MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

8.323: Relativistic Quantum Field Theory I

PROBLEM SET 3

REFERENCES: Peskin and Schroeder, Chapter 2

Problem 1: Space-time translations of a_k

(a) Show that for any operators A and B,

$$e^A B e^{-A} = B + [A\,,\,B] + \frac{1}{2}\,[A\,,\,[A\,,\,B]] + \frac{1}{3!}\,[A\,,\,[A\,,\,[A\,,\,B]]] + \ldots\ .$$

(b) Now apply this relation to the space-time translation operator on the Fock space of free scalar particles. These translations are generated by the 4-momentum operator,

$$\hat{P}^\mu = \int \frac{d^3k}{(2\pi)^3}\, k^\mu a_k^\dagger a_k\ ,$$

where $k_0 \equiv E_k = \sqrt{\vec{k}^2 + m^2}$, and $a_k^\dagger$ and a_k are the single particle creation and annihilation operators, normalized as in Peskin and Schroeder. Show that

$$e^{i\hat{P}\cdot x} a_k e^{-i\hat{P}\cdot x} = a_k e^{-ik\cdot x}\ .$$

Problem 2: "Smeared" fields and their variance

For a free scalar field $\phi(\vec{x})$ of mass m, consider the "smeared" field

$$\tilde{\phi}_a(\vec{x}, t) \equiv \frac{1}{\pi^{3/2}a^3} \int \mathrm{d}^3y\, \phi(\vec{y}, t) e^{-|\vec{y}-\vec{x}|^2/a^2}\ .$$

Note that $\phi_a(\vec{x}, t)$ corresponds to averaging the fundamental field $\phi(\vec{x}, t)$ over a region of size a with a Gaussian weight function. The vacuum expectation value of $\phi(\vec{x}, t)$ and $\tilde{\phi}_a(\vec{x}, t)$ are both zero, which means that a measurement of either quantity in the vacuum would yield zero on average. One would not always get zero, however, as the vacuum is not an eigenstate of these operators. To see the spread of values that one would get, one must calculate the variance

$$\sigma^2 \equiv \left\langle 0 \left| \left(\tilde{\phi}_a(\vec{x},t) - \langle \tilde{\phi}_a(\vec{x},t) \rangle \right)^2 \right| 0 \right\rangle \ .$$

(a) Write an expression for σ^2 as an integral over a single variable.

(b) Without evaluating the integral in general, show that in the limits of small a and large a, the leading term in σ^2 may be written as

$$\sigma^2 \approx \alpha a^\beta \ ,$$

and calculate α and β. You should discover that at large scales the average field approaches a classical variable whereas at small distances it is dominated by fluctuations.

Problem 3: Evaluation of $\langle 0|\phi(x)\phi(y)|0\rangle$ for spacelike separations

Problem 2.3 of Peskin and Schroeder. Alternatively — and this alternative might be more useful to your education — you can evaluate the expectation value by numerical integration, and draw a graph of the result. If you scale your axes with the appropriate powers of the mass m, the same graph can be valid for all values of m.