XXI. NETWORK SYNTHESIS

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A. SYNTHESIS OF RC NETWORKS BY MEANS OF COORDINATE TRANSFORMATIONS

In Quarterly Progress Report No. 64 (pp. 345-359), it was shown that the nodal parameter matrices of any two-element-kind network (not having mutual inductance) can always be simultaneously reduced to diagonal form by a real, nonsingular transformation. In particular, it was shown that this can be done irrespective of dynamical degeneracies existing in the network, which cause one or both of the parameter matrices to be singular. The diagonal form places in evidence the natural frequencies of the network. It was also shown how this diagonal form, together with the transformation matrix that effected the diagonalization, formed a basis for the expansion of the open-circuit impedance matrix of the network.

The present report takes up where the previous one left off and is directed toward the synthesis of two-terminal pair RC networks. However, the techniques developed and the general form of the results apply equally well to RL and LC networks. Specifically, some necessary conditions regarding compactness of residues, and a general synthesis procedure involving ideal transformers will be discussed. Finally, some comments regarding transformerless synthesis will be made.

1. The Compact Residue Requirement

In an n independent node RC network the nodal admittance matrix, Y, may always be diagonalized by a nonsingular, real congruent transformation M. We denote the complex-frequency variable by s, the nodal capacitance matrix by C, and the nodal conductance matrix by G; the rank of C is assumed to be $(n-\rho)$, and the rank of G to be $(n-\sigma)$, while n, σ, and ρ obey the constraint $(n-\sigma-\rho) \geqslant 0$. We have, then, Y and its diagonalized form, Y_d.

$$Y = [sC+G]$$

$$Y_d = M^tYM = M^t[sC+G]\ M = s[M^tCM] + [M^tGM]$$

$$Y_d = s\left[\begin{array}{c|c|c} U_\sigma & 0 & 0 \\ \hline 0 & U_{(n-\sigma-\rho)} & 0 \\ \hline 0 & 0 & 0_\rho \end{array}\right] + \left[\begin{array}{c|c|c} 0_\sigma & 0 & 0 \\ \hline 0 & \begin{matrix} g_{(\sigma+1)} & & \\ & \ddots & \\ & & g_{(n-\rho)} \end{matrix} & 0 \\ \hline 0 & 0 & U_\rho \end{array}\right] \tag{1}$$

where the superscript t denotes the transpose, U_k is a diagonal unit matrix of order k, and $g_{(\sigma+1)}$, $g_{(\sigma+2)}$ + ... $g_{(n-\rho)}$ are all real positive quantities.

The g's are the negatives of the natural frequencies of the network and it is convenient to relabel them in the following manner:

$$\begin{aligned} s_{(\sigma+1)} &= g_{(\sigma+1)} \\ s_{(\sigma+2)} &= g_{(\sigma+2)} \\ &\vdots \\ s_{(n-\rho)} &= g_{(n-\rho)}. \end{aligned} \tag{2}$$

Hence

$$Y_d = \left[\begin{array}{c:ccc:c} sU_\sigma & & 0 & & 0 \\ \hdashline & (s+s_{(\sigma+1)}) & & & \\ 0 & & \ddots & & 0 \\ & & & (s+s_{(n-\rho)}) & \\ \hdashline 0 & & 0 & & U_\rho \end{array}\right] \tag{3}$$

The open-circuit impedance matrix Z is the inverse of Y. Thus

$$Z = Y^{-1} = MY_d^{-1}M^t, \tag{4}$$

where the superscript −1 denotes the inverse. The entries in M are all real. M is given by

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & & & \vdots \\ \vdots & & & \vdots \\ m_{n1} & \cdots & \cdots & m_{nn} \end{bmatrix} \tag{5}$$

Equation 4 has the form of a Gramian, so that if we make the identification

$$m_{jq}m_{kq} = k_{jk}^{(q)} \tag{6}$$

a typical term in Z, z_{jk}, may be written

$$z_{jk}(s) = \sum_{q=1}^{\sigma} \frac{k_{jk}^{(q)}}{s} + \sum_{q=(\sigma+1)}^{(n-\sigma)} \frac{k_{jk}^{(q)}}{(s+s_q)} + \sum_{q=(n-\sigma+1)}^{n} k_{jk}^{(q)} \tag{7}$$

Equation 7 reveals $z_{jk}(s)$ in the form of a partial fraction expansion, the pole factors being the diagonal entries in Y_d, and the terms $k_{jk}^{(q)}$ being the residues in these poles. The terms in the summation at the extreme right in Eq. 7 are not really residues, for they define the constant term in the partial fraction expansion of z_{jk}, but for the sake of consistency we shall treat them as if they were residues.

Let us now consider a set of impedances z_{11}, z_{12}, and z_{22} and investigate the residues of these in their q^{th} pole. From (6) and (7) we have

$$\begin{aligned} k_{11}^{(q)} &= m_{1q}^2 \\ k_{12}^{(q)} &= m_{1q}m_{2q} \\ k_{22}^{(q)} &= m_{2q}^2 \end{aligned} \tag{8}$$

By inspection it is clear that these residues must satisfy the following condition:

$$k_{11}^{(q)}k_{22}^{(q)} - \left(k_{12}^{(q)}\right)^2 = m_{1q}^2 m_{2q}^2 - (m_{1q}m_{2q})^2 \equiv 0. \tag{9}$$

The equality stated by (9) is characterized by saying that the residues form a compact set or, more simply, that the residues are compact.

As shown in the previous report, the entries in Y_d, which are the pole factors of (7), are all dynamically distinct. They need not, however, be numerically distinct. The physical significance is simply that if a term in Y_d having a particular numerical value appears r times, in the network there are r different and independent ways of exciting a natural frequency having this particular numerical value.

The fact that the entries in Y_d can be numerically identical has an interesting implication. Let us assume that each of the impedances z_{11}, z_{12}, and z_{22} is written in the form of (7) and, for instance, that the natural frequencies s_1 and s_2 are numerically identical. Then, for the residues in these two poles, we have the relations

$$\begin{aligned} k_{11}^{(1)}k_{22}^{(1)} - \left(k_{12}^{(1)}\right)^2 &= 0 \\ k_{11}^{(2)}k_{22}^{(2)} - \left(k_{12}^{(2)}\right)^2 &= 0. \end{aligned} \tag{10}$$

On the other hand, we might wish to add together, in each of the three impedances, the residues relating to these numerically equal poles. We shall call such a residue sum a "net" residue. In our example each "net" residue is the sum of two terms. If we make use of the Schwartz inequality, we see that the "net" residues satisfy the following inequality:

$$\left[\left(k_{11}^{(1)}+k_{11}^{(2)}\right)\left(k_{22}^{(1)}+k_{22}^{(2)}\right)-\left(k_{12}^{(1)}+k_{12}^{(2)}\right)^2\right] \geq 0. \tag{11}$$

It follows that an inequality of the type given by (11) holds when there are any number of terms involved in the summations which define the "net" residues. Hence we have the result that the compactness of the residues in the dynamically independent poles insures that the so-called residue condition will obtain for the "net" residues.

In a synthesis problem the impedances z_{11}, z_{12}, and z_{22} (which we assume meet the so-called residue condition) are specified in advance. If the residues appear to be not compact, they can always be factorized into compact sets as the following example shows.

Suppose that we have

$$\begin{aligned} z_{11}(s) &= \frac{a_{11}^{(1)}}{s+s_1} \\ z_{12}(s) &= \frac{a_{12}^{(1)}}{s+s_1} \\ z_{22}(s) &= \frac{a_{22}^{(1)}}{s+s_1}, \end{aligned} \tag{12}$$

where $a_{11} > 0$, $a_{22} > 0$, and $a_{11}a_{22} - a_{12}^2 \geq 0$.

Make the following identifications:

$$\begin{aligned} a_{11}^{(1)} &= k_{11}^{(1)} + k_{11}^{(2)} \\ a_{12}^{(1)} &= k_{12}^{(1)} + k_{12}^{(2)} \\ a_{22}^{(1)} &= k_{22}^{(1)} + k_{22}^{(2)}. \end{aligned} \tag{13}$$

Let

$$k_{11}^{(1)} = a_{11}^{(1)} \qquad \therefore k_{11}^{(2)} = 0$$

$$k_{12}^{(1)} = a_{12}^{(1)} \qquad \therefore k_{12}^{(2)} = 0.$$

To satisfy the compactness requirement, we have

$$k_{11}^{(1)}k_{22}^{(1)} - \left(k_{12}^{(1)}\right)^2 = 0 \qquad \therefore k_{22}^{(1)} = \frac{\left(k_{12}^{(1)}\right)^2}{k_{11}^{(1)}} = \frac{\left(a_{12}^{(1)}\right)^2}{a_{11}^{(1)}}.$$

Thus

$$k_{22}^{(2)} = a_{22}^{(1)} - k_{22}^{(1)} = a_{22}^{(1)} - \frac{\left(a_{12}^{(1)}\right)^2}{a_{11}^{(1)}} \geq 0. \tag{14}$$

The values obtained simultaneously satisfy (10) and (13) and permit (12) to be recast in the form

$$\begin{aligned} z_{11}(s) &= \frac{k_{11}^{(1)}}{s+s_1} + \frac{0}{s+s_2} \\ z_{12}(s) &= \frac{k_{12}^{(1)}}{s+s_1} + \frac{0}{s+s_2} \\ z_{22}(s) &= \frac{k_{22}^{(1)}}{s+s_1} + \frac{k_{22}^{(2)}}{s+s_2}, \end{aligned} \tag{15}$$

where $s_2 = s_1$.

The example given above illustrates the simplest type of factorization that one can perform, but it is easy to see that it is not the only way of reducing a noncompact set to compact sets. Factorization into compact residue sets is by no means unique so that, in general, an infinite number of possibilities exist. However, the point is that factorization of a noncompact set into compact sets can always be accomplished. Hence in a synthesis problem in which a realizable set z_{11}, z_{12}, and z_{22} is prescribed, we may always accomplish a residue factorization and put each of the impedances in the form of Eq. 7.

2. Synthesis of a Given Set of Impedances by Using Ideal Transformers

Suppose that an n independent node-pair network exists whose nodal admittance matrix, Y, may be put into diagonal form by means of a congruent transformation with a matrix M. Thus, since

$$M^tYM = Y_d,$$

the open-circuit impedance matrix Z takes the familiar form

$$Y^{-1} = Z = MY_d^{-1}M^t.$$

The open-circuit equilibrium equations may then be written

$$\left[MY_d^{-1}M^t\right] I = V, \tag{16}$$

where I is the column matrix of terminal currents, and V is the column matrix of terminal voltages

$$I = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix} \qquad V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

A new set of current and voltage variables, I' and V', may be defined in terms of the original ones given in (16), as follows:

$$V' = M^{-1}V = \begin{bmatrix} v'_1 \\ v'_2 \\ \vdots \\ v'_n \end{bmatrix} \qquad I' = M^tI = \begin{bmatrix} i'_1 \\ i'_2 \\ \vdots \\ i'_n \end{bmatrix} \tag{17}$$

Note that the change of variables expressed by (17) leaves the power invariant, since $V'^tI' = V^tI$.

Premultiplying both sides of (16) by M^{-1} then yields an equation in the new variables

$$Y_d^{-1}I' = V'. \tag{18}$$

But we have

$$Y_d^{-1} = \begin{bmatrix} \frac{1}{y_1} & & & \\ & \frac{1}{y_2} & & \\ & & \ddots & \\ & & & \frac{1}{y_n} \end{bmatrix} \tag{19}$$

where the terms y_1, y_2 ... y_n represent the diagonal terms expressed in Eq. 3. Hence we have the relations

$$\begin{aligned} \frac{v'_1}{i'_1} &= \frac{1}{y_1} \\ \frac{v'_2}{i'_2} &= \frac{1}{y_2} \\ &\vdots \\ \frac{v'_n}{i'_n} &= \frac{1}{y_n}. \end{aligned} \tag{20}$$

Each of the impedances represented in (20) is simply a parallel RC combination. From Eq. 3 we note that none of the admittances is zero; hence none of the impedances is infinite.

At this point if we rewrite (17) in the following form, a straightforward physical interpretation becomes evident.

$$V' = M^{-1}V \qquad I = \left(M^t\right)^{-1} I'. \tag{21}$$

The entries in V are the original terminal voltages, and the entries in V' are linear combinations of these original voltages. From a physical point of view, we may think of the n original voltages as being associated to the primary windings of n ideal transformers, each of which has n secondary windings. The j^{th} row of M^{-1} represents the turns ratios (with respect to the primary) of the j^{th} secondary windings on the various transformers. Hence M^{-1} may be interpreted as a matrix of transformer turns ratios. The same conclusion is evident from recognizing that the currents in I' represent the n secondary winding currents. Thus the primary current in each transformer is related to the secondary currents through the turns ratios of the secondary windings, with respect to the primary.

If we define M^{-1} in the following manner

$$M^{-1} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & & & \vdots \\ \vdots & & & \vdots \\ \rho_{n1} & \cdots & \cdots & \rho_{nn} \end{bmatrix} \tag{22}$$

we can synthesize a network whose terminal pairs correspond to the terminals of the primary windings. Its open-circuit impedance matrix will be Z. The general development of such a circuit is given in Fig. XXI-1.

The resulting network is minimal in the number of R's and C's required, the total number of such elements being at most 2n. However, it is obviously extravagant in its utilization of transformers.

The application of this technique to the synthesis of a prescribed set of impedances z_{11}, z_{12}, and z_{22} is straightforward. As already shown, the residues in the given impedances can always be factorized into compact sets and the impedances put into the form of Eq. 7. Therefrom the entries in the first two rows of M, as well as the terms in Y_d, can be determined.

The remaining entries in M can be filled in an arbitrary manner, subject only to the restriction that M be nonsingular. A particularly easy way is to fill the (n-2)

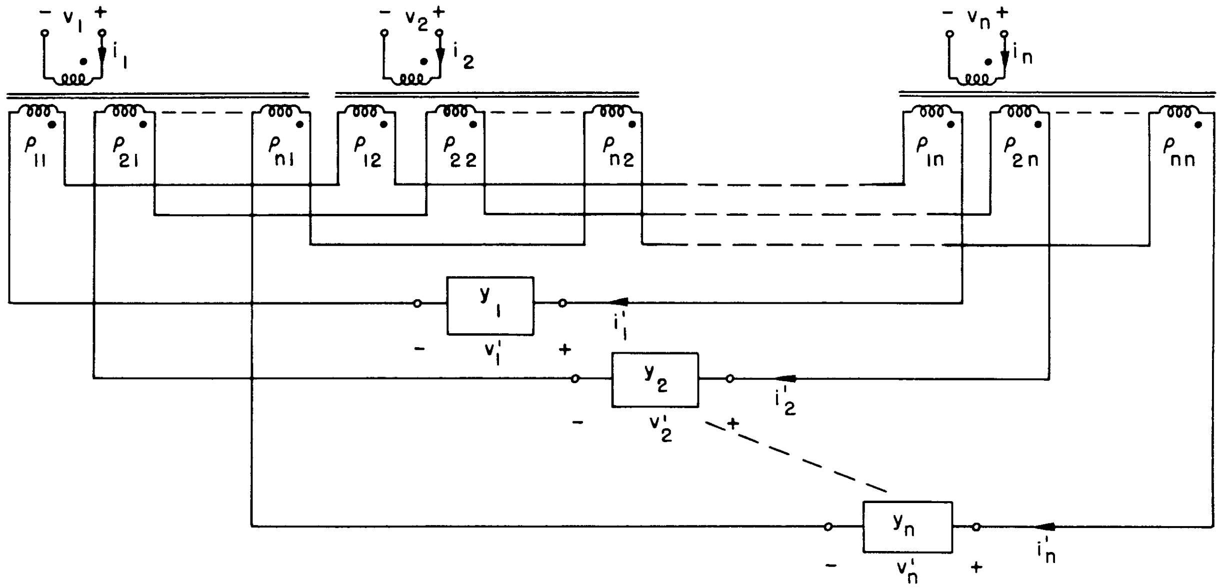

Fig. XXI-1. General development of a network realizing a prescribed impedance matrix. (All primary windings are considered as having one turn. Each admittance is a parallel RC combination.)

remaining diagonal positions with ones, and to place zeros in any positions not already filled. The inversion of M is then extremely simple and the number of transformer windings required is reduced considerably.

The technique, obviously, need not be limited to synthesizing two-terminal pair situations, for it is a very general scheme that lends itself readily to an n terminal pair synthesis.

3. Some Remarks on the Synthesis of a Grounded Two-Port without Ideal Transformers

In the following discussion we wish to point out a connection between the dynamical character and the topological character of a network that must be satisfied in a transformless network.

Suppose that a set of impedances z_{11}, z_{12}, and z_{22} is to be realized as a grounded two-port. Further, suppose that the given set of impedances meets whatever requirements are necessary.

If the network exists, a starlike tree may be defined on the network in such a fashion that the central node of the star corresponds to the ground node of the network, and that the two-ports of interest correspond to entries across two of the tree branches. If we assume a full network graph, the symmetry of the network graph makes each tree branch topologically equivalent to every other tree branch. Hence we may number the tree branches in an arbitrary way and assume that entries across tree branches 1 and

2 correspond to the two-ports of interest, without incurring any loss of generality.

The short-circuit nodal admittance matrix, based on the assumed starlike tree structure, may then be formed by using the appropriate cut-set matrix, α, and the diagonal branch admittance matrix, y_b. We assume that each branch in the network can be a parallel combination of a capacitance and conductance. Thus the admittance matrix will be a dominant matrix corresponding to a node-to-datum formulation.

$$Y = \alpha y_b \alpha^t, \tag{23}$$

where α = cut-set matrix for a starlike tree, and

$$y_b = \begin{bmatrix} (c_1s+g_1) & & & \\ & (c_2s+g_2) & & \\ & & \ddots & \\ & & & (c_bs+g_b) \end{bmatrix}$$

where the c's and g's are the branch capacitances and conductances, respectively.

However, we know from the dynamical properties of the system that Y may be diagonalized by a congruence transformation with M. We also know how the open-circuit impedance matrix Z, is related to M and Y_d. That is, we know that

$$\begin{aligned} M^tYM &= Y_d \\ Z &= MY_d^{-1}M^t. \end{aligned} \tag{24}$$

From the information supplied by the given two-port set of impedances, and by taking into account certain conditions that are known to be necessary for the realization of a grounded two-port, the terms in Y_d and the entries in the first two rows of M can be ascertained. We cannot just fill in the unknown entries in M arbitrarily and have a guarantee that the network will be realizable without transformers, for if we substitute (23) in (24), we have the requirement

$$[M^t\alpha][y_b][\alpha^tM] = Y_d. \tag{25}$$

Now the first two rows of M, and the entries in Y_d are known; α is presumed. Hence if we can find a solution (or solutions) of (25) which is such that the c's and g's (that form the entries in the diagonal matrix y_b) are non-negative, and the remaining entries in M are real, we shall have found a realization for the network without transformers. If we regard the product $[\alpha^tM]$ as a rectangular matrix of order $b \times n$, the problem reduces to transforming the diagonal form y_b to the diagonal form Y_d by means of a singular, congruent transformation.

Equation 25 relates the dynamical and topological properties of the network. Satisfaction of (25) requires, in a sense, that these two characteristics be compatible.

Expression 25 formulates the problem neatly, but its solution, subject to the restrictions mentioned above, is a knotty problem, requiring further investigation.

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B. ERRATUM

In Quarterly Progress Report No. 64, page 356, in the report entitled "Synthesis of Two-Element-Kind Networks by Means of Coordinate Transformations," Eq. (20) should read:

$$z_{jk}(s) = \sum_{q=1}^{\sigma} \frac{k_{jk}^{(q)}}{s} + \sum_{q=(\sigma+1)}^{(n-\lambda)} \frac{k_{jk}^{(q)}}{\left(s+\frac{\gamma_q}{s}\right)} + \sum_{q=(n-\lambda+1)}^{n} sk_{jk}^{(q)} \tag{20}$$

W. C. Schwab

C. NEW CANONIC REALIZATION PROCEDURES FOR RL IMPEDANCES

This report presents two new canonic realization procedures for RL impedances. The first procedure is appropriate to impedances of the form

$$Z = \frac{s\prod_{i=1}^{n-1}(s+\alpha_i)}{\prod_{i=1}^{n}(s+B_i)}, \tag{1}$$

and is based upon the cycle shown in Fig. XXI-2, for which the remainder impedance Z_r has the form

$$Z_r = H\frac{s\prod_{i=1}^{n-3}(s+\gamma_i)}{\prod_{i=1}^{n-2}(s+\delta_i)}.$$

The second procedure is appropriate to impedances of the form

$$Z = \frac{\prod_{i=1}^{n+1}(s+\alpha_i)}{\prod_{i=1}^{n}(s+B_i)}, \tag{2}$$

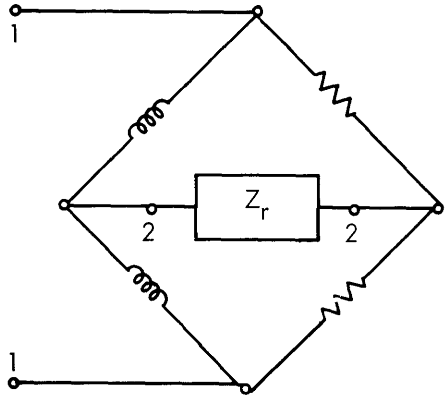

Fig. XXI-2.

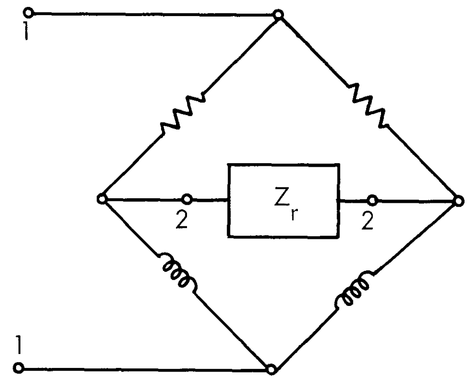

Fig. XXI-3.

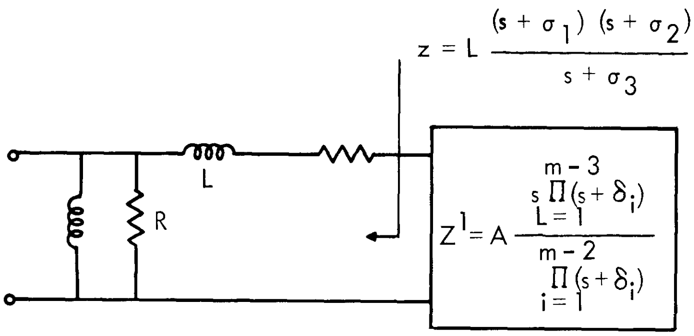

Fig. XXI-4.

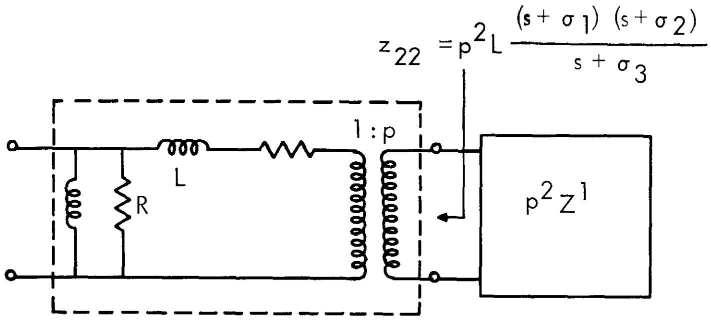

Fig. XXI-5.

and is based upon the cycle shown in Fig. XXI-3, for which Z_r has the form

$$Z_r = H \frac{\prod_{i=1}^{n-1} (s+\gamma_i)}{\prod_{i=1}^{n-2} (s+\delta_i)} .$$

The cycle of Fig. XXI-2 is carried out as follows. First, the Cauer cycle of Fig. XXI-4 is carried out, and then altered to that shown in Fig. XXI-5. Next, the turns ratio on the ideal transformer in Fig. XXI-5 is fixed to the value

$$\rho = \pm \frac{R}{L(\sigma_1 - \sigma_2)} \tag{3}$$

so that the boxed section becomes equivalent to a "zero section" S of the type shown in Fig. XXI-2. The cycle is completed by replacing the boxed section of Fig. XXI-5 with S. The element values of S are found most easily by computing z_{22} for the boxed section of Fig. XXI-5, and then realizing z_{22} on S by Foster's procedure.

The only question that arises in connection with the cycle above is: Does the choice or turns ratio (3) actually make possible an equivalence between the zero sections of Figs. XXI-2 and XXI-5? To understand that this is indeed the case, it is only necessary to realize z_{22} on both networks, compute z_{11} and z_{12} for each, and notice that corresponding impedances become equal when

$$\rho = \pm \frac{R}{L(\sigma_1 - \sigma_2)} .$$

We illustrate the cycle of Fig. XXI-2 by realizing the impedance

$$z = \frac{s(s+2)(s+4)}{(s+1)(s+3)(s+5)} = \frac{s^3 + 6s^2 + 8s}{s^3 + 9s^2 + 23s + 15} .$$

The altered Cauer cycle is shown in Fig. XXI-6. For the boxed section z_{22} is

$$z_{22} = \rho^2 \frac{8}{9} \frac{s^2 + \frac{27}{5} s + \frac{9}{2}}{s + \frac{15}{8}} = \rho^2 \frac{8}{9} \frac{\left(s + \frac{27}{10} + \frac{3}{10}\sqrt{31}\right)\left(s + \frac{27}{10} - \frac{3}{10}\sqrt{31}\right)}{s + \frac{15}{8}} .$$

The correct value for the turns ratio is

$$\rho = \pm \frac{1}{\frac{8}{9}\left(\frac{3}{5}\sqrt{31}\right)} ,$$

so that

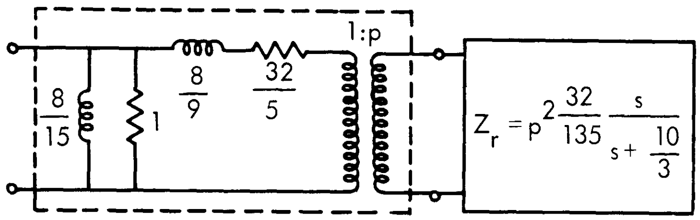

Fig. XXI-6.

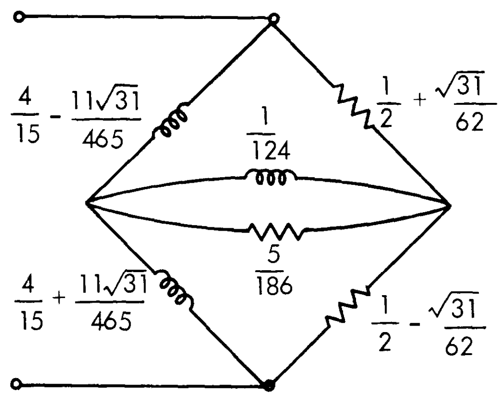

Fig. XXI-7.

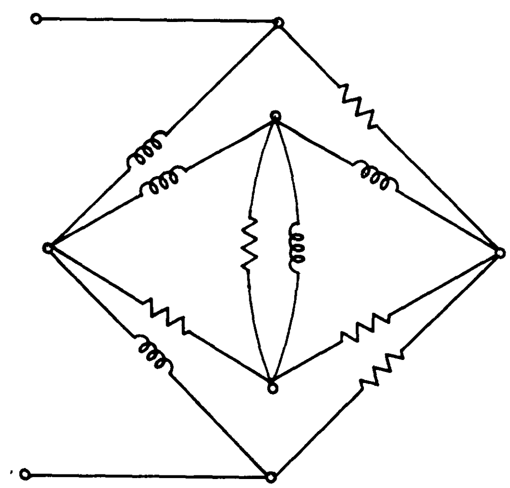

Fig. XXI-8.

$$z_{22} = \frac{75}{744} \frac{s^2 + \frac{27}{5}s + \frac{9}{2}}{s + \frac{15}{8}}, \qquad \text{and } Z_r = \frac{5}{186} \frac{s}{s + \frac{10}{3}}.$$

The zero section of Fig. XXI-2 now can be found by Foster's procedure. Z_r can be realized by inspection. The final realization is shown in Fig. XXI-7.

The most involved mathematical operation that must be performed to carry out the cycle of Fig. XXI-2 is the factorization of a second degree polynomial (i.e., the zeros of z_{22} must be found). For this reason, realization by the cycle of Fig. XXI-2 sometimes might be preferable to realization by the general Foster procedure, since the latter requires the factorization of an n^{th} degree polynomial. When the cycle of Fig. XXI-2 is repeated several times to realize an impedance of higher degree, the final realization consists of a sequence of bridges within one another. The realization of a 5^{th} degree impedance has the form shown in Fig. XXI-8.

The method for carrying out the cycle of Fig. XXI-3 is dual to the method for carrying out the cycle of Fig. XXI-2. This cycle is carried out as follows. First, the altered Cauer cycle shown in Fig. XXI-9 is carried out. Next, the turns ratio on the ideal transformer is brought to the value

$$\rho = \pm \frac{L(\sigma_1 - \sigma_2)}{R}, \tag{4}$$

so as to make the boxed section equivalent to a "zero section," zero section S' having the form shown in Fig. XXI-3. The cycle is completed by replacing the boxed section of Fig. XXI-9, by S'. The element values of S' are found most conveniently by computing y_{22} for the boxed section of Fig. XXI-9, and then realizing y_{22} on the zero section of Fig. XXI-3 by Foster's procedure.

The assertion upon which the success of the cycle hinges, is that the choice of turns ratio (4) makes possible an equivalence between the zero sections of Figs. XXI-3 and XXI-9. To prove that this assertion is correct, it is only necessary to realize y_{22} on both zero sections, compute y_{11} and y_{22} for each, and notice that corresponding pairs of admittances become equal when

$$\rho = \pm \frac{L(\sigma_1 - \sigma_2)}{R}.$$

We illustrate the cycle of Fig. XXI-3 by realizing the impedance

$$z = \frac{(s+1)(s+3)(s+8)}{(s+2)(s+6)} = \frac{s^3 + 12s^2 + 35s + 24}{s^2 + 8s + 12}.$$

The altered Cauer cycle is shown in Fig. XXI-10. For the boxed section y_{22} is

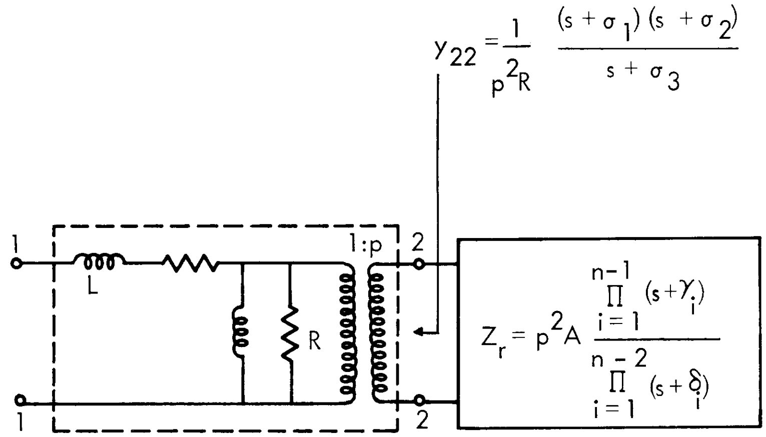

Fig. XXI-9.

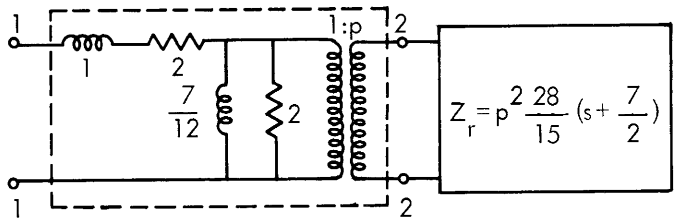

Fig. XXI-10.

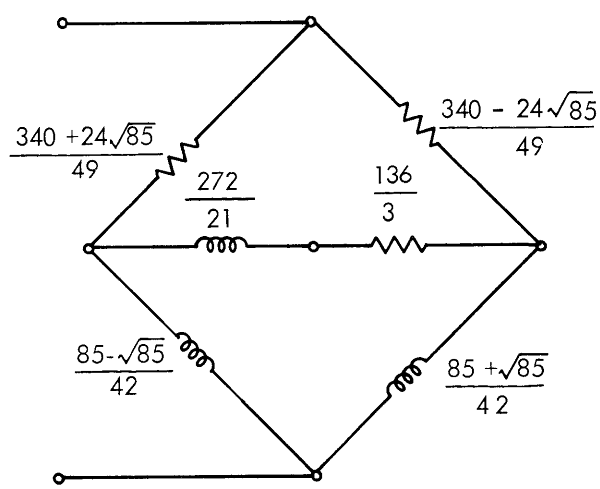

Fig. XXI-11.

$$y_{22} = \frac{1}{2\rho^2} \frac{s^2 + \frac{52}{7} + 48}{s(s+2)} = \frac{1}{2\rho^2} \frac{\left(s + \frac{26}{7} + \frac{2}{7}\sqrt{85}\right)\left(s + \frac{26}{7} - \frac{2}{7}\sqrt{85}\right)}{s(s+2)} .$$

The proper value for the turns ratio is

$$\rho = \pm \frac{1\left(\frac{4}{7}\sqrt{85}\right)}{2} = \pm \frac{2}{7}\sqrt{85},$$

so that

$$y_{22} = \frac{49}{680} \frac{s^2 + \frac{52}{7}s + 48}{s(s+2)}, \qquad \text{and } Z_r = \frac{272}{21}\left(s + \frac{7}{2}\right).$$

The element values of the zero section in Fig. XXI-3 now can be found by realizing y_{22} on the zero section, with Foster's procedure used; Z_r can be realized by inspection. The final realization of Z is shown in Fig. XXI-11.

As is the case for the cycle of Fig. XXI-2, the most involved mathematical operation that must be performed in carrying out the cycle of Fig. XXI-3 is the factoring of a 2^{nd} degree polynomial (i. e., the zeros of y_{22} must be found).

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