MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

8.323: Relativistic Quantum Field Theory I

PROBLEM SET 7 ERRATA AND CLARIFICATION

ERRATA TO *INFORMAL NOTES: DISTRIBUTIONS AND THE FOURIER TRANSFORM:*

There was one equation in the notes that was written with a "$<$" sign, but which should have been written with a "$\leq$" sign:

$$|f_\epsilon(t)| < |f(t)| \text{ should be } |f_\epsilon(t)| \leq |f(t)| \ .$$

The same equation, with the same error, appeared three times in the notes:

(1) The first displayed equation on p. 3.

(2) The fourth displayed equation on p. 5.

(3) The next to last displayed equation on p. 6.

ERRATA TO PROBLEM SET 7:

Eq. (1.9a) should read:

$$|g_\epsilon(t)| \leq g(t) \qquad \text{for } \epsilon > 0 \tag{1.9a}$$

On page 5, the fifth sentence in the first full paragraph should read:

> Eq. (2.9) does not define $O_4[w]$ as an integral, but is instead just a symbolic way of saying THAT $O_4[w]$ is the result of applying the distribution $:\phi^2(x^\mu):$ to the test function $w(x^\mu)$.

Problem 3 is removed from this problem set, but will appear on the next.

CLARIFICATION OF PROBLEMS 1(b) AND 1(c),PR OBLEM SET 7:

This problem was intended to illustrate the relationship between different methods of defining the Fourier transform of a function for which the usual integral definition diverges. The general theorem described in *Informal Notes: Distributions and the Fourier Transform* guarantees that the answers to parts (b) and (c) should be the same as the answer found in part (a), so if one uses this theorem there is nothing to do. So I want you not to use the theorem, but I should have been clearer about what steps I am expecting you to carry out explicitly.

In both cases (b) and (c), I would like you to explicitly compute the Fourier transform of the regulated function. That is, on part (b) I would like to you explicitly calculate

$\tilde{g}_\epsilon(\omega)$, as defined by Eq. (1.7). On part (c) I would like you to explicitly calculate $\tilde{g}_\epsilon^{(3)}(\omega)$, the Fourier transform of $g_\epsilon^{(3)}(t)$, as defined in Eq. (1.10).

In both cases (b) and (c), the hint stated in part (b) can be used. That means that the integrals appearing in Eqs. (1.8) and (1.11) can be evaluated by inserting the integral representations of $\tilde{g}_\epsilon(\omega)$ and $\tilde{g}_\epsilon^{(3)}(\omega)$ respectively, and then in each case you can reverse the order of integration. You should then carry out the integration over ω explicitly, leaving an integral over t.

In parts (a), (b), and (c), the integrals over t can be carried out and expressed in terms of the error function (with a complex argument), although you are not asked to do this. For part (a) you can leave the answer as an integral over t, and for parts (b) and (c) you can use Lebesgue's Dominated Convergence Theorem to show that in the limit as $\epsilon \to 0$, the expression is equal to the answer in part (a).

I should mention that I mistakenly told Joydip Kundu that I did not want you to use the hint to part (b) when answering part (c). I don't know what was discussed in recitation, but very likely Joydip passed on this incorrect advice to you. I'm sorry for any confusion, but over the vacation I forgot the details of which integrals could be done explicitly, and which could not. Please use these notes as a guide to what is intended.