

XXII. NOISE IN ELECTRON DEVICES*

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While our work on the general problem of noise in linear networks was declared closed in Quarterly Progress Report No. 52 and a Research Monograph is in the process of being published, related special problems continue to emerge. These are conveniently handled by the general methods developed in the monograph, and we shall continue to present applications of the general theory to cases of current interest.

A. GENERAL NOISE ANALYSIS OF PARAMETRIC AMPLIFIERS

In three-frequency parametric amplifiers that use one or more solid-state diodes for the nonlinear energy-storage element, the major contribution to the noise seems to be the thermal noise associated with the linear dissipative parts of the equivalent circuit

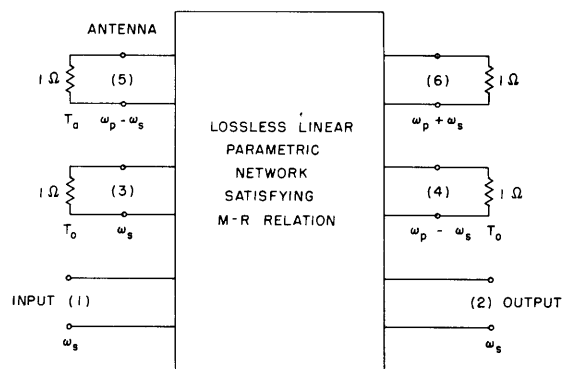


Fig. XXII-1. Small-signal schematic diagram of parametric amplifier.

and the noise contributed by the idler termination. Under such conditions the amplifier noise performance can be analyzed by using a scattering matrix representation similar to the one that was successful in the noise analysis of electron beam amplifiers (1, 2). From this approach, information for the necessary steps leading to noise optimization can be obtained with little effort. The representation is easily extended to the analysis of parametric amplifiers with more than one idler frequency, frequency-up-or-down converters, and parametric amplifiers under double-sideband operation.

A system under a large-signal pumping excitation at frequency ω_p and a small-signal excitation at frequency ω_s leads to excitations at frequencies $m\omega_p + \omega_s$, where

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m may range over all positive and negative integers. We shall follow usual analyses by assuming that the loss of the system can be separated from the pumped nonlinear part of the system and can be represented by linear circuit elements. The lossless nonlinear part of the system is assumed to satisfy the Manley-Rowe relations. The linear operation of a three- (signal, idler, pump) frequency, negative-resistance parametric amplifier with the idler circuit coupled to the antenna and with a parasitic excitation at the upper sideband $\omega_p + \omega_s$ can then be represented by the circuit of Fig. XXII-1. Terminal pair (1) is the input, (2) the output, the 1-ohm resistors at terminal pairs (3), (4), and (6) represent the losses at signal, idler, and parasitic upper-sideband frequencies at room temperature T_o , terminal pair (5) represents the idler coupling to the antenna with a noise at the idler frequency represented by the temperature T_a . The normalized incident waves a (a_1, a_2, \dots, a_6) and reflected waves b (b_1, b_2, \dots, b_6), normalized to transmission lines of 1-ohm characteristic impedance, are related by

$$b = Sa \quad (1)$$

Here, the lower sideband $\omega_p - \omega_s$ is characterized by the complex amplitudes at the negative frequencies ($-\omega_p + \omega_s$). The network has to satisfy the Manley-Rowe relation, and therefore

$$b^\dagger Pb = a^\dagger Pa \quad (2)$$

where (\dagger) indicates a Hermitian conjugate and

$$P = \text{diag} \left(\frac{1}{\omega_s}, \frac{1}{\omega_s}, \frac{1}{\omega_s}, -\frac{1}{\omega_p - \omega_s}, -\frac{1}{\omega_p - \omega_s}, \frac{1}{\omega_p + \omega_s} \right) \quad (3)$$

Combining Eqs. 1 and 2 and noting that a may be chosen arbitrarily, we obtain

$$S^\dagger PS = P \quad (4)$$

Equation 4 multiplied by SP^{-1} from the left and $S^{-1}P^{-1}$ from the right gives

$$SP^{-1}S^\dagger = P^{-1} \quad (5)$$

where

$$P^{-1} = \text{diag} (\omega_s, \omega_s, \omega_s, -\omega_p + \omega_s, -\omega_p + \omega_s, \omega_p + \omega_s)$$

The 22 element of the matrix equation, Eq. 5 is

$$\begin{aligned} \omega_s \left(|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 \right) - (\omega_p - \omega_s) \left(|S_{24}|^2 + |S_{25}|^2 \right) \\ + (\omega_p + \omega_s) |S_{26}|^2 = \omega_s \end{aligned} \quad (6)$$

The noise figure of the parametric amplifier is

$$F - 1 = \frac{\overline{|b_2|_i^2}}{\overline{|b_2|_o^2}} \quad (7)$$

where

$$\overline{|b_2|_i^2} = \overline{|S_{23}a_3 + S_{24}a_4 + S_{25}a_5 + S_{26}a_6|^2} \quad (8)$$

and

$$\overline{|b_2|_o^2} = \overline{|S_{21}|^2} kT_o \Delta f \quad (9)$$

Noting that all crosscorrelations of the a's are zero and

$$\overline{|a_3|^2} = \overline{|a_4|^2} = \overline{|a_6|^2} = kT_o \Delta f \quad (10)$$

$$\overline{|a_5|^2} = kT_a \Delta f \quad (11)$$

and using Eq. 6, we have

$$\begin{aligned} F - 1 = & \frac{\omega_s}{\omega_p - \omega_s} \frac{T_a}{T_o} \left[1 - \frac{1 - |S_{22}|^2}{|S_{21}|^2} \right] + \left[\frac{\omega_s}{\omega_p - \omega_s} \frac{T_a}{T_o} + 1 \right] \left| \frac{S_{23}}{S_{21}} \right|^2 \\ & + \left[\frac{\omega_p + \omega_s}{\omega_p - \omega_s} \frac{T_a}{T_o} + 1 \right] \left| \frac{S_{26}}{S_{21}} \right|^2 + \frac{T_o - T_a}{T_o} \left| \frac{S_{24}}{S_{21}} \right|^2 \end{aligned} \quad (12)$$

Equation 12 shows the harmful effects of the parasitic upper sideband (S_{26}), the loss at the signal frequency (S_{23}), and the loss at the idler frequency (S_{24}). The best noise figure is achieved if none of these noise contributions enter into Eq. 12. Furthermore, noting that

$$\frac{\overline{|S_{21}|^2}}{1 - |S_{22}|^2} = G_e$$

which is the exchangeable gain (3), we have for the optimum noise figure

$$F - 1 = \frac{\omega_s}{\omega_p - \omega_s} \frac{T_a}{T_o} \left(1 - \frac{1}{G_e} \right) \quad (13)$$

The optimum noise measure under any input termination (3) is

$$M_e = \frac{\omega_s}{\omega_p - \omega_s} \frac{T_a}{T_o} \quad (14)$$

and therefore (see ref. 3) the excess noise figure at high gain of any over-all two terminal-pair amplifier employing the parametric amplifier in conjunction with other "noisier" amplifiers cannot be better than Eq. 14.

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References

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