitive to the phase and amplitude errors that delta modulation will introduce.

A great amount of frame-to-frame redundancy exists in the color television signal, and this can be exploited by using delays that can store entire frames. The highlight brightness for the blue and red signals is much lower, and this considerably reduces the critical flicker frequency. This fact can be exploited by transmitting fewer frames per second for these signals. Also, the vertical resolution in the color pictures is greater than necessary. It is more difficult to resolve the line structure in the Blue and Red components of the picture, than for the luminance component. Low-cost line delays may be used to further exploit the redundancy in vertical resolution. Such techniques seem to be very promising, and further investigation is called for.

The transmission of a low-resolution monochrome signal by several kinds of differential PCM schemes was also studied. We found that for the same bit rate a 3-bit DPCM system produces a picture that is less noisy than either ΔM or 2-bit DPCM pictures, but subjectively the ΔM picture is preferred, because of the sharpness resulting from the permitted higher sampling rate. For a higher quality of transmission, the 3-bit DPCM system would be more efficient, as indicated by Fig. XXII-3.

A. K. Bhushan

References

1. This work was done at Centre 321, Bell Telephone Laboratories, Inc., Holmdel, New Jersey, and was submitted as a thesis in partial fulfillment of the requirements for the degree of Master of Science, Department of Electrical Engineering, M.I.T., June 1967, with the title, "Efficient Transmission and Coding of Color Pictures."

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C. SUBJECTIVE EFFECT OF ADDITIVE WHITE PICTORIAL NOISE WITH VARIOUS PROBABILITY DISTRIBUTIONS

The purpose of this study was to find out how strongly the objectionability of pictorial noise depends on its probability distribution.

The noisy pictures were generated on a digital computer. A picture with a cameraman as a central object was used as the original, to which noise with various probability distributions was added.

For the simulation on the computer the picture was divided into 256×256 points, and the intensity at these points quantized to $2^{10} = 1024$ levels. The noise was generated by using the random-number generator subroutine of the computer in conjunction with suitable transformations. To the picture samples were then added the noise samples, thus producing the output noisy picture. The noise samples were independent of each other and of the original picture samples.

The signal-to-noise ratio, S/N, is defined as

$$\frac{S}{N} = 10 \log_{10} \frac{S_{max}^2}{\sigma^2} = 20 \log_{10} \frac{S_{max}}{\sigma},$$

where

S_{max} = peak signal amplitude = 1023

σ^2 = variance of the noise.

Table XXII-5 shows the signal-to-noise ratios that were used and the corresponding values of σ and σ^2.

Table XXII-6 shows the probability distributions that were used, together with their variances.

Table XXII-5. Signal-to-noise ratios.

S/N (db)	σ	σ^2
26	51.2	2620
22	81.3	6610
20	102.3	10485
18	129	16650
14	204	41620

Table XXII-6. Probability distributions.

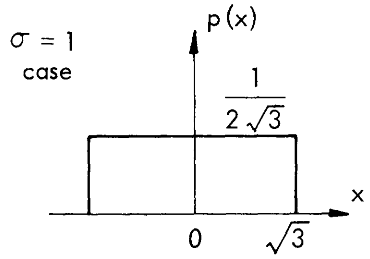
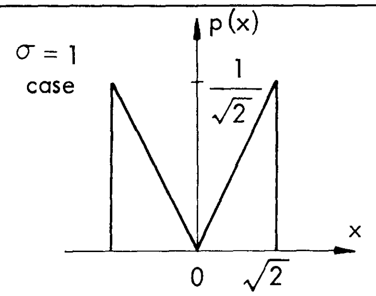
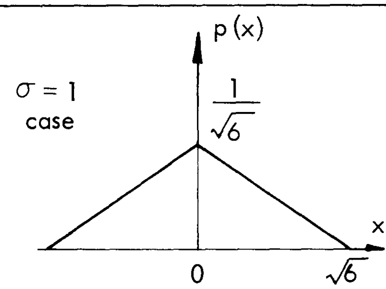
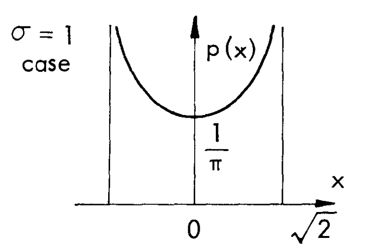
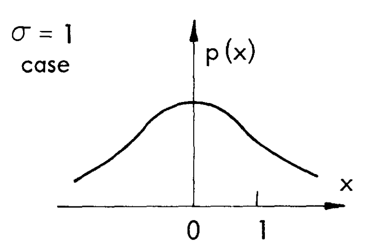
Uniform Distribution		$p(x) = \begin{cases} \frac{1}{2 \cdot a} & \lvert x \rvert < a \\ 0 & \lvert x \rvert > a \end{cases}$ $\sigma^2 = \frac{1}{3} \cdot a^2$
Inverse Triangular Distribution		$p(x) = \begin{cases} \frac{1}{a^2} \cdot x & 0<x<a \\ -\frac{1}{a^2} \cdot x & -a<x<0 \\ 0 & \text{otherwise} \end{cases}$ $\sigma^2 = \frac{1}{2} \cdot a^2$
Triangular Distribution		$p(x) = \begin{cases} \frac{1}{a} - \frac{1}{a^2} \cdot x & 0<x<a \\ \frac{1}{a} + \frac{1}{a^2} \cdot x & -a<x<0 \\ 0 & \text{otherwise} \end{cases}$ $\sigma^2 = \frac{1}{6} \cdot a^2$
" Sine " Distribution		$p(x) = \begin{cases} \frac{1}{\pi} \cdot \frac{1}{\sqrt{a^2 - x^2}} & \lvert x \rvert < a \\ 0 & \lvert x \rvert > a \end{cases}$ $\sigma^2 = \frac{1}{2} \cdot a^2$
Gaussian Distribution		$p(x) = \frac{1}{\sqrt{2\pi}\; a} \cdot e^{-\frac{x^2}{2a^2}}$ $\sigma^2 = a^2$

Table XXII-7 shows the values of a (see Table XXII-6) for the various probability distributions and the different signal-to-noise ratios.

The fourth moments were also computed; their values are shown in Table XXII-8.

Table XXII-7. Values of a.

Distribution \ S/N	26	22	20	18	14
Uniform	88.8	141.0	177.8	223.9	354.0
Inverse triangular	72.4	115.0	144.9	182.4	288.8
Triangular	125.4	199	251	316	500
"Sine"	72.4	115.0	144.9	182.4	288.8
Gaussian	51.2	81.3	102.3	129	204

Table XXII-8. Values of x^4. (These values have to be multiplied by 10^6.)

Distribution \ S/N	26	22	20	18	14
Uniform	12.37	78.8	198.0	500.0	3120
Inverse triangular	9.13	58.1	146.3	368	2310
Triangular	1.83	11.7	29.3	74.0	462
"Sine"	10.3	65.5	165	416	2600
Gaussian	20.6	131	330	832	5200

By inspection of the noisy pictures, we found that the difference between the objectionabilities of noises having the same variance but different probability distributions was hardly noticeable, although the fourth moments of the noises were quite different. It seems therefore, that for pictures with additive white noise, the signal-to-noise ratio can be used as a good measure of picture quality for a wide class of noise probability distributions, including at least the five that we studied.

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D. STOCHASTIC MODEL FOR WEATHER MAPS

An investigation was undertaken[1] to see if a two-state Markov source[2] was an adequate model for weather maps in the following sense. Segments of weather maps were quantized into an $n \times n$ array of 1's and 0's. The array was then scanned in line-by-line fashion. The one-dimensional sequence of zeros and ones thus generated was modeled as a stochastic source, for which line-to-line dependencies in the original picture were ignored.

The two-state Markov model implies that the run lengths are independent and geometrically distributed. Three statistical tests were performed to test the hypotheses that the one runs were independent, the zero runs were independent, and the one runs were independent of the zero runs. The results of the last two tests were consistent with the hypothesis of independence at approximately the .05 level of significance. The first hypothesis was rejected, however, at the same significance level. The test of the one runs was repeated, only those one runs being counted were preceded by a one run of length 3 or larger. This was motivated by the fact that the pictures present regions of high detail and other regions of low detail. If a different run-length density were used within each region, the resulting model might be more accurate. This test (at the .05 level) was consistent with the hypothesis of independence. The experimental probability transition functions were calculated and compared with the geometric run-length probabilities of the two-state Markov model. The Chi-Square test of significance was used in this comparison. The tests indicated a significant deviation; however, graphs of the data revealed that the geometric approximation was apparently quite good.

Sequences of run lengths from the pictures were coded by using Huffman coding. The probability of overflow and the expected amount of overflow were determined from the data. We chose to encode $T = 1800$ cells at a time. If a maximum of $RT = .575 \times 1800$ bits were used in encoding the original 1800 cells, the expected overflow would be 7 bits. The entropy of the pictures as calculated from the model was .455 bits. A bound to the probability of overflow was derived which is exponentially decreasing in T, the number of cells encoded for a given rate R (=number of bits allowed in encoding T cells/T). From this, a bound to the expected over-flow was calculated. Both of these bounds were quite loose when compared with the actual data for $T = 1800$.

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References

1. J. Woods, "Run-Length Coding of Weather Maps," S. M. Thesis, Department of Electrical Engineering, M. I. T., June 1967.
2. J. Capon, "A Probabilistic Model for Run-Length Coding of Pictures," IRE Trans., Vol. IT-5, No. 4, No. 4, pp. 157-163, December 1959.